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Getting ≤ Started

REVIEW OF TERMS AND CONNECTIONS

WORDS You Need to Communicate Effectively

1. Match each term with the best example or description on the right.

a) linear equation
b) x- and y-intercepts
c) slope
d) linear inequality
e) dependent variable
f) domain
g) range
h) discrete
i) continuous
j) independent variable
k) quadrant I

i) the value 3 in the equation $y = 3x + 1$ **c)** slope
ii) $\{1, 2, 3\}$ in the solution set $\{(1, 5), (2, 6), (3, 7)\}$ **f)** domain
iii) in a relationship, the variable graphed on the y-axis **e)** dependent variable
iv) $2y = 3x + 7$ **a)** linear equation
v) $3 \leq x + 5$ **d)** linear inequality
vi) term used to describe a solution set from the set of real numbers **i)** continuous (solid line)
vii) $\left(\frac{5}{4}, 0\right)$ and $(0, -5)$ for the graph of $y = 4x - 5$ **b)** x- and y-intercepts
viii) $\{5, 6, 7\}$ in the solution set $\{(1, 5), (2, 6), (3, 7)\}$ **g)** range
ix) in a relationship, the variable graphed on the x-axis **j)** independent variable
x) term used to describe a solution set from the set of integers **h)** discrete (dotted line)
xi) the part of the coordinate plane where $x > 0$ and $y > 0$ **k)** quadrant I

Answers

1.

REVIEW OF TERMS AND CONNECTIONS

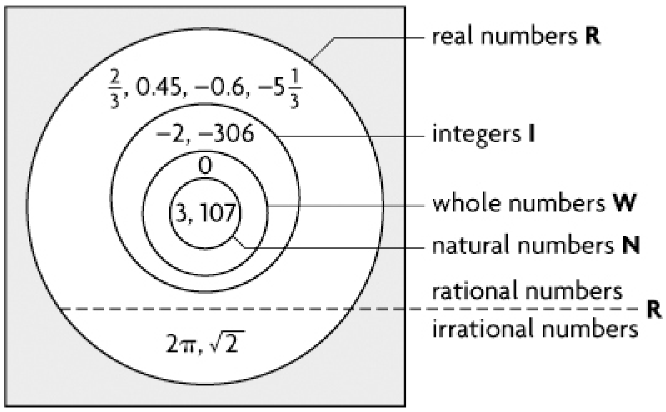
CONNECTIONS You Need for Success

Applying Number Concepts

Number Classification

When working with linear equations and linear inequalities, the domain and range may be restricted to a specific set of numbers. Knowing how the sets of numbers are different is important when interpreting and solving problems and when graphing. For example:

- If $\{(x, y) \mid x \in W, y \in W\}$, the variables x and y are from the set of whole numbers and the problem has a whole-number solution, such as (4, 6). If the problem is represented graphically, the graph will be in the first quadrant.
- If $\{(x, y) \mid x \in R, y \in R\}$, the variables x and y are from the set of real numbers and the problem has real-number solutions, such as $(-4, 5.7)$ or $(\sqrt{2}, \sqrt{8})$. If the problem is represented graphically, the graph could be in any of the four quadrants.



2. Give an example of an ordered pair that could be in each solution set.

a) $\{(x, y) \mid x \in I, y \in I\}$

b) $\{(m, p) \mid m \in R, p \in R\}$

c) $\{(k, j) \mid k \in N, j \in N\}$

d) $\{(x, y) \mid x \geq 0, x \in R, y \geq 0, y \in R\}$

Answers

2. Answers will vary, e.g.,

a) $(-2, 7)$ **b)** $(-0.12, 0.75)$ **c)** $(1, 95)$ **d)** $\left(\frac{4}{5}, 1\frac{7}{8}\right)$

REVIEW OF TERMS AND CONNECTIONS

Working with Linear Equations and Linear Inequalities

Rearranging Equations

The slope-intercept form of a linear equation, $y = mx + b$, is useful for visualizing the graph of the equation. For example, when $5x + (-3)y + 6 = 0$ is rearranged as $y = \frac{5x}{3} + 2$, you know that the graph will go through point (0, 2) (the y-intercept) and it will have a positive slope of $\frac{5}{3}$.

When using graphing technology, you often need to create an equivalent form of the linear equation to isolate the variable y, so that you can enter the equation into the graphing technology. For example, to enter the equation $x + 5y = 3$, you must rearrange it to $y = -\frac{x}{5} + \frac{3}{5}$ or $y = \frac{3}{5} - \frac{x}{5}$.

3. Rearrange each equation to isolate y.

a) $6x + 2y - 7 = 0$ **b)** $12 - x + 4y = 0$ **c)** $\frac{2x}{3} + \frac{y}{3} = 4$

Rearranging Inequalities

Rearranging inequalities is much like rearranging equations. However, there are times when you must reverse the inequality sign to keep the inequality true. If you divide or multiply both sides of the inequality by a negative number as you isolate the variable, the sign must be reversed. For example:

$7 - 7x \leq 2 + 3x$	
$-7x - 3x \leq 2 - 7$	Divide both sides by -10.
$\frac{-10x}{-10} \geq \frac{-5}{-10}$	Reverse the sign from $\leq$ to $\geq$.
$x \geq \frac{5}{10}$	

4. Rearrange each equation to isolate x.

a) $5x + 2x - 7 < 14$ **b)** $10 - 4x + 3x > 0$ **c)** $\frac{x}{3} - \frac{5x}{3} \leq 8$

Answers

3. a) $y = -3x + 3.5$
b) $y = \frac{x}{4} - 3$
c) $y = -2x + 12$

4. a) $x < 3$
b) $x < 10$
c) $x \geq -6$

③ b) $12 - x + 4y = 0$

$\frac{4y}{4} = \frac{x - 12}{4}$

$y = \frac{x}{4} - \frac{12}{4}$

$y = \frac{x}{4} - 3$

REVIEW OF TERMS AND CONNECTIONS

Verifying Solutions to Equations

When verifying the solution to a linear equation or a system of linear equations, the left side–right side format is usually used. If the solution is not valid, you will end up with an untrue statement. The following example shows how to verify that point (1, 3) is a solution to the system of linear equations $x + y = 4$ and $3x - y = 0$.

Test (1, 3) for $x + y = 4$:

LS	RS
1 + 3 4	4

4 = 4 is true.

Test (1, 3) for $3x - y = 0$:

LS	RS
3(1) − 3 0	0

0 = 0 is true.

Therefore, (1, 3) is a solution to the system of linear equations $x + y = 4$ and $3x - y = 0$.

5. Verify that point (4, 5) is a solution to each equation or system.

a) $3x + 5y = 20$ **b)** $4x = 21 - y$ **c)** $6x - 2y = 14$ and $3x + y = 17$

Answers

5. a) No
b) Yes
c) Yes

REVIEW OF TERMS AND CONNECTIONS

PRACTISING

6. Isolate the variable in each equation or inequality.

a) $\frac{x}{2} + 5 = 3$ **b)** $2(y - 1) = 3(y + 1)$ **c)** $-2w \geq 2 + 7 + w$

7. Simplify and evaluate each expression for $a = 2$, $d = 3$, and $v = 4$.

a) $a + 5a + 3 - 6a + 3a + 1$
b) $7 + d + 2(3d - 2)$
c) $5(v + 2) - 13$

8. Represent each problem algebraically, and then solve it.

a) Ian looked inside a pencil box and said, "There are more than three pencils." Freda looked inside the same box and said, "But there are fewer than a dozen pencils." What are the possible numbers of pencils inside the box?

b) Rick's exercise routine consists of two types of exercises: lifting free weights and using cardio machines. On Saturday, Rick spent 78 min exercising. Rick spent double the time lifting weights as he did using the cardio machines. How much time did he spend on each type of exercise?

9. For each problem in question 8, state

a) what number set the domain and range belong to, and
b) whether the solution is discrete or continuous

Answers

6. a) $x = -4$
b) $y = -5$
c) $w \leq -3$

7. a) $3a + 4 = 10$
b) $7d + 3 = 24$
c) $5v - 3 = 17$

8. a) Let p represent the number of pencils.

$3 < p < 12$ or $p > 3$ and $p < 12$

There are 4, 5, 6, 7, 8, 9, 10, or 11 pencils.

b) Let w represent lifting weights, and let c represent using the cardio machines.

$w + c = 78$ and $w = 2c$

Rick spent 52 min lifting weights and 26 min using the cardio machines.

9.

6 a)

$$\frac{x}{2} + 5 = 3$$

$$\frac{x}{2} = 3 - 5$$

$$2 \cdot \frac{x}{2} = -2 \cdot 2$$

$$x = -4$$

c)

$$-2w \geq 2 + 7 + w$$

$$-2w \geq 9 + w$$

$$-2w - w \geq 9$$

$$\frac{-3w}{-3} \geq \frac{9}{-3}$$

$$w \leq -3$$

8 Let x = # of mins on cardio

Let 2x = # of mins on weights

$$x + 2x = 78$$

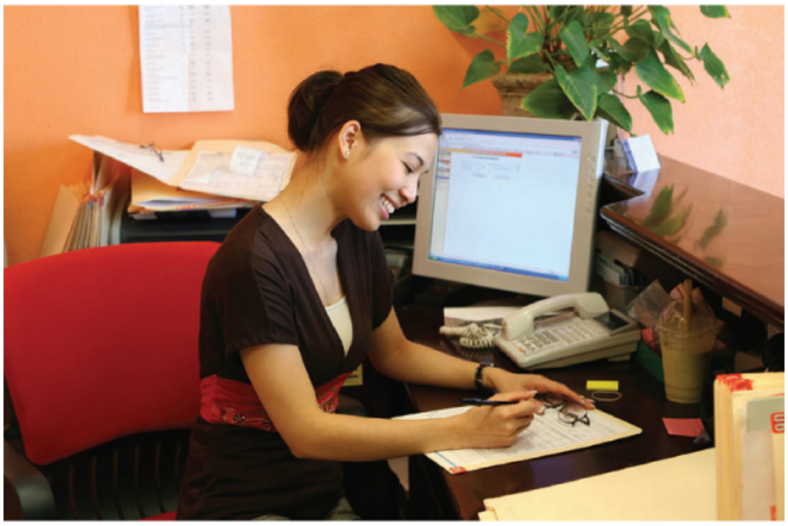
$$\frac{3x}{3} = \frac{78}{3}$$

$x = 26$ mins.

5 Getting Started

Balancing Two Part-Time Jobs

Marnie has two part-time jobs. She earns minimum wage working at the information desk in a hospital and $4 above minimum wage helping with her mother's house-cleaning business. Marnie works in whole-hour increments only. She enjoys the work at the hospital more than house cleaning and does not work more than 15 h a week, since she often has a lot of homework and she plays on her school's volleyball team.



? How many hours can Marnie work at the hospital and earn at least $160 a week?

A. Predict the solution to the problem. Explain your prediction to a partner.

B. If Marnie works 5 h at the hospital, how many hours will she work for her mother? Will she earn at least $160? Explain.

C. If Marnie works 10 h at the hospital, how many hours will she work for her mother? Will she earn at least $160? Explain.

D. What type of **linear inequality** is being described in the problem? Justify your choice.

linear inequality

A linear inequality is a relationship between two linear expressions in which one expression is less than (<), greater than (>), less than or equal to (≤), or greater than or equal to (≥) the other expression.

Answers

A. Answers will vary, e.g., I predict that Marnie could work 8 h at the hospital and 7 h for her mother because

8($9) + 7($13) = $163

B. 10 h or fewer; yes, she will earn $165 or less at a $9 minimum wage because

5($9) + 10($13) = $175

C. 5 h or fewer; no, she will earn $155 or less at a $9 minimum wage because

10($9) + 5($13) = $155

D.

? How many hours can Marnie work at the hospital and earn at least $160 a week?

E. Use a variable to represent the number of hours that Marnie works at the hospital. Use the same variable to create an expression for the number of hours that she works for her mother.

F. To what set of numbers—natural, whole, integer, rational, or real—does the variable you defined in part E belong? Justify your decision.

G. Write a linear inequality to represent Marnie's situation. Explain why your linear inequality makes sense.

H. Solve the linear inequality, and represent the solution on a number line.

I. Describe how your prediction in part A compares with your solution in part H.

Answers

E. Let x represent the number of hours that Marnie works at the hospital. Let $15 - x$ represent the number of hours that Marnie works for her mother.

F. whole numbers, since she works in whole-hour increments

G. $9x + 13(15 - x) \geq 160$

This makes sense, because x is the number of hours that Marnie works at the hospital, and she gets a minimum wage of \$9/h. Similarly, $(15 - x)$ is the number of hours that she works for her mother, and she gets \$13/h (\$4/h more than minimum wage). The sum of the two must be greater than or equal to \$160 in order to solve the problem.

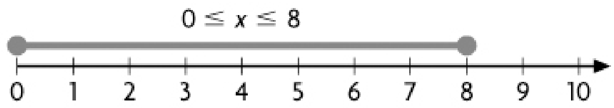
H. $9x + 13(15 - x) \geq 160$

$$9x + 195 - 13x \geq 160$$

$$-4x \geq -35$$

$$x \leq 8.25$$

Since 8.25 is not a whole number, $x \leq 8$.



I. My prediction in prompt A was one possible solution to the problem. I could have given any whole number of hours from 0 to 8.