

Questions from homework

③ $\frac{\cos(-11\pi)}{2 - \cot\left(\frac{43\pi}{6}\right)}$

$\frac{\cos(\pi)}{2 - \cot\left(\frac{43\pi}{6}\right)}$

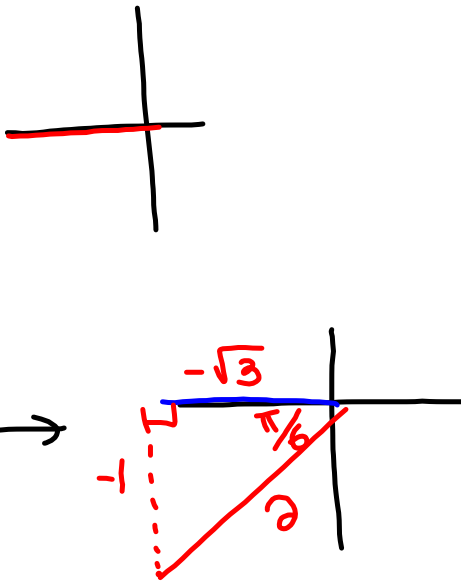
$\frac{(-1)}{2 - \left(\frac{-\sqrt{3}}{-1}\right)}$

$\frac{-1}{(2 - \sqrt{3})(2 + \sqrt{3})}$

$\frac{-2 - \sqrt{3}}{4 - 3}$

$\frac{-2 - \sqrt{3}}{1}$

$-2 - \sqrt{3}$



Questions from homework

$$\textcircled{5} \quad \csc^2\left(\frac{4\pi}{3}\right)\tan\left(\frac{15\pi}{4}\right) + \cos\left(-\frac{13\pi}{6}\right) - \sin\left(\frac{15\pi}{2}\right) + \cos(-4\pi)$$

$$\csc^2\left(\frac{4\pi}{3}\right)\tan\left(\frac{15\pi}{4}\right) + \cos\left(\frac{11\pi}{6}\right) - \sin\left(\frac{15\pi}{2}\right) + \cos(2\pi)$$

Q3 Q4 Q4

$$\left(-\frac{2}{\sqrt{3}}\right)^2\left(-\frac{1}{1}\right) + \left(\frac{+\sqrt{3}}{2}\right) - (-1) + 1$$

$$\left(\frac{4}{3}\right)\left(-\frac{1}{1}\right) + \frac{\sqrt{3}}{2} + \frac{2}{1}$$

$$-\frac{4}{3} + \frac{\sqrt{3}}{2} + \frac{2}{1}$$

Introduction to Trigonometric Equations

trigonometric equation

- an equation involving trigonometric ratios

Focus on...

- algebraically solving first-degree and second-degree trigonometric equations in radians and in degrees
- verifying that a specific value is a solution to a trigonometric equation
- identifying exact and approximate solutions of a trigonometric equation in a restricted domain
- determining the general solution of a trigonometric equation

Did You Know?

In equations, mathematicians often use the notation $\cos^2 \theta$. This means the same as $(\cos \theta)^2$.

Let's start with basic LINEAR trigonometric equations...

Solve: $\sin \theta = 0.9659$, $-360^\circ < \theta < 720^\circ$

...Pre-Calculus 110

- Reference angle?
- Which quadrants?
- Any co-terminal angles acceptable?

- If the domain is in degrees, give solutions in degrees.
- If the domain is in radians, give solutions in radians.

$$\bar{\theta} = \sin^{-1}(0.9659)$$

$$\bar{\theta} = 75^\circ$$

(Always use a positive when finding $\bar{\theta}$)

Where is $\sin \theta > 0$

Q1	Q2
$\theta = \bar{\theta}$	$\theta = 180^\circ - \bar{\theta}$
$\theta = 75^\circ$	$\theta = 180^\circ - 75^\circ = 105^\circ$

$$\theta = 75^\circ - 360^\circ = -285^\circ \quad \theta = 105^\circ - 360^\circ = -255^\circ$$

$$\theta = 75^\circ + 360^\circ = 435^\circ \quad \theta = 105^\circ + 360^\circ = 465^\circ$$

Solve: $\sec \theta = -1.3054$, $-2\pi \leq \theta \leq 2\pi$ (Radians)

$$\cos \theta = \frac{1}{-1.3054}$$

$$\cos \theta = -0.7660$$

$$\bar{\theta} = \cos^{-1}(0.7660)$$

$$\bar{\theta} = 0.6982$$

Where is $\cos \theta < 0$

Q2	Q3
$\theta = \pi - \bar{\theta}$	$\theta = \pi + \bar{\theta}$
$\theta = 3.14 - 0.6982$	$\theta = 3.14 + 0.6982$
$\theta = 2.4418$	$\theta = 3.8382$
$\theta = 2.4418 - 6.28$	$\theta = 3.8382 - 6.28$
$= -3.8382$	$= -2.4418$

$$\textcircled{4} \text{ f) } \csc \theta = -1.57, \quad 0 \leq \theta < 2\pi$$

$$\sin \theta = \frac{1}{-1.57} \quad \text{where is } \sin \theta < 0$$

$$\sin \theta = -0.6369$$

$$\bar{\theta} = \sin^{-1}(0.6369)$$

$$\bar{\theta} = 0.6905 \text{ rads}$$

Q3

$$\theta = \pi + \bar{\theta}$$

$$\theta = 3.14 + 0.6905$$

$$\theta = 3.8305$$

Q4

$$\theta = 2\pi - \bar{\theta}$$

$$\theta = 6.28 - 0.6905$$

$$\theta = 5.5895$$

(Exact Values)

EX. $\sqrt{2} \cos \theta + 1 = 0, -360^\circ \leq \theta \leq 720^\circ$

$$\sqrt{2} \cos \theta = -1$$

$$\cos \theta = -\frac{1}{\sqrt{2}}$$

(Special Angles) $\rightarrow \bar{\theta} = 45^\circ$

where is $\cos \theta < 0$

Q2	Q3
$\theta = 180^\circ - 45^\circ$	$\theta = 180^\circ + 45^\circ$
$\theta = 135^\circ$	$\theta = 225^\circ$
$= 495^\circ$	$= 585^\circ$
$= -225^\circ$	$= -135^\circ$

(Exact Value)

Ex. $\sin x + 1 = 0, -2\pi \leq x \leq 4\pi$

$$\sin x = -1$$

$$\text{(Unit Circle)} \rightarrow x = \frac{3\pi}{2}$$

$$\frac{3\pi}{2} - \frac{2\pi}{1}$$

$$\frac{3\pi}{2} - \frac{4\pi}{2}$$

$$\frac{-\pi}{2}$$

$$\frac{3\pi}{2} + \frac{2\pi}{1}$$

$$\frac{3\pi}{2} + \frac{4\pi}{2}$$

$$\frac{7\pi}{2}$$

$$x = \frac{-\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}$$

Your Turn

Solve each trigonometric equation in the specified domain.

a) $3 \cos \theta - 1 = \cos \theta + 1, -2\pi \leq \theta \leq 2\pi$

b) $4 \sec x + 8 = 0, 0^\circ \leq x < 360^\circ$

a) $3 \cos \theta - 1 = \cos \theta + 1, -2\pi \leq \theta \leq 2\pi$

$2 \cos \theta = 2$

$0 - 2\pi = -2\pi$

$\cos \theta = 1$

(Unit Circle) →

$\theta = 0 \text{ and } 2\pi \text{ and } -2\pi$

b) $4 \sec x + 8 = 0, 0^\circ \leq x < 360^\circ$

$4 \sec x = -8$

$\sec x = -2$

$\cos x = -\frac{1}{2}$

where is $\cos \theta < 0$

(special Angles) →

$\bar{x} = 60^\circ$

Q2

$x = 180^\circ - 60^\circ$

$x = 120^\circ$

Q3

$x = 180^\circ + 60^\circ$

$x = 240^\circ$

Homework

Page 211 #1-5

Check-Up:

Solve:

$$\textcircled{1} \cot \theta = 0.7834, \quad -\frac{\pi}{2} < \theta < -\pi$$

$$\textcircled{2} 3\cos x + 5 = 6, \quad -360^\circ \leq x \leq 720^\circ$$

$$\textcircled{3} 2\csc x (1 - \csc x) = 0, \quad -4\pi < x < 4\pi$$

$$\textcircled{1} \cot \theta = 0.7834, \quad -\frac{\pi}{2} < \theta < -\pi$$

$$\textcircled{2} 3\cos x + 5 = 6, \quad -360^\circ \leq x \leq 720^\circ$$

$$\textcircled{3} 2\csc x(1 - \csc x) = 0, \quad -4\pi < x < 4\pi$$

Let's move onto QUADRATIC trigonometric equations...

...Pre-Calculus 110

- What strategies can we use to solve quadratic equations?
- Quadratic trigonometric equations will ultimately become TWO linear trigonometric equations.

Solve: $2x^2 + x = 1$ $\frac{2x-1}{2} = \frac{-2}{1}$ Solve: $2\sin^2 x + \sin x = 1, 0 \leq x \leq 4\pi$

$$\begin{aligned} 2x^2 + x - 1 &= 0 \\ (2x^2 + 2x)(x-1) &= 0 \\ 2x(x+1) - 1(x+1) &= 0 \\ (x+1)(2x-1) &= 0 \end{aligned}$$

$$\begin{array}{l|l} x+1=0 & 2x-1=0 \\ x=-1 & 2x=1 \\ & x=\frac{1}{2} \end{array}$$

$$\begin{aligned} 2\sin^2 x + \sin x - 1 &= 0 \\ (2\sin^2 x + 2\sin x)(\sin x - 1) &= 0 \\ 2\sin x(\sin x + 1) - 1(\sin x + 1) &= 0 \\ (\sin x + 1)(2\sin x - 1) &= 0 \end{aligned}$$

$$\begin{aligned} \sin x + 1 &= 0 \\ \sin x &= -1 \\ x &= \frac{3\pi}{2} \end{aligned}$$

$$x = \frac{3\pi}{2} + \frac{4\pi}{2} = \frac{7\pi}{2}$$

$$\begin{aligned} 2\sin x - 1 &= 0 \\ 2\sin x &= 1 \\ \sin x &= \frac{1}{2} \\ \bar{x} &= \frac{\pi}{6} \end{aligned}$$

Where is $\sin \theta > 0$

Q1	Q2
$x = \bar{x}$	$x = \pi - \bar{x}$
$x = \frac{\pi}{6}$	$x = \frac{5\pi}{6}$
$x = \frac{\pi}{6} + \frac{12\pi}{6}$	$x = \frac{5\pi}{6} + \frac{12\pi}{6}$
$x = \frac{13\pi}{6}$	$x = \frac{17\pi}{6}$

Ex. $\cos^2 \theta - \frac{1}{2} \cos \theta = 0, -2\pi \leq \theta \leq 4\pi$ (Common Factor)

$$(\cos \theta) \left(\cos \theta - \frac{1}{2} \right) = 0$$

$$\cos \theta = 0 \quad \left| \quad \cos \theta - \frac{1}{2} = 0$$

$$\theta = \frac{\pi}{2} \text{ and } \frac{3\pi}{2}$$

$$\theta = -\frac{3\pi}{2} \text{ and } -\frac{\pi}{2}$$

$$\theta = \frac{5\pi}{2} \text{ and } \frac{7\pi}{2}$$

$$\cos \theta = \frac{1}{2}$$

$$\theta = \frac{\pi}{3}$$

Where is $\cos \theta > 0$

Q1	Q4
$\theta = \frac{\pi}{3}$	$\theta = 2\pi - \frac{\pi}{3}$
$\theta = \frac{5\pi}{3}$	$\theta = \frac{5\pi}{3}$
$\theta = -\frac{5\pi}{3}$	$\theta = -\frac{\pi}{3}$
$\theta = \frac{7\pi}{3}$	$\theta = \frac{11\pi}{3}$

Ex. $6 \sin^2 x - \sin x = 2, -2\pi \leq \theta \leq 4\pi$ (Decomposition)

Your TurnSolve for θ .

$$\begin{array}{l} -2 \times 1 = -2 \\ -2 + 1 = -1 \end{array}$$

$$\cos^2 \theta - \cos \theta - 2 = 0, 0^\circ \leq \theta < 360^\circ$$

(Simple Trinomial)

Give solutions as exact values where possible. Otherwise, give approximate measures to the nearest thousandth of a degree.

$$\cos^2 \theta - \cos \theta - 2 = 0$$

$$(\cos \theta - 2)(\cos \theta + 1) = 0$$

$$\cos \theta - 2 = 0 \quad | \quad \cos \theta + 1 = 0$$

$$\cos \theta = 2 \quad | \quad \cos \theta = -1$$

Not Possible

$$\theta = 180^\circ$$

General Solution of a Trigonometric Equation

Solve: $3\cos^2 \theta - \cos \theta = 2; \theta \in \mathbb{R}$ (Decomposition)

Determine the general solution for $\sin^2 x - 1 = 0$ over the real numbers if x is measured in radians.

$$\sin^2 x - 1 = 0 \quad (\text{Diff of Squares})$$

$$(\sin x - 1)(\sin x + 1) = 0$$

$$\begin{array}{l|l} \sin x - 1 = 0 & \sin x + 1 = 0 \\ \sin x = 1 & \sin x = -1 \\ x = \frac{\pi}{2} & x = \frac{3\pi}{2} \end{array}$$

$$x = \frac{\pi}{2} \pm 2\pi n, n \in \mathbb{N} \quad x = \frac{3\pi}{2} \pm 2\pi n, n \in \mathbb{N}$$

$$\text{or } x = \frac{\pi}{2} \pm \pi n, n \in \mathbb{N}$$

Did You Know?

$2n$, where $n \in \mathbb{I}$, represents all even integers.

$2n + 1$, where $n \in \mathbb{I}$, is an expression for all odd integers.

$$x = \frac{\pi}{2} + 2\pi n, \text{ where } n \in \mathbb{I}$$

$$x = \frac{3\pi}{2} + 2\pi n, \text{ where } n \in \mathbb{I}$$

or

$$x = \frac{\pi}{2} + \pi n, \text{ where } n \in \mathbb{I}$$

or

$$(2n + 1)\left(\frac{\pi}{2}\right), n \in \mathbb{I}$$

Determine the general solution for $\cos^2 x - 1 = 0$, where the domain is real numbers measured in degrees.

Practice Problems:

Pages 212 - 214

#7-9, 11-13, 16, 18, 22

Check-Up problem...

Solve:

$$\sin x \sec x + 2 \sin x = 0, \quad x \in R \quad (x \text{ is measured in radians})$$

Unit Review...

What topics have we covered??

Review...

- C4 a)** Determine all solutions for the equation $2 \sin^2 \theta = 1 - \sin \theta$ in the domain $0^\circ \leq \theta < 360^\circ$.
- b)** Are your solutions exact or approximate? Why?
- c)** Show how you can check one of your solutions to verify its correctness.

A grandfather clock shows a time of 7 o'clock. What is the exact radian measure of the angle between the hour hand and the minute hand?

Determine the angular velocity of the minute hand on a clock.

Solve: $6 \sin^2 \theta - 3 \sin \theta = 0$, $0 \leq \theta \leq 360^\circ$

[A] $0^\circ, 30^\circ, 180^\circ, 330^\circ, 360^\circ$

[B] $0^\circ, 30^\circ, 180^\circ, 150^\circ, 360^\circ$

[C] $30^\circ, 90^\circ, 120^\circ, 270^\circ$

[D] $0^\circ, 180^\circ, 210^\circ, 330^\circ, 360^\circ$

If $\csc \theta < 0$ and $\tan \theta > 0$, then which of the following could be a possible measure of angle θ ?

[A] $\frac{11\pi}{6}$

[B] $\frac{4\pi}{3}$

[C] $\frac{3\pi}{4}$

[D] $\frac{\pi}{2}$

What is the principal angle of $-\frac{25\pi}{4}$?

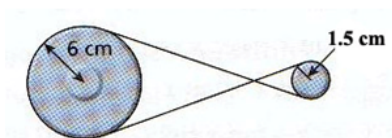
[A] $\frac{3\pi}{4}$

[B] $\frac{\pi}{4}$

[C] $-\frac{\pi}{4}$

[D] $\frac{7\pi}{4}$

If the belt in the pulley system below travels 30 cm, what is the angle of rotation of the smaller pulley?



[A] $\frac{\pi}{9}$ radians

[B] 20°

[C] 20 radians

[D] 5°

Nibbles the hamster is running at 0.02 m/s on an exercise wheel of radius 8 cm. What is the angular velocity of this wheel?

[A] 0.15 rad/minute

[B] 240 rad/minute

[C] 0.25 rad/minute

[D] 15 radians/minute

Solve: $2(1 - \sin \theta)^2 + \sin \theta = 2(3 - 4 \sin^2 \theta)$, $-360^\circ \leq \theta \leq 720^\circ$

Little Johnny has a rock tied to the end of a piece of rope 1.5 m long and he is swinging it around his head in a circular pattern. Mrs. Centripetal, his physics teacher, is watching Johnny out the window of her physics lab and notes that the rock is making 12 revolutions every 48 seconds.

- (a) Determine the angular velocity with which little Johnny is twirling the rope above his head. [2]

- (b) The rock comes flying from the rope 3 minutes after Mrs. Centripetal started to time little Johnny. How far did the rock travel during the 3 minutes? [2]

Attachments

Worksheet - Sketching Angles in Radians.doc