Differentiation Rules

Product Rule:

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

Express the product rule verbally if you are considering a function of the form...

f(x) = (First) X (Second)

In words, ***the Product Rule*** says that the ***derivative of a product of two functions is:*** ***the first function times the derivative of the second function, plus the derivative of the first function times the second function***

Get in the habit of verbalizing the rule as you differentiate...it will help when the functions get more complicated.

8) Find the <u>equation</u> of the tangent line

$$y = (2-\sqrt{x})(1+\sqrt{x}+3x) \quad \text{at} \quad (1,5)$$

1) Differentiate

$$y = (2-x^{1/2})(1+x^{1/2}+3x)$$

$$y' = (2-x^{1/2})\left(\tfrac{1}{2}x^{-1/2}+3\right) + \left(-\tfrac{1}{2}x^{-1/2}\right)(1+x^{1/2}+3x)$$

$$y' = (2-\sqrt{x})\left(\frac{1}{2\sqrt{x}}+3\right) + \left(\frac{-1}{2\sqrt{x}}\right)(1+\sqrt{x}+3x)$$

2) Sub in x-value and solve for slope

$$y' = (2-\sqrt{1})\left(\frac{1}{2\sqrt{1}}+3\right) + \left(-\frac{1}{2\sqrt{1}}\right)(1+\sqrt{1}+3(1))$$

$$y' = \left(\frac{1}{1}\right)\left(\frac{7}{2}\right) + \left(\frac{-1}{2}\right)\left(\frac{5}{1}\right)$$

$$y' = \frac{7}{2} + \frac{-5}{2} = \frac{2}{2} = \boxed{1}$$

slope of tangent "m"

3) Find the equation:

$$y - y_1 = m(x - x_1)$$

$$y - 5 = 1(x - 1)$$

$$y - 5 = x - 1$$

$$\boxed{0 = x - y + 4}$$

2) $f(t) = (6+t^{-2})(8t^{10}-5t^3)$

$$f'(t) = (6+t^{-2})(80t^9-15t^2) + (-2t^{-3})(8t^{10}-5t^3)$$

$$= 480t^9 - 90t^2 + 80t^7 - 15t^0 - 16t^7 + 10t^0$$

$$= \boxed{480t^9 + 64t^7 - 90t^2 - 5}$$

Differentiate the following function and simplify your answer:

$$h(t) = (t^3 - 5t)(6\sqrt{t} - t^{-5}) = (t^3 - 5t)(6t^{1/2} - t^{-5})$$

$$h'(t) = (t^3 - 5t)(3t^{-1/2} + 5t^{-6}) + (3t^2 - 5)(6t^{1/2} - t^{-5})$$

$$h'(t) = 3t^{5/2} + 5t^{-3} - 15t^{1/2} - 25t^{-5} + 18t^{5/2} - 3t^{-3} - 30t^{1/2} + 5t^{-5}$$

$$h'(t) = 21t^{5/2} - 45t^{1/2} + 2t^{-3} - 20t^{-5}$$

$$h'(t) = 21t^{5/2} - 45t^{1/2} + \frac{2}{t^3} - \frac{20}{t^5}$$

$$f(x) = (7x^3 - x^2 + 5)(x^9 + 3x - 5)$$

Quotient Rule:

The Quotient Rule If f and g are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

Express the quotient rule verbally if you are considering a function of the form...

$$f(x) = \frac{\text{(First)}}{\text{(Second)}}$$

In words, ***the Quotient Rule*** says that the ***derivative of a quotient is:***
the denominator times the derivative of the numerator, minus the numerator times the derivative of the denominator, all divided by the square of the denominator.

Examples:

Differentiate the following functions and simplify your answers:

$$F(x) = \frac{x^2 + 2x - 3}{x^3 + 1}$$

$$F'(x) = \frac{(x^3+1)(2x+2) - (x^2+2x-3)(3x^2)}{(x^3+1)^2}$$

$$= \frac{2x^4 + 2x^3 + 2x + 2 - 3x^4 - 6x^3 + 9x^2}{(x^3+1)^2}$$

$$= \boxed{\frac{-x^4 - 4x^3 + 9x^2 + 2x + 2}{(x^3+1)^2}}$$

$$F(x) = \frac{\sqrt{x}}{1+2x} = \frac{x^{1/2}}{1+2x}$$

$$F'(x) = \frac{(1+2x)\left(\frac{1}{2}x^{-1/2}\right) - (x^{1/2})(2)}{(1+2x)^2}$$

$$= \frac{\frac{1}{2}x^{-1/2} + 1x^{1/2} - 2x^{1/2}}{(1+2x)^2}$$

$$2\sqrt{x} \; = \frac{\frac{1}{2\sqrt{x}} - \frac{\sqrt{x}}{1} \cdot 2\sqrt{x}}{2\sqrt{x}\,(1+2x)^2}$$

$$\boxed{= \frac{1 - 2x}{2\sqrt{x}\,(1+2x)^2}}$$

Differentiate the following functions, do not simplify your answers:

$$f(x) = \frac{8 - 9x^7}{3x - 7}$$

$$f'(x) = \frac{(3x-7)(-63x^6) - (8-9x^7)(3)}{(3x-7)^2}$$

$$f(x) = \frac{x^3 - 7x^2 + 2}{x^8 - 4x^5}$$

$$f'(x) = \frac{(x^8-4x^5)(3x^2-14x) - (x^3-7x^2+2)(8x^7-20x^4)}{(x^8-4x^5)^2}$$

Homework

$$f(x)=\frac{\left(10x^{-5}+x\right)\left(3x^{3}+5\right)}{\left(-2x^{6}+\sqrt[3]{x}\right)}$$

$$f(x)=\frac{(x-7)\left(2x^{6}-x^{4}+5\right)}{\left(6x-x^{5}\right)\left(4x^{3}+2\right)}$$