4.4 Solving Problems Using Obtuse Triangles

GOAL

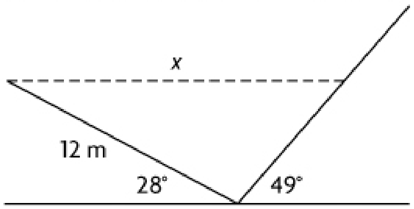
Solve problems that can be modelled by one or more obtuse triangles.

EXPLORE...

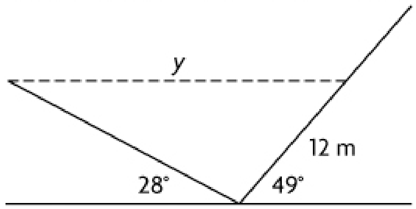
- The cross-section of a canal has two slopes and is triangular in shape. The angles of inclination for the slopes measure 28° and 49°. When the canal is full of water, the length of one of the slopes is 12 m. What is the width of the surface of water when the canal is full?

SAMPLE ANSWER

There are two possible diagrams that can be drawn from the given information.

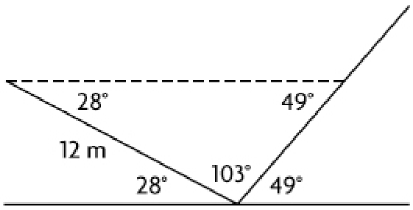


Case 1



Case 2

Consider Case 1:

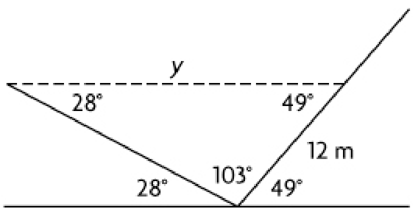


Using alternate interior angles with parallel lines and supplementary angles on a line, all the angle measures inside the triangle can be determined.

$$\frac{12}{\sin 49°} = \frac{x}{\sin 103°}$$

$$15.4926\ldots \text{ m} = x$$

Consider Case 2:



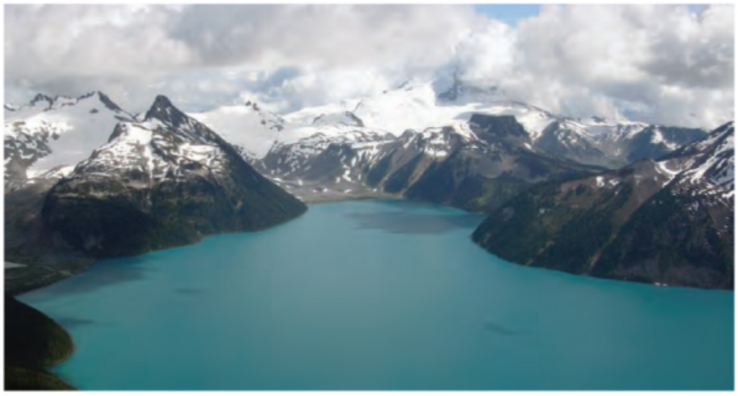
Case 2

$$\frac{12}{\sin 28°} = \frac{y}{\sin 103°}$$

$$24.905\ldots \text{ m} = y$$

LEARN ABOUT the Math

A surveyor in a helicopter would like to know the width of Garibaldi Lake in British Columbia. When the helicopter is hovering at 1610 m above the forest, the surveyor observes that the angles of depression to two points on opposite shores of the lake measure 45° and 82°. The helicopter and the two points are in the same vertical plane.



This view of the south part of Garibaldi Lake was captured from the Panorama Ridge trail.

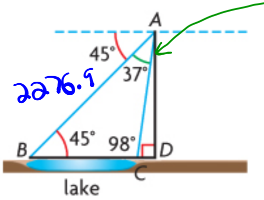
❓ What is the width of Garibaldi Lake?

EXAMPLE 1 Visualizing a triangle to solve a problem

Determine the width of the lake, to the nearest metre.

Emily's Solution: Using the sine law

$82° - 45° = 37°$



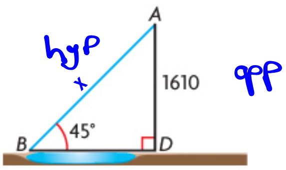
I drew a diagram to represent the situation.

I used parallel lines to determine the measure of ∠ *B*. Then I calculated the remaining angle in the base to be 98°, since the measures of angles in a triangle add to 180°.

(Alternate interior)

$180° - 45° - 37° = 98°$

I calculated the angle at the helicopter, between the sight lines, by subtraction.



In △*ABD*:

$$\frac{\sin 45°}{1} = \frac{1610}{x}$$

$$\frac{x \cancel{\sin 45}}{\cancel{\sin 45}} = \frac{1610}{\sin 45}$$

$$x = 2276.9$$

I used the primary trigonometric ratios to determine the length of *AB*. *AB* is a side in both △*ABD* and △*ABC*.

In △*ABC*:

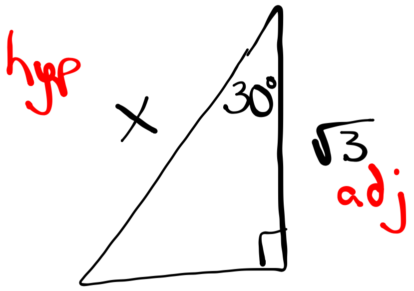
$$\frac{2276.9}{\sin 98°} = \frac{a}{\sin 37°}$$

$$\frac{2276.9 \sin 37}{\sin 98} = \frac{a \cancel{\sin 98}}{\cancel{\sin 98}}$$

$$1383.7 = a$$

I used the sine law to determine the width of the lake, *BC*.

The width of the lake is about 1384 m.



$$\cos\theta = \frac{adj}{hyp}$$

$$\frac{\cos 30^\circ}{1} = \frac{\sqrt{3}}{x}$$

$$\frac{x\cancel{\cos 30^\circ}}{\cancel{\cos 30^\circ}} = \frac{\sqrt{3}}{\cos 30^\circ}$$

$$x = 2$$

Reflecting

A. Could Emily have used the cosine law to calculate the width of the lake?

B. Does Emily need to worry about the ambiguous case when using the sine law in this situation? Explain.

Answers

A. Emily could have used the cosine law if she had first calculated the length of the other sight line using sin 82°. Then she would have had a situation that could have been solved using the cosine law.

B. No, Emily does not need to worry about the ambiguous case. The ambiguous case arises only in a SSA situation.

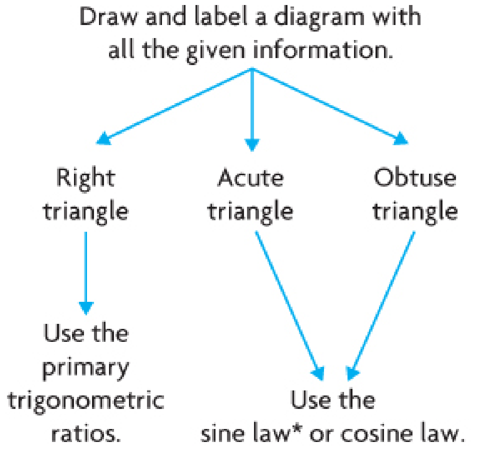
In Summary

Key Idea

- The sine law, the cosine law, the primary trigonometric ratios, and the sum of the measures of the angles in a triangle may all be useful when solving problems that can be modelled using obtuse triangles.

Need to Know

- When solving problems that involve trigonometry, the following decision tree may be useful for choosing an appropriate strategy.

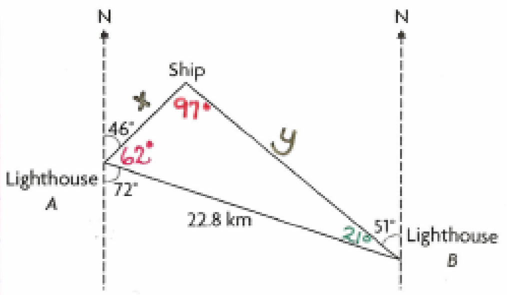


* When you know the lengths of two sides and the measure of an angle that is not contained by the two sides, the case may be ambiguous.

Complete Section 4.4 Worksheet

SOLUTIONS => 4.4 Solving Problems Using Obtuse Triangles (WORKBOOK)

1. Two lighthouses, A and B, are 22.8 Km apart. From lighthouse A, the compass heading for lighthouse B is S72°E. The Keeper in each lighthouse sees the same ship. The heading of the ship from lighthouse A is N46°E. The heading of the ship from lighthouse B is N51°W. How far, to the nearest tenth of a Kilometer, is the ship from each lighthouse?



$180° - 72° - 46° = \boxed{62°}$

$72° - 51° = \boxed{21°}$

$180° - 62° - 21° = \boxed{97°}$

Distance from Ship to Lighthouse A => "x"

$$\frac{x}{\sin 21°} = \frac{22.8}{\sin 97°}$$

$$\frac{x \cancel{\sin 97°}}{\cancel{\sin 97°}} = \frac{22.8 \sin 21°}{\sin 97°}$$

$$x = 8.2 \text{ Km}$$

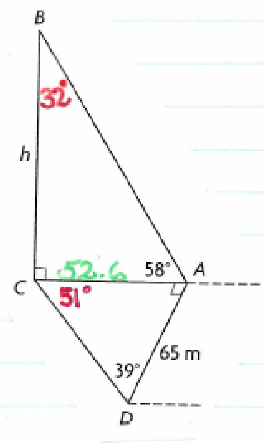
Distance from Ship to Lighthouse B => "y"

$$\frac{y}{\sin 62°} = \frac{22.8}{\sin 97°}$$

$$\frac{y \cancel{\sin 97°}}{\cancel{\sin 97°}} = \frac{22.8 \sin 62°}{\sin 97°}$$

$$y = 20.3 \text{ Km}$$

2. Calculate the height, h, to the nearest tenth of a meter.



$\angle ACD = 180° - 90° - 39°$
$= \boxed{51°}$

To find "h", we need to find the length of AC first:

$$\frac{x}{\sin 39°} = \frac{65}{\sin 51°}$$

$$\frac{x \cancel{\sin 51°}}{\cancel{\sin 51°}} = \frac{65 \sin 39°}{\sin 51°}$$

$$\boxed{x = 52.6 \text{ m}}$$

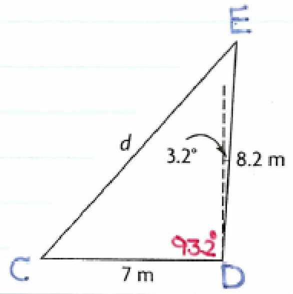
$\angle CBA = 180° - 90° - 58°$
$= \boxed{32°}$

$$\frac{h}{\sin 58°} = \frac{52.6}{\sin 32°}$$

$$\frac{h \cancel{\sin 32°}}{\cancel{\sin 32°}} = \frac{52.6 \sin 58°}{\sin 32°}$$

$$h = 84.2 \text{ m}$$

3. An 8.2m tall telephone pole stands on level ground and leans 3.2° from the vertical. When the pole's shadow is 7m long, what is the distance, d, from the top of the pole to the tip of the shadow, to the nearest tenth of a meter?



$\angle CDE = 90° + 3.2°$

$= 93.2°$

$d^2 = c^2 + e^2 - 2ce \cos D$

$d^2 = (8.2)^2 + (7)^2 - 2(8.2)(7)\cos 93.2°$

$d^2 = 67.24 + 49 - 114.8(-0.0558)$

$d^2 = 116.24 + 6.4058$

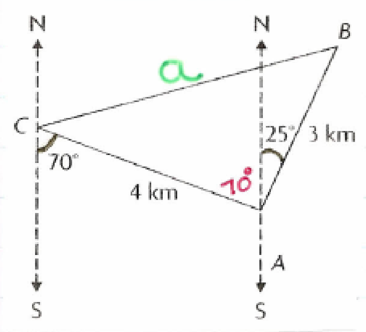
$d^2 = 122.6458$

$d = \sqrt{122.6458}$

$d = 11.1$ m

It is 11.1m from the top of the pole to the tip of the shadow.

4. Jasleen leaves her campsite at C and hikes 4 Km in a S70°E direction to A. She then turns and hikes 3 Km in a N25°E direction to B. How far is Jasleen from the campsite? Round your answer to the nearest tenth of a kilometer.



$\angle CAN = 70°$ (alternate interior)

$a^2 = b^2 + c^2 - 2bc\cos A$

$a^2 = (4)^2 + (3)^2 - 2(4)(3)\cos 95°$

$a^2 = 16 + 9 - 24(-0.0872)$

$a^2 = 25 + 2.0928$

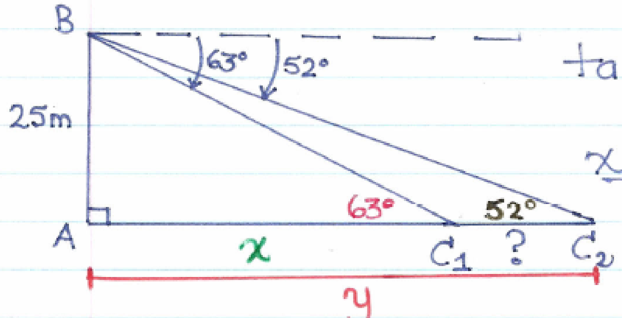
$a^2 = 27.0928$

$a = \sqrt{27.0928}$

$a = 5.2\text{ Km}$

5. From the top of a 25 m building, the angle of depression to one parked car is 63° and the angle of depression to another parked car is 52°. The cars are parked in the same line of sight.

The distance between the two cars, to the nearest tenth of a meter is?



$\tan 63° = \dfrac{25}{x}$

$\dfrac{x \cancel{\tan 63°}}{\cancel{\tan 63°}} = \dfrac{25}{\tan 63°}$

$x = 12.7\text{m}$

$\tan 52° = \dfrac{25}{y}$

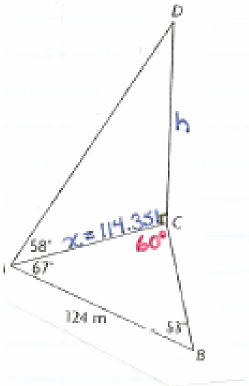
$\dfrac{y \cancel{\tan 52°}}{\cancel{\tan 52°}} = \dfrac{25}{\tan 52°}$

$y = 19.5\text{ m}$

$\angle BC_1A = 63°$
$\angle BC_2A = 52°$

The distance between the cars is: $19.5\text{m} - 12.7\text{m}$
$= 6.8\text{m}$

6. The diagram shows the measurements Roberta made to determine the height, CD, of a skyscraper, using a baseline AB.



a) The first thing Roberta needed to know, based on her measurements, was the measure of $\angle ACB$.

$\angle ACB = 180° - 67° - 53°$
$= 60°$

b) Roberta then used this angle measure to determine the length of AC from one end of her baseline to the foot of the building.

$$\frac{x}{\sin 53°} = \frac{124}{\sin 60°}$$

$$\frac{x \cancel{\sin 60°}}{\cancel{\sin 60°}} = \frac{124 \sin 53°}{\sin 60°}$$

$x = 114.351$ m

c) Finally, Roberta was able to determine the height of the skyscraper, using the angle of elevation she had measured.

$$\tan 58° = \frac{h}{114.351}$$

$114.351 \tan 58° = h$

$183\text{m} = h$

Attachments

4Ws4e2.mp4