Questions from Quiz

① a) $\lim_{x\to 3} \frac{(\sqrt{x^2+7}-\sqrt{x+13})(\sqrt{x^2+7}+\sqrt{x+13})}{(x-3)(\sqrt{x^2+7}+\sqrt{x+13})}$

$\lim_{x\to 3} \frac{x^2+7-x-13}{(x-3)(\sqrt{x^2+7}+\sqrt{x+13})}$

$\lim_{x\to 3} \frac{\cancel{(x-3)}(x+2)}{\cancel{(x-3)}(\sqrt{x^2+7}+\sqrt{x+13})} = \boxed{\frac{5}{8}}$

g) $\lim_{x\to 0} \frac{\frac{1}{x+5}-\frac{1}{5}}{x^2+5x}$

L'Hopital's Rule $\lim_{x\to 0} \frac{\frac{-1}{(x+5)^2}}{2x+5}$ $\lim_{x\to 0} \frac{-\frac{1}{25}}{5} = \boxed{\frac{-1}{125}}$

g) $\lim_{x\to 0} \frac{5\cancel{(x+5)}\frac{1}{\cancel{x+5}} - \frac{1}{\cancel{5}}5(x+5)}{x^2+5x\,(5)(x+5)}$

$\lim_{x\to 0} \frac{5-x-5}{5x(x+5)(x+5)}$

$\lim_{x\to 0} \frac{-\cancel{x}}{5\cancel{x}(x+5)^2} = \boxed{-\frac{1}{125}}$

① h) $\lim_{x\to b} \frac{x^2-b^2}{x^8-b^8}$

$\lim_{x\to b} \frac{(x^2-b^2)}{(x^4-b^4)(x^4+b^4)}$

$\lim_{x\to b} \frac{\cancel{(x^2-b^2)}}{\cancel{(x^2-b^2)}(x^2+b^2)(x^4+b^4)} = \frac{1}{(2b^2)(2b^4)} = \boxed{\frac{1}{4b^6}}$

① f) $\lim_{x\to 0} \frac{\sin 7x}{2x}$

$\lim_{x\to 0} \left(\frac{\sin 7x}{7x}\right)\left(\frac{7}{2}\right) = \boxed{\frac{7}{2}}$

④ $\lim_{x\to 1} \frac{\sqrt[6]{x}-1}{\sqrt[4]{x}-1}$

L'Hopital's Rule

$\lim_{x\to 1} \frac{\frac{1}{6}x^{-5/6}}{\frac{1}{4}x^{-3/4}}$ $\lim_{x\to 1} \frac{\frac{1}{6}}{\frac{1}{4}} \cdot \frac{4}{6} = \boxed{\frac{2}{3}}$

Bonus:

$y = ax^2 + bx + c$ $\quad 4 = 2a(1) + b$ $\quad 8 = 2a(-1) + b$

$y' = 2ax + b$ $\quad 2a + b = 4$ $\quad -2a + b = 8$

$2a + b = 4$

$(+)\ -2a + b = 8$

$2b = 12$

$\boxed{b = 6}$

$2a + 6 = 4$

$2a = -2$

$\boxed{a = -1}$

$y = -x^2 + 6x + c$ passes through $(2, 15)$

$15 = -4 + 12 + c$

$\boxed{7 = c}$

$\therefore\ y = -x^2 + 6x + 7$ is the equation

d) $\displaystyle \lim_{x \to -1^-} \frac{|x+1|}{x^2 - 1}$

$\displaystyle \lim_{x \to -1^-} \frac{|x+1|}{(x+1)(x-1)}$

$\displaystyle \lim_{x \to -1^-} \frac{\cancel{|-1.0001+1|}^{-1}}{(\cancel{-1.0001+1})(-1.0001-1)} = \frac{-1}{-2} = \boxed{\frac{1}{2}}$

Questions from Homework

⑥ a) $f(x) = \sqrt{x} - \sqrt{1-x} = x^{1/2} - (1-x)^{1/2}$

$$F(x) = \frac{x^{3/2}}{3/2} - \frac{(1-x)^{3/2}}{3/2}(-1) + C$$

$$F(x) = \frac{2}{3}x^{3/2} + \frac{2}{3}(1-x)^{3/2} + C$$

⑥ b) $f(x) = \frac{1}{x} - \frac{1}{1-x}$

$$F(x) = \ln|x| - \ln|1-x|(-1) + C$$

$$F(x) = \ln|x| + \ln|1-x| + C$$

$$F(x) = \ln|x-x^2| + C$$

⑥ c) $f(x) = \frac{1}{\sqrt{1-x}} + \frac{1}{\sqrt{x}}$

$$f(x) = (1-x)^{-1/2} + (x)^{-1/2}$$

$$F(x) = \frac{(1-x)^{1/2}}{1/2}(-1) + \frac{x^{1/2}}{1/2} + C$$

$$F(x) = -2\sqrt{1-x} + 2\sqrt{x} + C \checkmark$$

$$F(x) = 2(\sqrt{x} - \sqrt{1-x}) + C$$

Warm Up

Determine the general antiderivative of the following:

$$f(x) = 2x^2 - x + 7$$

$$\boxed{F(x) = \frac{2x^3}{3} - \frac{1x^2}{2} + 7x + C}$$

$$f(x) = \cos x - \sin x$$

$$F(x) = \sin x - (-\cos x) + C$$

$$\boxed{F(x) = \sin x + \cos x + C}$$

$$f(x) = -3e^{-x} + 6e^{2x}$$

$$F(x) = \frac{-3}{-1}e^{-1x} + \frac{6}{2}e^{2x} + C$$

$$F(x) = \frac{3}{e^x} + 3e^{2x} + C$$

$$f(x) = \frac{2}{x^2} - \frac{5}{x} + x = 2x^{-2} - \frac{5}{x} + x$$

$$F(x) = \frac{2x^{-1}}{-1} - 5\ln|x| + \frac{x^2}{2} + C$$

$$\boxed{F(x) = \frac{-2}{x} - 5\ln|x| + \frac{1x^2}{2} + C}$$

Differential Equations

An equation that involves the derivative of a function is called a differential equation:

As discussed previously, in applications of calculus it is very common to have a situation where it is required to find a function, given knowledge about its derivatives.

Find all functions g such that: ← Antiderivative

$$g'(x) = 4\sin x - 3x^5 + 6\sqrt[4]{x^3}$$

$$f(x) = -4\cos x - \frac{x^6}{2} + \frac{24}{7}x^{7/4} + C$$

$$g'(x) = 4\sin x - 3x^5 + 6x^{3/4}$$

$$g(x) = -\frac{4}{1}\cos x - \frac{3x^6}{6} + \frac{6x^{7/4}}{7/4} + C$$

$$g(x) = -4\cos x - \frac{1}{2}x^6 + \frac{24}{7}x^{7/4} + C$$

Identifying a unique solution for an antiderivative

Examples:
Determine the function with the given derivative whose graph satisfies the initial condition provided.

Find f if given f '(x): and f (0) = - 2

$$f'(x) = e^x + \frac{20}{1+x^2}$$

$f(x) = e^x + \frac{20}{1}\tan^{-1} x + C$

$f(x) = e^x + 20\tan^{-1} x + C$

if $f(0) = -2$

$-2 = e^0 + 20\tan^{-1}(0) + C$

$-2 = 1 + 20(0) + C$

$-2 = 1 + C$

$\boxed{-3 = C}$

$f(x) = e^x + 20\tan^{-1} x - 3$

Find f if given f ''(x): and f (0) = 4, and f (1) = 1

$$f''(x) = 12x^2 + 6x - 4$$

$f'(x) = \frac{12x^3}{3} + \frac{6x^2}{2} - 4x + C$

$\boxed{f'(x) = 4x^3 + 3x^2 - 4x + C}$

$f(x) = \frac{4x^4}{4} + \frac{3x^3}{3} - \frac{4x^2}{2} + Cx + d$

$\boxed{f(x) = x^4 + x^3 - 2x^2 + cx + d}$

if $f(0) = 4$ then:

$4 = (0)^4 + (0)^3 - 2(0)^2 + c(0) + d$

$\boxed{4 = d}$

if $f(1) = 1$ then:

$1 = (1)^4 + (1)^3 - 2(1)^2 + c(1) + 4$

$1 = 1 + 1 - 2 + c + 4$

$\boxed{-3 = c}$

$\boxed{f(x) = x^4 + x^3 - 2x^2 - 3x + 4}$

$f(x) = x^4 + x^3 - 2x^2 - 3x + 4$

Practice Problems...

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Antiderivatives involving chain rule...

Remember how the Chain Rule works...

$$f(x)=[g(x)]^n$$

$$f'(x)=n[g(x)]^{n-1}g'(x)$$

Let's look at the following:

$$f'(x)=(x^2-3)^5(2x)$$

$$f'(x)=x^2\sqrt{x^3-1}$$

$$f'(x)=\frac{3x}{\sqrt{1-5x^2}}$$

$$f'(x)=\frac{\cos 8x}{(1+\sin 8x)^4}$$