Equations in Standard Form

$$y = a\sin\left[b(x-h)\right]+k$$ or $$y=a\cos\left[b(x-h)\right]+k$$

a = ***Amplitude*** → influences how tall the sine curve is.

$b = \dfrac{360}{P} = \dfrac{2\pi}{P}$ → influences how often the pattern repeats. $P = \dfrac{360^\circ}{b} = \dfrac{2\pi}{b}$

h = ***Horizontal Translation*** → Influences how far to the left or the right that the graph will shift. Phase Shift

- If h is positive → Shift Right
- If h is negative → Shift Left

} Inside Brackets

k = ***Vertical Translation*** → influences how far up and down the graph will shift.

- If k is positive → Shift Up
- If k is negative → Shift Down
- equation of sin axis : $y = k$

State ***a, b, c, d, and P*** from the following sinusoidal equations:

$$2y+6=4\sin(4x+\frac{\pi}{2})-2$$

$$\frac{2y}{2}=\frac{4\sin(4x+\frac{\pi}{2})}{2}-\frac{8}{2}$$

$\frac{\pi}{2} \div 4$

$\frac{\pi}{2} \times \frac{1}{4}$

$$y=2\sin(4x+\frac{\pi}{2})-4$$

$$y=2\sin\left[4\left(x+\frac{\pi}{8}\right)\right]-4$$

$a=2$ $b=4$ $h=-\frac{\pi}{8}$ $k=-4$

$\text{Amp}=2$ $P=\frac{2\pi}{4}=\frac{\pi}{2}$ sin axis: $y=-4$

Sketching Sinusoidal Functions using Mapping

Development of a standard form for sinusoidal functions...

Standard Form ⟶ $y = a\sin\left[b(x-h)\right]+k$

1. Reflection: If a < 0 the graph will be reflected in the x-axis.

2. Amplitude: The amplitude of the graph will be equal to | a | .

3. Period: The period of the graph will be equal to $\frac{360^\circ}{b}$ or $\frac{2\pi}{b}$

4. Horizontal Phase Shift: (Translation) The graph will shift 'h" units to the right.

5. Vertical Translation: The graph will shift k1" units up.

The Mapping Rule: $(x,y)\rightarrow\left[\frac{x}{b}+c, ay+d\right]$

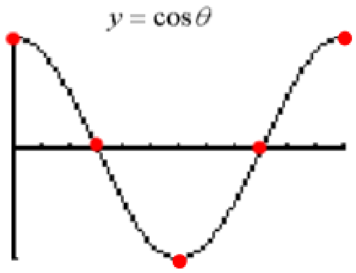
$(x,y)\rightarrow\left[\frac{1}{b}x+h, ay+k\right]$

$$y = 3\cos[2(\theta - 135°)] + 2$$

① Put in standard form
② State parameters

$a = 3$ $b = 2$ $h = 135°$ $k = 2$

$P = 180°$



Mapping:

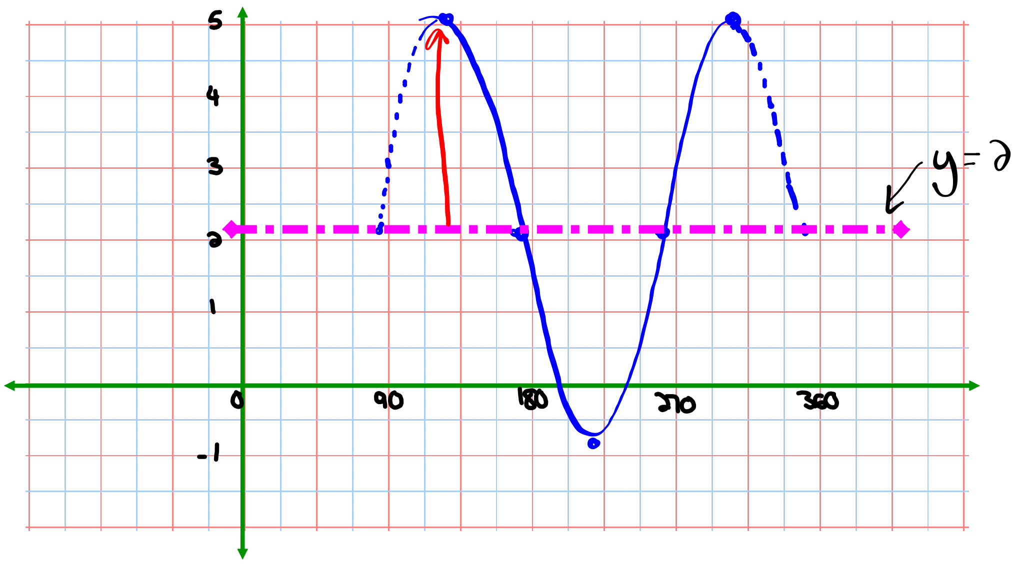
$$(\theta, y) \rightarrow \left[\frac{1}{2}\theta + 135°,\ 3y + 2\right]$$

$y = \cos\theta$

θ	y
0	1
90	0
180	-1
270	0
360	1

New points after mapping

θ	y
135°	5
180°	2
225°	-1
270°	2
315°	5



DOMAIN	$\{\theta \mid \theta \in R\}$
RANGE	$\{y \mid -1 \leq y \leq 5, y \in R\}$
AMPLITUDE	3
PERIOD	$P = \frac{360°}{b} = \frac{360°}{2} = 180°$
PHASE SHIFT	135°
VERTICAL TRANSLATION	2
EQUATION OF SINUSOIDAL AXIS	$y = 2$

Use Mapping to Graph

$$3y=-6\cos(3x-\pi)-9$$

$y = -2\cos(3x-\pi)-3$

$y = -2\cos\left[3\left(x-\frac{\pi}{3}\right)\right]-3$

$a=-2$ $b=3$ $h=\frac{\pi}{3}$

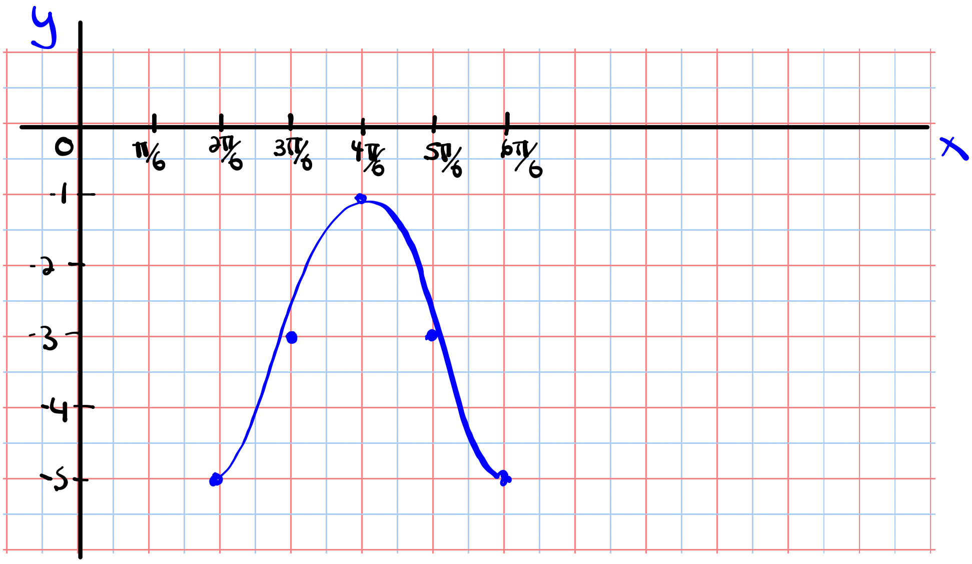
Amp $=2$ $P=\frac{2\pi}{3}$ $k=-3$

$(x,y)\rightarrow\left[\frac{1}{3}x+\frac{\pi}{3}, -2y-3\right]$

θ	y
0	1
$\pi/2$	0
π	-1
$3\pi/2$	0
2π	1

New points after mapping

θ	y
$2\pi/6 = \pi/3$	-5
$3\pi/6 = \pi/2$	-3
$4\pi/6 = 2\pi/3$	-1
$5\pi/6$	-3
$6\pi/6 = \pi$	-5



Domain	$\{x \mid x\in R\}$
Range	$\{y \mid -5\le y\le -1, y\in R\}$
Amplitude	2
Period	$\frac{2\pi}{3}$
Phase Shift	$\pi/3$
Vertical Translation	-3
Equation of Sinusoidal Axis	$y=-3$

Use Mapping to Graph

$$\frac{1}{2}(y+1)=3\sin\left(\frac{1}{2}\theta-90^\circ\right)+2$$

Remember...Put in standard form first!!

ϑ	y
0	
90	
180	
270	
360	

[redacted]

New points after mapping

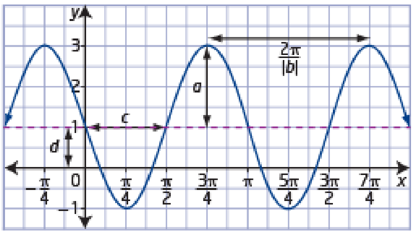
ϑ	y

Key Ideas

- You can determine the amplitude, period, phase shift, and vertical displacement of sinusoidal functions when the equation of the function is given in the form $y = a \sin b(x - c) + d$ or $y = a \cos b(x - c) + d$.

For: $y = a \sin b(x - c) + d$
$y = a \cos b(x - c) + d$

How does changing each parameter affect the graph of a function?



Vertical stretch by a factor of $|a|$
- changes the amplitude to $|a|$
- reflected in the x-axis if $a < 0$

Horizontal stretch by a factor of $\frac{1}{|b|}$
- changes the period to $\frac{360°}{|b|}$ (in degrees) or $\frac{2\pi}{|b|}$ (in radians)
- reflected in the y-axis if $b < 0$

Horizontal phase shift represented by c
- to right if $c > 0$
- to left if $c < 0$

Vertical displacement represented by d
- up if $d > 0$
- down if $d < 0$

$$d = \frac{\text{maximum value} + \text{minimum value}}{2}$$

- You can determine the equation of a sinusoidal function given its properties or its graph.

Homework

Finish worksheet

Attachments

Sketching Sinusoidal Functions.pdf