Warm Up

$$\lim_{x\to 0}\frac{(x+2)^4-16}{x}$$

$$\lim_{x\to 0}\frac{((x+2)^2-4)((x+2)^2+4)}{x}$$

$$\lim_{x\to 0}\frac{((x+2)-2)((x+2)+2)((x+2)^2+4)}{x}$$

$$\lim_{x\to 0}\frac{\cancel{(x)}(x+4)((x+2)^2+4)}{\cancel{x}} = (4)(8) = \boxed{32}$$

C.D. $(1+x)$

$$\lim_{x\to 0}\frac{\cancel{(1+x)}\frac{1}{\cancel{(1+x)}}-1\,(1+x)}{x\,(1+x)}$$

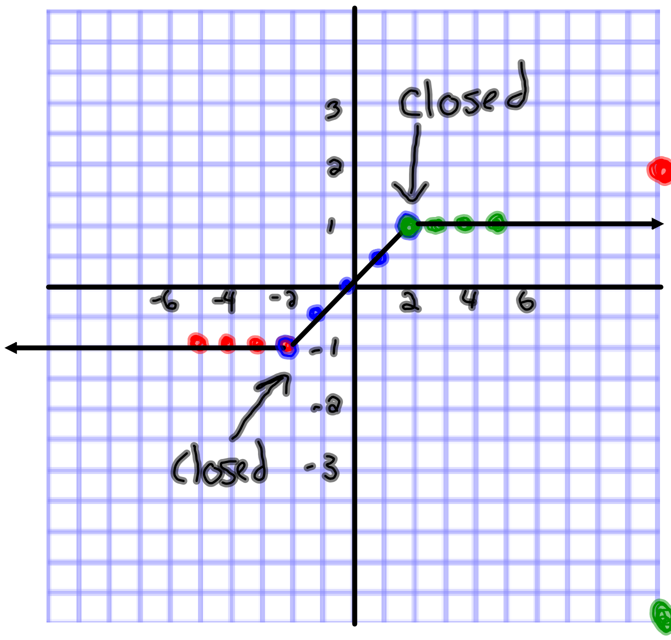
$$\lim_{x\to 0}\frac{1-(1+x)}{x(1+x)}$$

$$\lim_{x\to 0}\frac{1-1-x}{x(1+x)}$$

$$\lim_{x\to 0}\frac{-\cancel{x}}{\cancel{x}(1+x)} = \frac{-1}{1} = \boxed{-1}$$

Questions From Homework

⑧ $F(x) = \begin{cases} -1 & \text{if } x \leq -2 \\ \frac{1}{2}x & \text{if } -2 < x < 2 \\ 1 & \text{if } x \geq 2 \end{cases}$



-1

x	y
-2	-1
-3	-1
-4	-1
-5	-1

$\frac{1}{2}(x)$

x	y
-2	-1
-1	-0.5
0	0
1	0.5
2	1

1

x	y
2	1
3	1
4	1
5	1

a) $\lim_{x \to -2^-} f(x) = -1$

$\lim_{x \to -2^+} f(x) = -1$

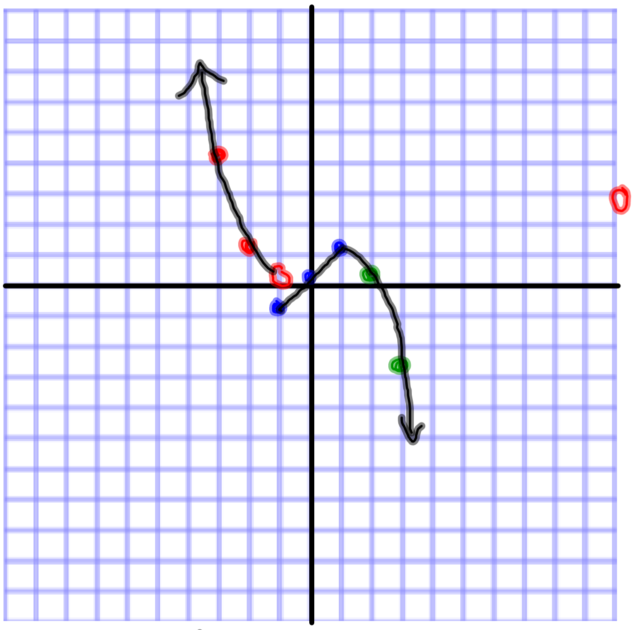
$\lim_{x \to 2^-} f(x) = 1$

$\lim_{x \to 2^+} f(x) = 1$

c) It is a continuous function

Questions From Homework

9) $$f(x) = \begin{cases} (x+1)^2 & \text{if } x < -1 \\ x & \text{if } -1 \le x \le 1 \\ 2x - x^2 & \text{if } x > 1 \end{cases}$$



$(x+1)^2$

x	$f(x)$
-1	0
-2	1
-3	4

x

x	$f(x)$
-1	-1
0	0
1	1

$2x - x^2$

x	$f(x)$
1	1
2	0
3	-3

$$\lim_{x \to -1^-} f(x) = 0$$

$$\lim_{x \to -1^+} f(x) = -1$$

$$\lim_{x \to -1} f(x) = DNE$$

$$f(-1) = -1$$

Continuity

Definition

- We noticed in the preceding section that…
  - the limit of a function as x approaches a

  can often be found simply by…
  - calculating the value of the function at a.
- Functions with this property are called *continuous* at a:

1 Definition A function f is **continuous at a number a** if

$$\lim_{x \to a} f(x) = f(a)$$

- This definition implicitly requires three things if f is continuous at a:
  1. $f(a)$ is defined
     - That is, a is in the domain of f
  2. $f(x)$ has a limit as x approaches a
  3. This limit is actually equal to $f(a)$.

In English! Continuity

- **Graph must be defined at that point**
- **Limit from left and right must be equal**
- **Limit must be the same as the defined height of the function**

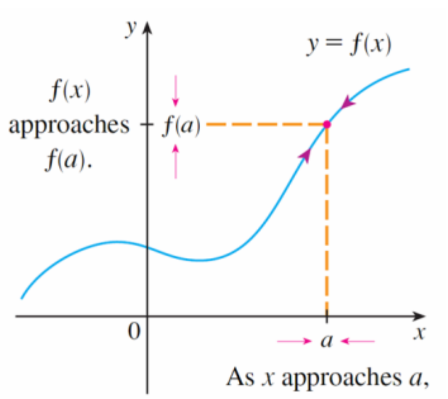


FIGURE 1

Let's simplify things...

A function whose graph has holes or breaks is considered discontinuous at these particular points.

If you have to lift your pencil from the page to sketch the graph, it is discontinuous anywhere you lift your pencil

Try this one...

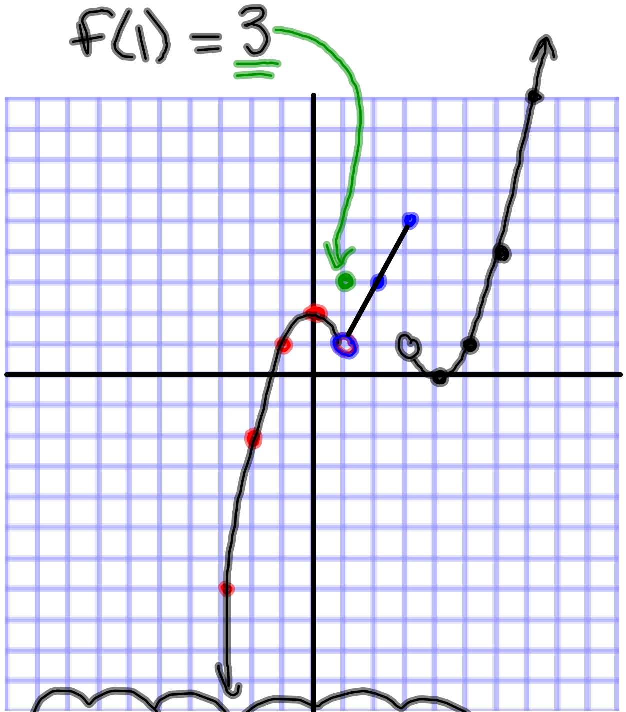
$$f(x) = \begin{cases} 2 - x^2 & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ 2x - 1 & \text{if } 1 < x \le 3 \\ (x-4)^2 & \text{if } x > 3 \end{cases}$$

Evaluate:

$\lim_{x \to 1} f(x) = 1$

$\lim_{x \to 3} f(x) = \text{DNE}$

$f(1) = \underline{\underline{3}}$



The function is discontinuous at: $x = 1, 3$

$2-x^2$

x	$f(x)$
1	1
0	2
-1	1
-2	-2

3

x	$f(x)$
1	3

$2x-1$

x	$f(x)$
1	1
2	3
3	5

$(x-4)^2$

x	$f(x)$
3	1
4	0
5	1
6	4

Homework