# Questions From Homework

2 h) $h(y) = \left(\frac{y}{3}\right)^2 = \frac{y^2}{9} = \frac{1}{9}y^2$

$h'(y) = \frac{2}{9}y$

j) $f(x) = \sqrt[3]{x^2} = x^{2/3}$

$f'(x) = \frac{2}{3}x^{-1/3} = \frac{2}{3x^{1/3}} = \frac{2}{3\sqrt[3]{x}}$

3 e) $y = \sqrt{x^3}$, $x = 8$

$y = x^{3/2}$

$y' = \frac{3x^{1/2}}{2} = \frac{3\sqrt{x}}{2}$

$y'(8) = \frac{3\sqrt{8}}{2} = \frac{3\sqrt{4 \cdot 2}}{2} = \frac{6\sqrt{2}}{2} = \boxed{3\sqrt{2}}$

Slope of the tangent

4 c) $xy = 1$ $\qquad$ $\underset{(5,\ 1/5)}{x_1,\ y_1}$

$y = \frac{1}{x} = x^{-1}$

1 Differentiate:

$y' = -x^{-2} = \frac{-1}{x^2}$

2 Sub in x-value

$y'(5) = \frac{-1}{(5)^2}$

$y'(5) = \boxed{\frac{-1}{25}}$

Slope of the tangent "m"

3 Find Equation:

$y - y_1 = m(x - x_1)$

$y - \frac{1}{5} = \frac{-1}{25}(x - 5)$

$25 \cdot y - 25 \cdot \frac{1}{5} = 25 \cdot \frac{-x}{25} + 25 \cdot \frac{5}{25}$

$25y - 5 = -x + 5$

$\boxed{x + 25y - 10 = 0}$

2 b) $y = \frac{3}{\sqrt[4]{x}} = \frac{3}{x^{1/4}} = 3x^{-1/4}$

$y' = -\frac{3}{4}x^{-5/4} = \frac{-3}{4x^{5/4}} = \frac{-3}{4\sqrt[4]{x^5}}$

# Questions From Homework

⑤ $f(x) = \boxed{\frac{1}{x}}$ | $f(x+h) = \boxed{\frac{1}{x+h}}$

$$f'(x) = \lim_{h \to 0} \frac{\boxed{f(x+h)} - \boxed{f(x)}}{h}$$

$$f'(x) = \lim_{h \to 0} \frac{\frac{1}{x+h} - \frac{1}{x}}{h}$$

multiply by: $(x)(x+h)$

$$f'(x) = \lim_{h \to 0} \frac{x - (x+h)}{h(x)(x+h)}$$

$$f'(x) = \lim_{h \to 0} \frac{x - x - h}{h(x)(x+h)}$$

$$f'(x) = \lim_{h \to 0} \frac{-\cancel{h}}{\cancel{h}(x)(x+h)} = \boxed{\frac{-1}{x^2}}$$

## Example:

**Find the slope of the tangent line to the graph of the given function at the given x value.**

$$g(x) = \sqrt[5]{x} \qquad x = 32$$

$g(x) = x^{1/5}$

① Differentiate:

$$g'(x) = \frac{1}{5}x^{-4/5} = \frac{1}{5x^{4/5}} = \frac{1}{5\sqrt[5]{x^4}}$$

② Sub in x-value

$$g'(32) = \frac{1}{5(32)^{4/5}} = \frac{1}{5(16)} = \boxed{\frac{1}{80}}$$

Slope of the tangent "m"

## Example:

### Find the equation of the tangent line to the curve $f(x) = x^6$ at the point (-2, 64)

Remember that the equation of a line is found by using the point-slope formula... $y - y_1 = m(x - x_1)$

The curve is the graph of the function $f(x) = x^6$ and we know that the slope of the tangent line at (-2, 64) is the derivative $f'(-2)$

- Find derivative
- Fill in x value and solve for slope
- Use equation of a line formula and solve

① Differentiate:

$$f(x) = x^6$$
$$f'(x) = 6x^5$$

② Sub in x-value:

$$f'(-2) = 6(-2)^5$$
$$= 6(-32)$$
$$\boxed{= -192}$$

"m"

③ Find Equation:

$$y - y_1 = m(x - x_1)$$
$$y - 64 = -192(x + 2)$$
$$y - 64 = -192x - 384$$
$$\boxed{192x + y + 320 = 0}$$

# Sums and Differences

- These next rules say that <u>the derivative of a sum (difference) of functions is the sum (difference) of the derivatives</u>:

**The Sum Rule** If $f$ and $g$ are both differentiable, then

$$\frac{d}{dx}[f(x) + g(x)] = \frac{d}{dx}f(x) + \frac{d}{dx}g(x)$$

**The Difference Rule** If $f$ and $g$ are both differentiable, then

$$\frac{d}{dx}[f(x) - g(x)] = \frac{d}{dx}f(x) - \frac{d}{dx}g(x)$$

Demonstrate what this all means...

Differentiate each of the following:

1. $f(x) = 2x^4 + \sqrt{x}$

$= 2x^4 + x^{1/2}$

$f'(x) = 8x^3 + \frac{1}{2}x^{-1/2}$

$= 8x^3 + \frac{1}{2x^{1/2}}$

2. $f(x) = 6x^4 - 5x^3 - 2x + 17$

$f'(x) = 24x^3 - 15x^2 - 2x^0 + 0$

$= 24x^3 - 15x^2 - 2$

3. $f(x) = (2x^3 - 5)^2$

$= (2x^3-5)(2x^3-5)$

$= 4x^6 - 10x^3 - 10x^3 + 25$

$= 4x^6 - 20x^3 + 25$

$f'(x) = 24x^5 - 60x^2$

4. $g(x) = 4x^3 - \frac{6}{x^2} + \sqrt[3]{x}$

$= 4x^3 - 6x^{-2} + x^{1/3}$

$g'(x) = 12x^2 + 12x^{-3} + \frac{1}{3}x^{-2/3}$

$= 12x^2 + \frac{12}{x^3} + \frac{1}{3x^{2/3}}$

# Homework

① g) $y = \dfrac{x+1}{\sqrt{x}} = \dfrac{x+1}{x^{1/2}} = x^{-1/2}(x+1) = x^{1/2} + x^{-1/2}$

$y' = \dfrac{1}{2}x^{-1/2} - \dfrac{1}{2}x^{-3/2}$