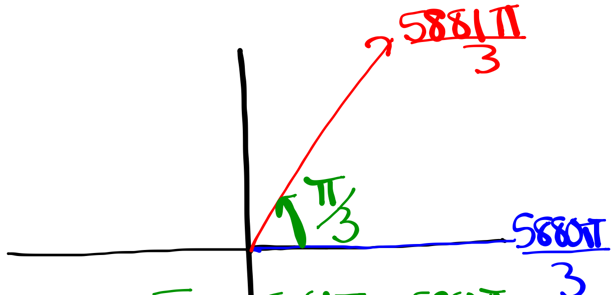
Sketch the following and determine a negative angle co-terminal with:

$(i)\ \dfrac{5881\pi}{3}$

$\dfrac{5880\pi}{3}, \dfrac{5881\pi}{3}, \dfrac{5882\pi}{3}$

1960π



$\bar{\theta} = \dfrac{5881\pi}{3} - \dfrac{5880\pi}{3}$

$\bar{\theta} = \dfrac{\pi}{3}$

Negative coterminal angle:

$\dfrac{5881\pi}{3} - \dfrac{1962\pi}{1}$

$\dfrac{5881\pi}{3} - \dfrac{5886\pi}{3}$

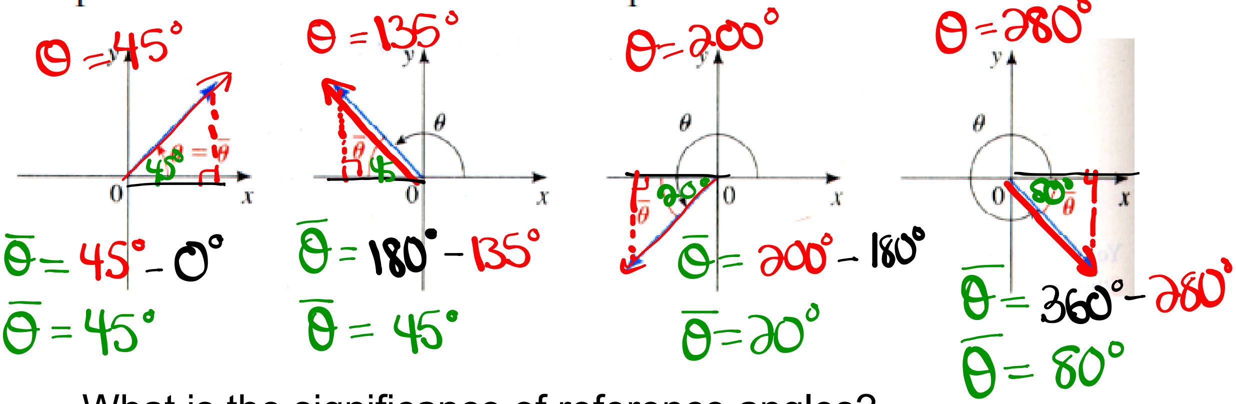
$\dfrac{-5\pi}{3}$

Reference Triangles:

Definition 17 *The reference angle $\bar{\theta}$ of an angle θ in standard position is the acute angle (between 0 and 90°) the terminal side makes with the x-axis.*

arm

The picture below illustrates this concept.

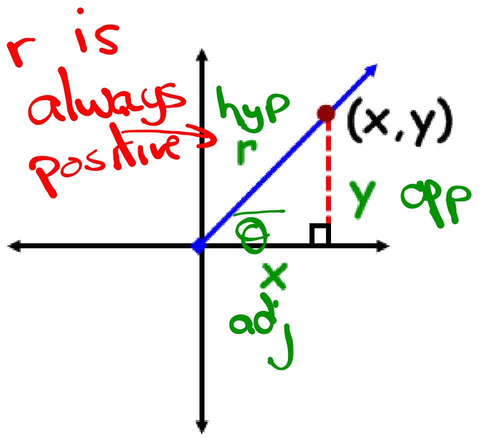


What is the significance of reference angles?

Angles on the Cartesian Plane

- **Reference Angle** - an acute angle formed between the terminal arm and the x-axis.

- **Reference Triangle** - a triangle formed by drawing a perpendicular line from a point on the terminal to the x-axis.



TRIG RATIOS on the CARTESIAN PLANE

$$\sin\theta = \frac{y}{r} = \frac{o}{h} \qquad \csc\theta = \frac{r}{y} = \frac{h}{o}$$

$$\cos\theta = \frac{x}{r} = \frac{a}{h} \qquad \sec\theta = \frac{r}{x} = \frac{h}{a}$$

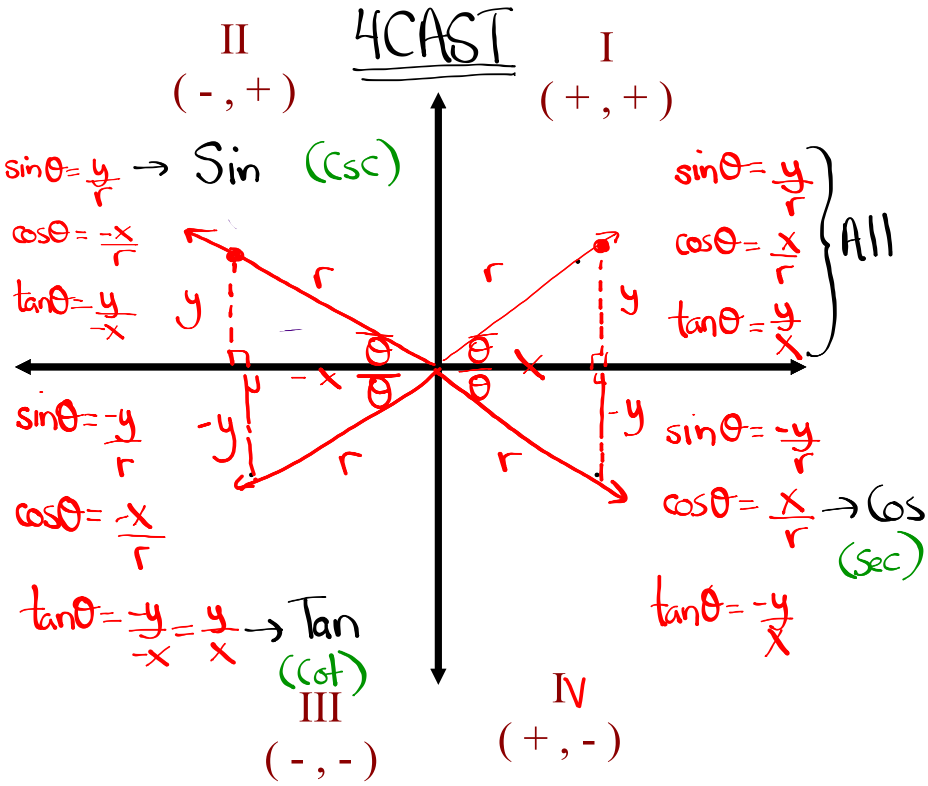
$$\tan\theta = \frac{y}{x} = \frac{o}{a} \qquad \cot\theta = \frac{x}{y} = \frac{a}{o}$$

"Primary" "Reciprocal"

Notice what will happen if the rotation moves into other quadrants?

TRIG RATIOS IN ALL 4 QUADRANTS

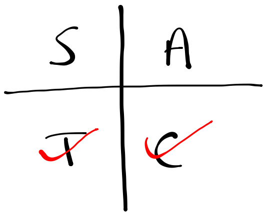
What primary trig ratios are POSITIVE in...



Where is θ if...

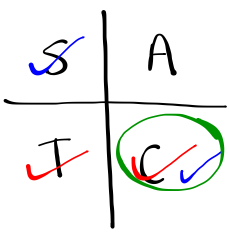
csc θ<0

sinθ<0



Quad 3 + 4

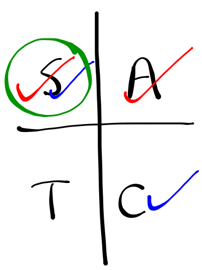
sinθ<0 & tanθ<0



Quad 4

cscθ>0 & cotθ<0

sinθ >0 + tanθ<0



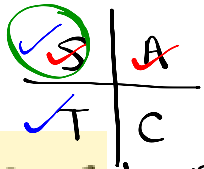
Quad 2

Homework

$\sin\theta > 0$

$\cos\theta < 0$

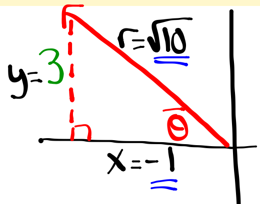
Quad 2



$\sec\theta < 0$

If $\sec\theta = -\sqrt{10}$ and $\sin\theta > 0$, determine the value of $\csc\theta = \frac{h}{o} = \frac{r}{y}$

$\sec\theta = \frac{-\sqrt{10}}{1} = \frac{hyp}{adj} = \frac{r}{x}$



$r = \sqrt{10}$ (always +)

$x = -1$

① Find y:

$x^2 + y^2 = r^2$

$(-1)^2 + y^2 = (\sqrt{10})^2$

$1 + y^2 = 10$

$y^2 = 9$

$y = \pm 3$

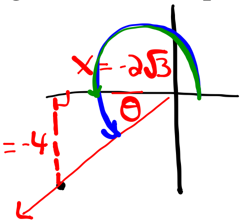
$y = 3$ (Q2)

② $\csc\theta = \frac{hyp}{opp} = \frac{r}{y} = \frac{\sqrt{10}}{3}$

Determine the measure (in radians) ~~of an angle~~ whose terminal arm passes through the ordered pair $(-2\sqrt{3}, -4)$

$x = -2\sqrt{3}$

$y = -4$



① Find $\bar{\theta}$

$\tan\bar{\theta} = \frac{y}{x} = \frac{-4}{-2\sqrt{3}} = \frac{2}{\sqrt{3}}$

$\tan\bar{\theta} = 1.1547$

convert calculator to rads $\rightarrow$ $\bar{\theta} = \tan^{-1}(1.1547)$

$\bar{\theta} = 0.86$ rads

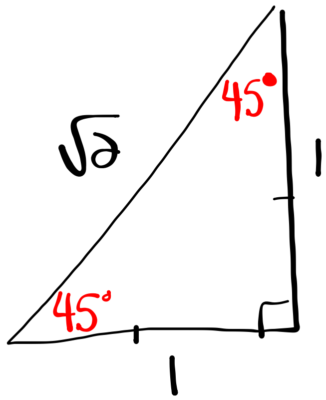
② Find θ

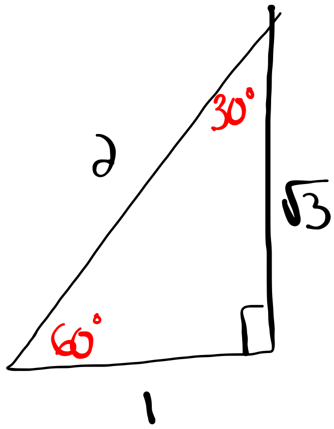
$\theta = \pi + \bar{\theta}$

$\theta = 3.14 + 0.86$

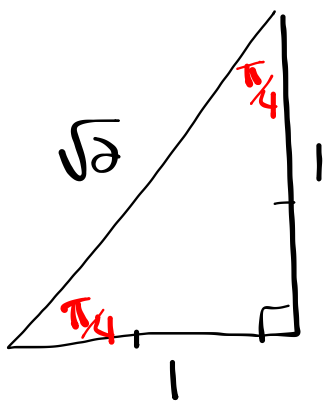
$\theta = 4$ rads

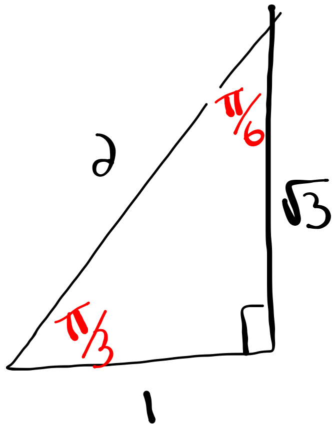
In Degrees



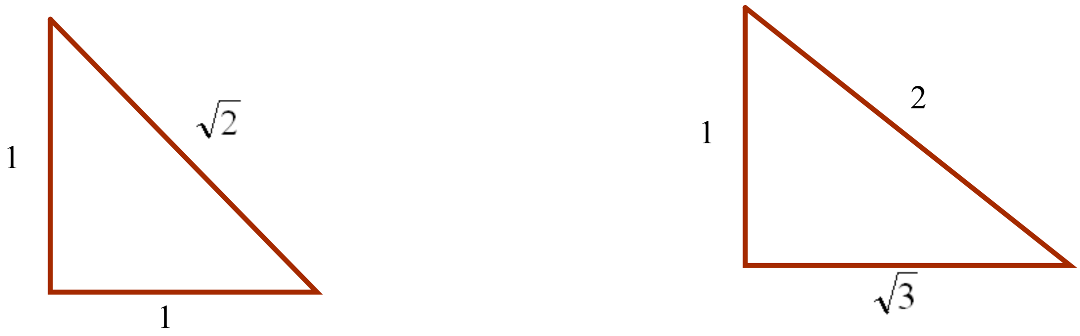


In Radians

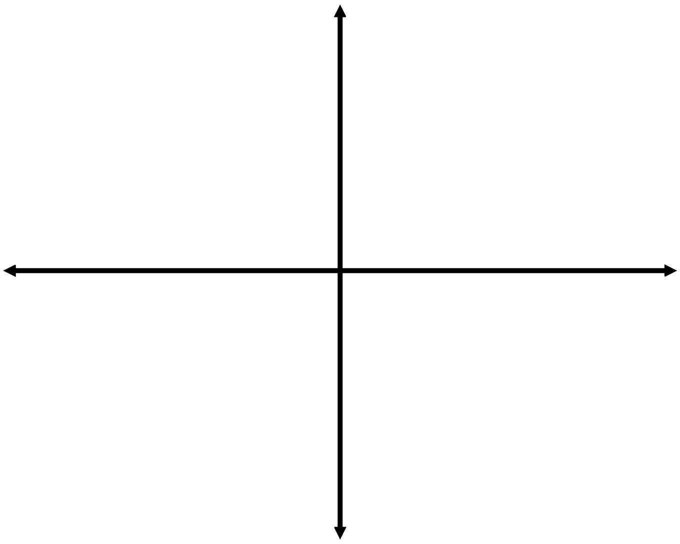




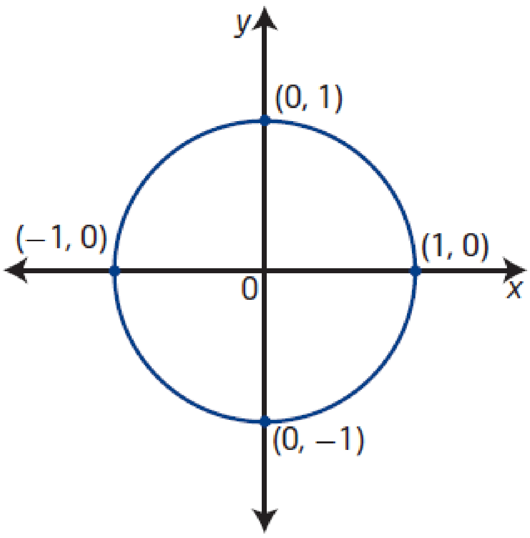
Special Angles (in radians)



Quadrantal Angles

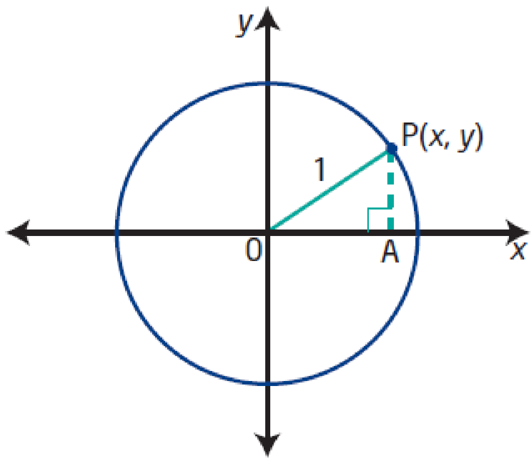


Unit Circle



unit circle

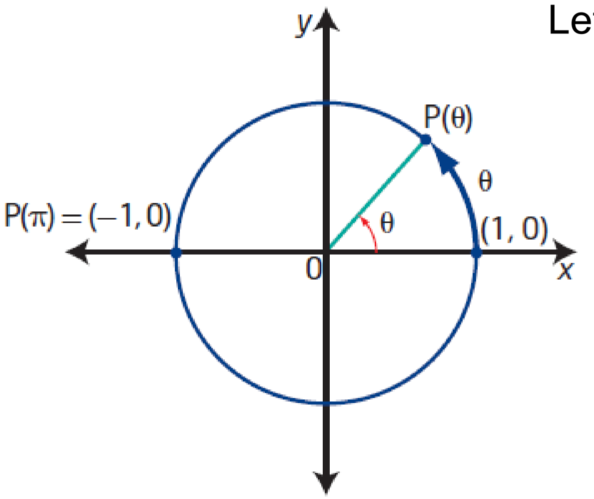
- a circle with radius 1 unit
- a circle of radius 1 unit with centre at the origin on the Cartesian plane is known as *the* unit circle



The equation of the unit circle is $x^2 + y^2 = 1$.

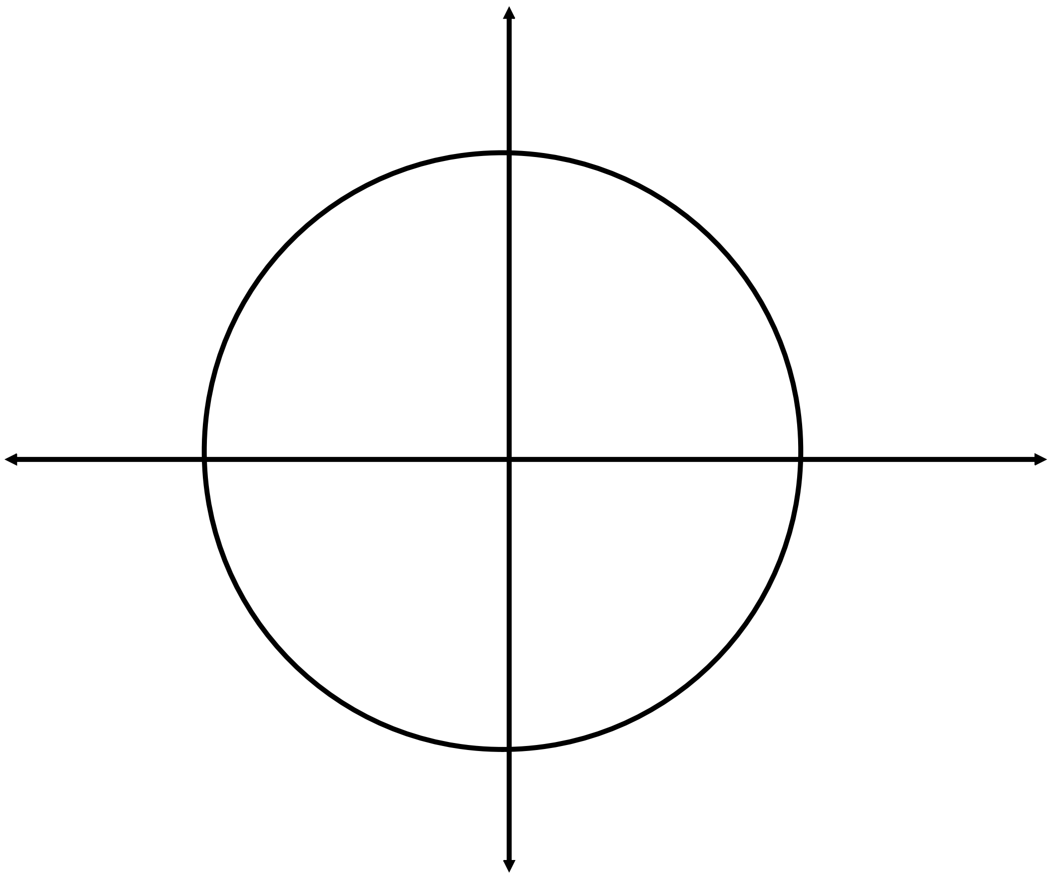
Determine the equation of a circle with centre at the origin and radius 6.

Special Angles on the Unit Circle:

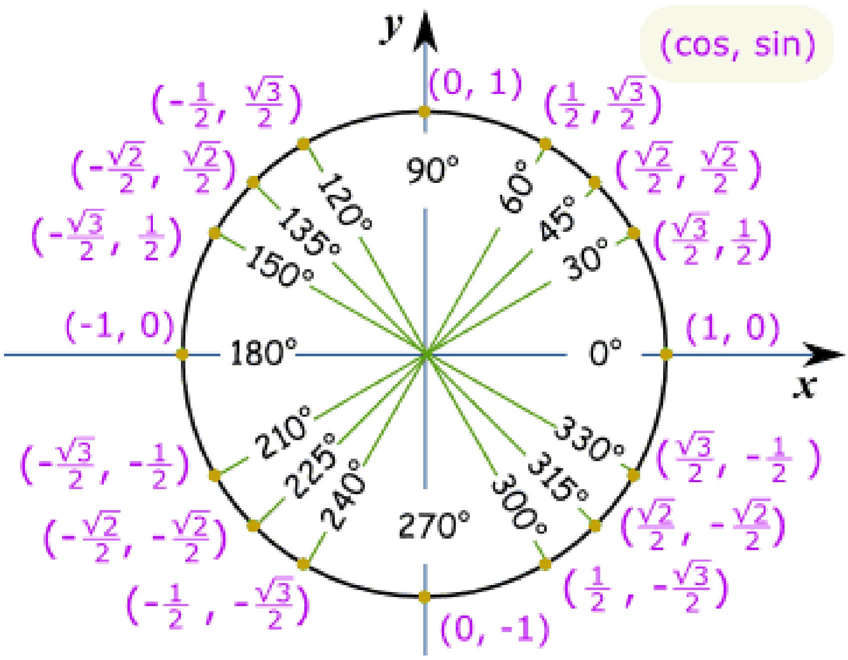


Let's use $\frac{\pi}{4}$ as our reference angle

Construct reference triangles
for all multiples of $\pi/4$
between 0 and 2π

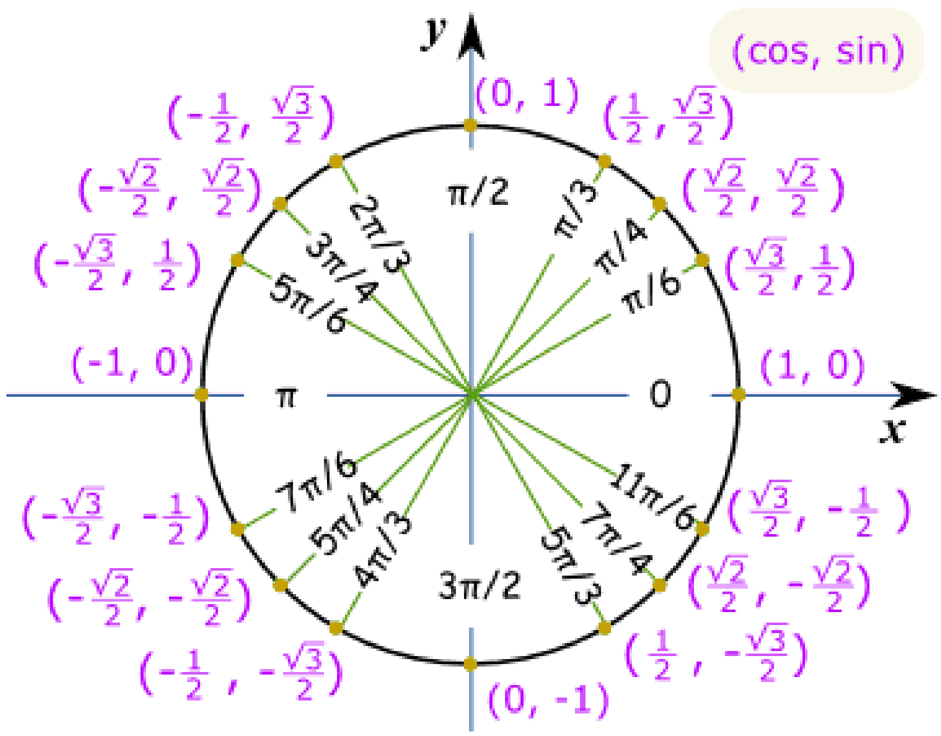


Unit Circle of Special Angles in Degrees



This is lovely...so what is it used for????

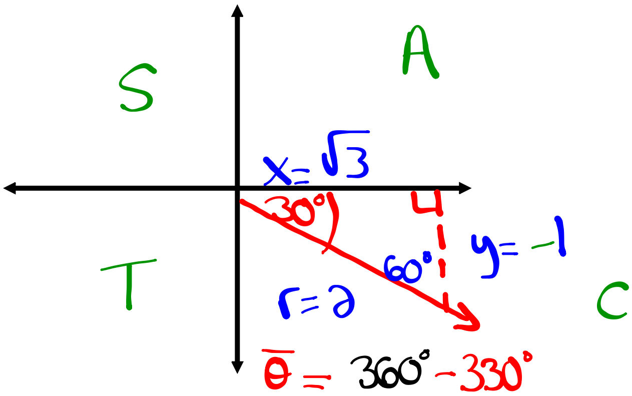
Unit Circle of Special Angles in Radians



$690° - 360° = 330°$

Solving Trig Expressions by Sketching Angles

Ex. Evaluate the $\sin 690° = \sin 330° = \frac{-1}{2}$



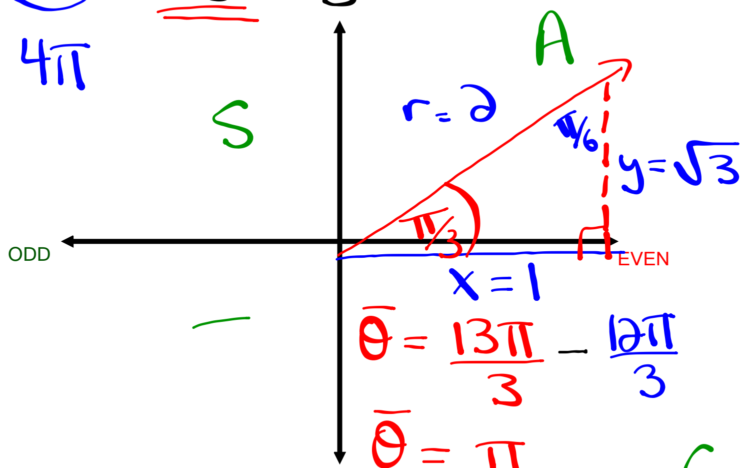
$\bar{\theta} = 360° - 330°$

$\bar{\theta} = 30°$

Ex. $\cos\frac{13\pi}{3} = \frac{1}{2}$

$\frac{12\pi}{3}, \frac{13\pi}{3}, \frac{14\pi}{3}$

4π



$\bar{\theta} = \frac{13\pi}{3} - \frac{12\pi}{3}$

$\bar{\theta} = \frac{\pi}{3}$

1

Homework

Evaluate each Trig Expression (provide a sketch of each angle)

1. $\tan\frac{17\pi}{6}$

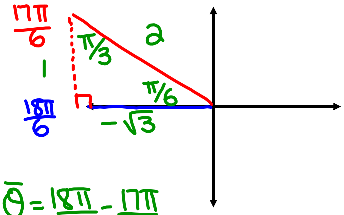
2. $\sin\frac{15\pi}{4}$

3. $\cos\left(-\frac{21\pi}{4}\right)$

Ex. $\tan\frac{17\pi}{6} = \frac{1}{-\sqrt{3}} = -\frac{\sqrt{3}}{3}$

$\frac{16\pi}{6}, \frac{17\pi}{6}, \frac{18\pi}{6}$

3π



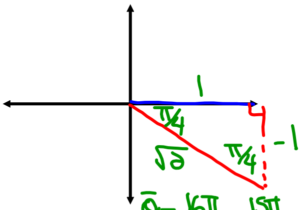
$\bar{\theta} = \frac{18\pi}{6} - \frac{17\pi}{6}$

$\bar{\theta} = \frac{\pi}{6}$

Ex. $\sin\frac{15\pi}{4} = \frac{-1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

$\frac{14\pi}{4}, \frac{15\pi}{4}, \frac{16\pi}{4}$

4π



$\bar{\theta} = \frac{16\pi}{4} - \frac{15\pi}{4}$

$\bar{\theta} = \frac{\pi}{4}$

Ex. $\cos\left(-\frac{21\pi}{4}\right) = -\frac{1}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$

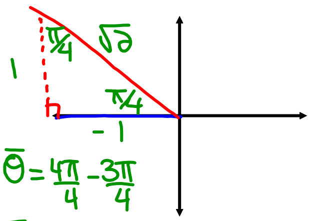
$\frac{-21\pi}{4} + \frac{6\pi}{1}$

$\frac{-21\pi}{4} + \frac{24\pi}{4} = \frac{3\pi}{4}$

$\cos\frac{3\pi}{4}$

$\frac{2\pi}{4}, \frac{3\pi}{4}, \frac{4\pi}{4}$

1π

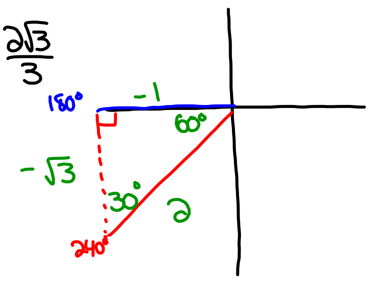


$\bar{\theta} = \frac{4\pi}{4} - \frac{3\pi}{4}$

$\bar{\theta} = \frac{\pi}{4}$

Extra

$\csc 240^\circ = \frac{2}{-\sqrt{3}} = -\frac{2\sqrt{3}}{3}$



$\bar{\theta} = 240° - 180°$

$\bar{\theta} = 60°$

Attachments

Worksheet - Sketching Angles in Radians.doc