5.1 Graphing Linear Inequalities in Two Variables

GOAL

Solve problems by modelling linear inequalities in two variables.

EXPLORE...

- For which inequalities is (3, 1) a possible solution? How do you know?

a) $13 - 3x > 4y$
b) $2y - 5 \leq x$
c) $y + x < 10$
d) $y \geq 9$

SAMPLE ANSWER

(3, 1) is a possible solution for parts b) and c). When (3, 1) is substituted for x and y into each inequality, they make a true statement.

Methods of graphing: $2x + y = 8$

① x and y intercepts:

x-int $(y=0)$

$2x + (0) = 8$

$\frac{2x}{2} = \frac{8}{2}$

$x = 4$

$\boxed{(4,0)}$

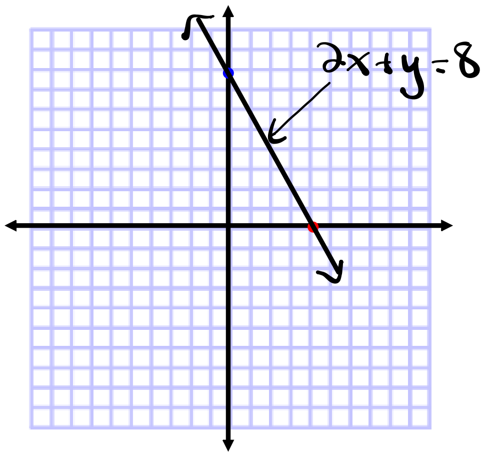
y-int $(x=0)$

$2(0) + y = 8$

$0 + y = 8$

$y = 8$

$\boxed{(0,8)}$



② $y = mx + b$

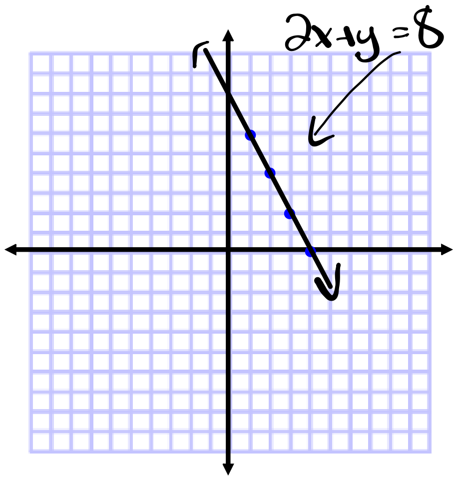
$2x + y = 8$

$y = -2x + 8$

$m = \frac{-2}{1}$ $\frac{\text{rise}}{\text{run}}$

$b = \text{y-int} = 8$

$(0,8)$



③ Table of values

$y = -2x + 8$

x	y
-2	12
-1	10
0	8
1	6
2	4

INVESTIGATE the Math

Amir owns a health-food store. He is making a mixture of nuts and raisins to sell in bulk. His supplier charges \$25/kg for nuts and \$8/kg for raisins.

? What quantities of nuts and raisins can Amir mix together if he wants to spend less than \$200 to make the mixture?

A. Suppose that Amir wants to spend exactly \$200 to make the mixture. Work with a partner to create an equation that represents this situation.

B. To what set of numbers does the domain and range of the two variables in your equation belong? Use this information to help you graph the equation on a coordinate plane.

C. Explain why the graph is a line segment, not a ray or a line.

D. What region of the coordinate plane includes points representing quantities of nuts and raisins that Amir could use if he wants to spend less than \$200? How do you know?

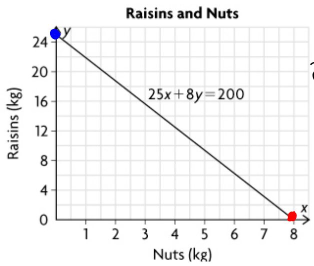
E. There are many possible solutions to Amir's problem. Plot at least three points that represent reasonable solutions to Amir's problem. Explain why you chose these points.

Answers

A. Let x represent the number of kilograms of nuts, and let y represent the number of kilograms of raisins:

$25x + 8y = 200$

B. The domain and range belong to the set of real numbers greater than or equal to 0.

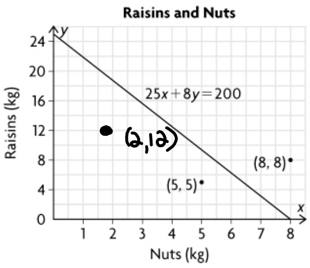


x-int (y=0)
$25x+8(0)=200$
$25x+0=200$
$\frac{25x}{25}=\frac{200}{25}$
$x=8$
$(8,0)$

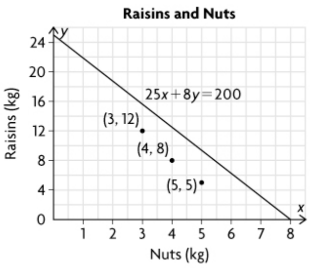
y-int (x=0)
$25(0)+8y=200$
$0+8y=200$
$\frac{8y}{8}=\frac{200}{8}$
$y=25$
$(0,25)$

C. The graph is a line segment because it has endpoints at the x-axis and y-axis.

D. The region below the line segment includes points representing quantities of nuts and raisins that Amir could use. Answers will vary, e.g., I picked a point in the region above and below the line and tested both algebraically:



E. Answers will vary, e.g., I chose these points because they are in the region below the line. I know that points in the region below the line will solve the problem. However, even though points such as (0, 0), (7, 0) and (0, 24) are below the line, they wouldn't make very good mixtures because they either involve only one quantity or no quantities at all.



Test a point above the line.	Test a point below the line.
(8, 8), or 8 kg of nuts and 8 kg of raisins:	(5, 5), or 5 kg of nuts and 5 kg of raisins:
$25x + 8y \rightarrow 25(8) + 8(8) = 264$	$25x + 8y \rightarrow 25(5) + 8(5) = 165$
So, 8 kg of nuts and 8 kg of raisins would cost \$264, which is more than \$200.	So, 5 kg of nuts and 5 kg of raisins would cost \$165, which is less than \$200.

Reflecting

F. Discuss and then decide whether the **solution set** for Amir's problem is represented by
   i) points in the region above the line segment.
   ii) points in the region below the line segment.
   iii) points on the line segment.

G. Why might the line segment be considered a boundary of the solution set?

H. Why might you use a dashed line segment for this graph instead of a solid line segment?

solution set
The set of all possible solutions.

Answers

F. i) Points in the region above the line segment are not part of the solution set because they result in costs greater than \$200.

ii) Points in the region below the line segment represent the solution set because they result in costs less than \$200.

iii) Points on the line segment are not part of the solution set because they result in costs that are exactly \$200.

G. The line segment might be considered a boundary because the solution set is completely on one side of it. It separates the points that are in the solution set from the points that are not.

H. A dashed line segment shows that the points on the line segment are not part of the solution set.

APPLY the Math

EXAMPLE 1 Solving a linear inequality graphically when it has a continuous solution set in two variables

Graph the solution set for this linear inequality:
$-2x + 5y \geq 10$

Robert's Solution: Using graph paper

Linear equation that represents the boundary:
$-2x + 5y = 10$

I knew that the graph of the linear equation $-2x + 5y = 10$ would form the boundary of the linear inequality $-2x + 5y \geq 10$.

The variables represent numbers from the set of real numbers.
$x \in R$ and $y \in R$

The domain and range are not stated and no context is given, so I assumed that the domain and range are the set of real numbers. This means that the solution set is **continuous**.

continuous
A connected set of numbers. In a continuous set, there is always another number between any two given numbers. Continuous variables represent things that can be measured, such as time.

y-intercept:

$$-2x + 5y = 10$$
$$-2(0) + 5y = 10$$
$$\frac{5y}{5} = \frac{10}{5}$$
$$y = 2$$

The y-intercept is at (0, 2).

I knew that I needed to plot and join only two points to graph the linear equation. I decided to plot the two intercepts.

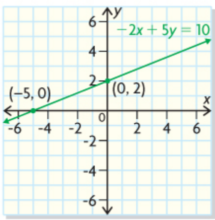
To determine the y-intercept, I substituted 0 for x.

x-intercept:

$$-2x + 5y = 10$$
$$-2x + 5(0) = 10$$
$$\frac{-2x}{-2} = \frac{10}{-2}$$
$$x = -5$$

The x-intercept is at $(-5, 0)$.

To determine the x-intercept, I substituted 0 for y.



Since the linear inequality has the possibility of equality ($\geq$), and the variables represent real numbers, I knew that the **solution region** includes all the points on its boundary. That's why I drew a solid green line through the intercepts.

solution region
The part of the graph of a linear inequality that represents the solution set; the solution region includes points on its boundary if the inequality has the possibility of equality.

Test (0, 0) in $-2x + 5y \geq 10$.

LS	RS
$-2(0) + 5(0)$	10
0	

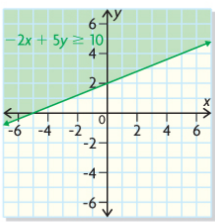
Since 0 is not greater than or equal to 10, (0, 0) is not in the solution region.

I needed to know which **half plane**, above or below the boundary, represents the solution region for the linear inequality.

To find out, I substituted the coordinates of a point in the half plane below the line. I used (0, 0) because it made the calculations simple.

I already knew that the solution region includes points on the boundary, so I didn't need to check a point on the line.

half plane
The region on one side of the graph of a linear relation on a Cartesian plane.



Since my test point below the boundary was not a solution, I shaded the half plane that did not include my test point. This was the region above the boundary.

I used green shading to show that the solution set belongs to the set of real numbers.

Since the domain and range are in the set of real numbers, I knew that the solution set is continuous. Therefore, the solution region includes all points in the shaded area and on the solid boundary.

Communication Tip
If the solution set to a linear inequality is continuous and the sign includes equality ($\leq$ or $\geq$), a solid green line is used for the boundary, and the solution region is shaded green.

$-2x + 5y \geq 10$

- Boundary line will be solid
- $x \in R$ and $y \in R$

① $-2x + 5y = 10$

② x-int $(y=0)$

$-2x + 5(0) = 10$

$-2x + 0 = 10$

$\frac{-2x}{-2} = \frac{10}{-2}$

$x = -5$

$(-5, 0)$

y-int $(x=0)$

$-2(0) + 5y = 10$

$0 + 5y = 10$

$\frac{5y}{5} = \frac{10}{5}$

$y = 2$

$(0, 2)$

③ $-2x + 5y \geq 10$

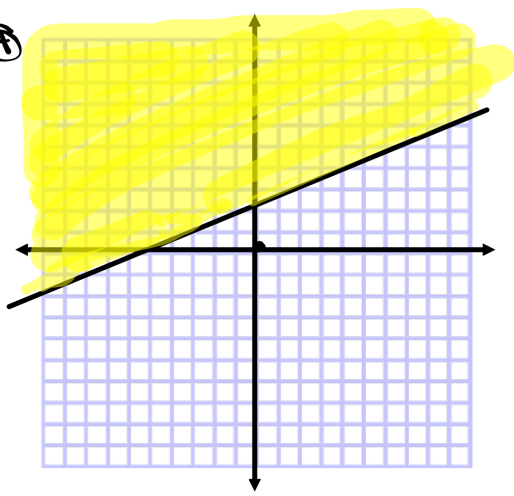
$-2(0) + 5(0) \geq 10$

$0 + 0 \geq 10$

$0 \geq 10$

(False)

④

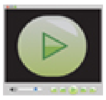


EXAMPLE 1 Solving a linear inequality graphically when it has a continuous solution set in two variables

Graph the solution set for this linear inequality:
$-2x + 5y \geq 10$

Your Turn

Compare the graphs of the following relations. What do you notice?
$-2x + 5y \geq 10$ $-2x + 5y = 10$ $-2x + 5y < 10$



Answer

They all have the same line or boundary, but the inequality $-2x + 5y \geq 10$ has a solution region that includes the boundary and the half plane above it. The equation $-2x + 5y = 10$ has a solution that includes only the values on the line. The inequality $-2x + 5y < 10$ has a solution region that is the half plane below the boundary and does not include values on the boundary.

EXAMPLE 2 Graphing linear inequalities with vertical or horizontal boundaries

Graph the solution set for each linear inequality on a Cartesian plane.

a) $\{(x, y) \mid x - 2 > 0, x \in \mathrm{R}, y \in \mathrm{R}\}$

b) $\{(x, y) \mid -3y + 6 \geq -6 + y, x \in \mathrm{I}, y \in \mathrm{I}\}$

Wynn's Solution

a) $x - 2 > 0$

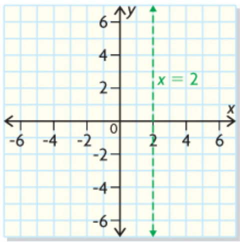
$x > 2$

I isolated x so I could graph the inequality.

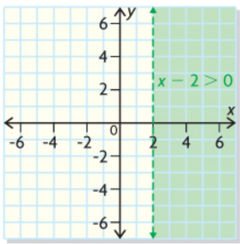
The variables represent numbers from the set of real numbers.

$x \in \mathrm{R}$ and $y \in \mathrm{R}$

The domain and range are stated as the set of real numbers. The solution set is continuous, so the solution region and its boundary will be green in my graph.



I drew the boundary of the linear inequality as a dashed green line because I knew that the linear inequality (>) does not include the possibility of x being equal to 2.



I needed to decide which half plane to shade. For x to be greater than 2, I knew that any point to the right of the boundary would work.

The solution region includes all the points in the shaded area because the solution set is continuous. The solution region does not include points on the boundary.

Communication Tip
If the solution set to a linear inequality is continuous and the sign does not include equality (< or >), a dashed green line is used for the boundary and the solution region is shaded green.

b) $-3y + 6 \geq -6 + y$

$-4y \geq -12$

$\dfrac{-4y}{-4} \leq \dfrac{-12}{-4}$

$y \leq 3$

Since the linear inequality has only one variable, y, I isolated the y.

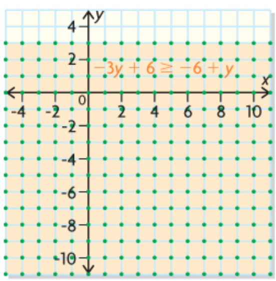
As I rearranged the linear inequality, I divided both sides by -4. That's why I reversed the sign from $\geq$ to $\leq$.

The variables represent integers.

$x \in \mathrm{I}$ and $y \in \mathrm{I}$

The domain and range are stated as being in the set of integers. I knew this means that the solution set is **discrete**.

discrete
Consisting of separate or distinct parts. Discrete variables represent things that can be counted, such as people in a room.



$\{(x, y) \mid -3y + 6 \geq -6 + y, x \in \mathrm{I}, y \in \mathrm{I}\}$

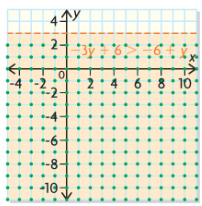
I knew that points with integer coordinates below the line $y = 3$ were solutions, so I shaded the half plane below it orange.

I knew the linear inequality includes 3, so points on the boundary with integer coordinates are also solutions to the linear inequality.

I stippled the boundary and the orange half plane with green points to show that the solution set is discrete.

The solution region includes only the points with integer coordinates in the shaded region and along the boundary.

Communication Tip
If the solution set to a linear inequality is discrete, the solution region is shaded orange and stippled with green points. If the sign includes equality (≥ or ≤), the boundary is also stippled. An example of this is shown to the left. If equality is not possible (< or >), the boundary is a dashed orange line. An example of this is shown below.



EXAMPLE 2 Graphing linear inequalities with vertical or horizontal boundaries

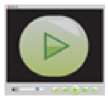
Graph the solution set for each linear inequality on a Cartesian plane.

a) $\{(x, y) \mid x - 2 > 0, x \in \mathrm{R}, y \in \mathrm{R}\}$

b) $\{(x, y) \mid -3y + 6 \geq -6 + y, x \in \mathrm{I}, y \in \mathrm{I}\}$

Your Turn

How can you tell if the boundary of a linear inequality is vertical or horizontal without graphing the linear inequality? Explain.



Answer

If, after simplifying a linear inequality, there is only one variable, you know that the inequality will have a vertical or horizontal boundary. If the variable is x, the boundary will be vertical. If the variable is y, the boundary will be horizontal.

Assignment: pages 221- 222
Questions 2, 4, 5abc, 6bd

SOLUTIONS => 5.1 Graphing Linear Inequalities in Two Variables.

2. Consider the graph of this inequality.
$2x + 3y > 5$.

Make each of the following decisions, and provide your reasoning.

a) whether the boundary should be dashed, stippled, or solid.

Since the inequality excludes the possibility of equality (>), the boundary should be dashed.

b) Whether the half plane above or below the boundary should be shaded.

Test Point (0,0):

L.S.	R.S
$2x+3y$	5
$2(0)+3(0)$	
$0+0$	
0	

Since $0<5$, (0,0) is not in the solution region.

* Since (0,0) is not in the solution region, you would shade the half plane above the boundary.

c) whether each point is in its solution region:

i) (1,1)

L.S	R.S.
$2x+3y$	5
$2(1)+3(1)$	
$2+3$	
5	

Since 5 is not greater than 5, (1,1) is not in the solution region.

ii) (1,0)

L.S	R.S
$2x+3y$	5
$2(1)+3(0)$	
$2+0$	
2	

Since 2 is not greater than 5, (1,0) is not in the solution region.

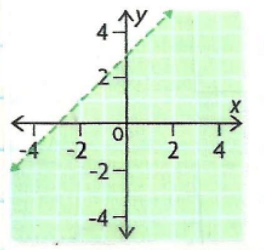
iii) (1,2)

L.S	R.S.
$2x+3y$	5
$2(1)+3(2)$	
$2+6$	
8	

Since 8 is greater than 5, (1,2) is in the solution region.

4. Match each graph with its linear inequality, and justify your match.

a)

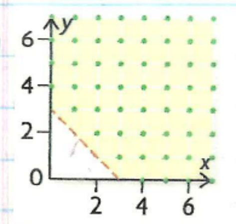


Match: ii) $\{(x,y) \mid x - y > -3, x \in R, y \in R\}$

dashed line

Continuous (not stippled)

b)

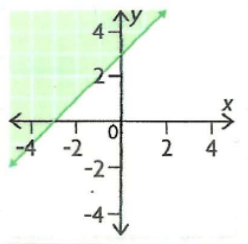


Match: i) $\{(x,y) \mid x - 3 > -y, x \in W, y \in W\}$

dashed line

Discrete (stippled)

c)



Match: iii) $\{(x,y) \mid y - 3 \geq x, x \in R, y \in R\}$

solid line

Continuous (not stippled)

5. Graph the solution set for each linear inequality.

a) $y > -2x + 8$
Dashed line

① Equation of boundary: $y = -2x + 8$

② Boundary's x-int and y-int:

For $y=0$,
$0 = -2x + 8$
$\frac{\cancel{2}x}{\cancel{2}} = \frac{8}{2}$
$x = 4$
x-int => (4,0)

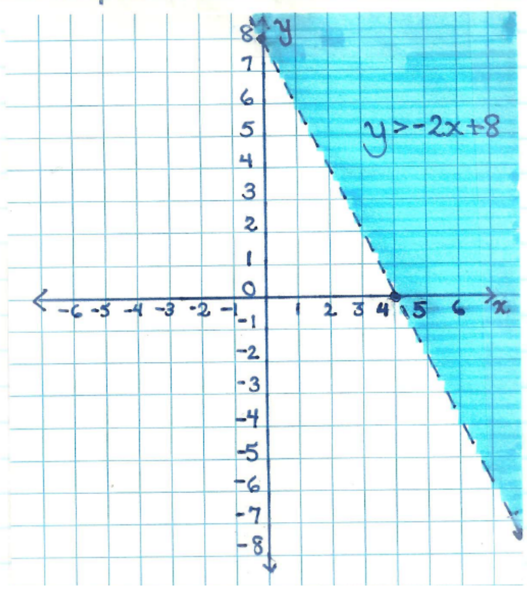
For $x=0$,
$y = -2(0) + 8$
$y = 0 + 8$
$y = 8$
y-int => (0,8)

③ Test Point (0,0):

L.S	R.S.
y	$-2x+8$
0	$-2(0)+8$
	$0+8$
	8

Since $0<8$, (0,0) is not in the solution region.

④ Graph:



b) $-3y \leq 9x + 12$

↓ Solid line

① Equation of boundary: $-3y = 9x + 12$

② Boundary's x-int and y-int:

For $y = 0$,

$$-3(0) = 9x + 12$$
$$0 = 9x + 12$$
$$\frac{-12}{9} = \frac{\cancel{9}x}{\cancel{9}}$$
$$-\frac{4}{3} = x$$
$$-1.3 \doteq x$$

x-int => $(-1.3, 0)$

For $x = 0$,

$$-3y = 9(0) + 12$$
$$-3y = 0 + 12$$
$$\frac{\cancel{-3}y}{\cancel{-3}} = \frac{12}{-3}$$
$$y = -4$$

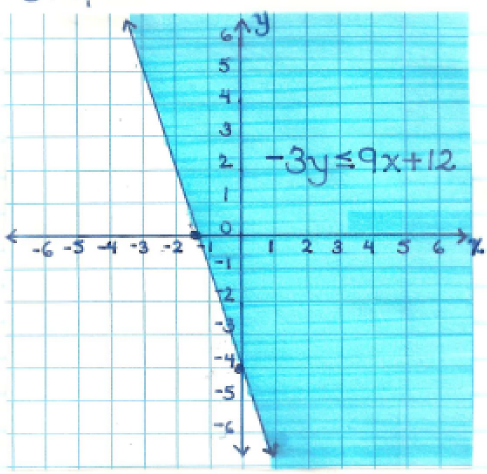
y-int => $(0, -4)$

③ Test Point $(0,0)$:

L.S.	R.S.
$-3y$	$9x+12$
$-3(0)$	$9(0)+12$
0	$0+12$
	12

Since $0 \leq 12$, $(0,0)$ is in the solution region.

④ Graph:



c) $y < 6$
Dashed Line

① Equation of boundary: $y = 6$

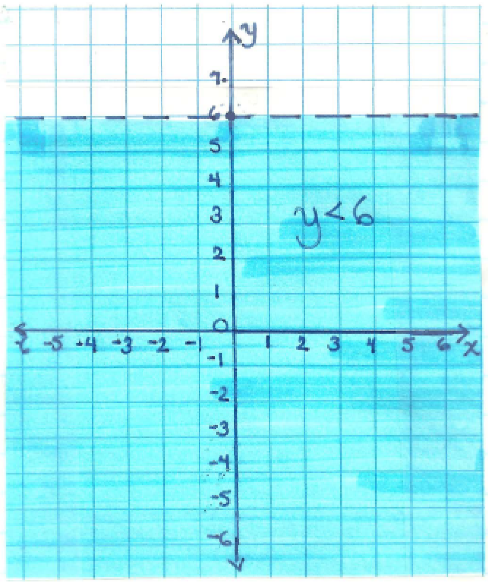
② Skip ⇒ Horizontal Line

③ Test Point (0, 0):

L.S.	R.S.
y	6
0	

Since $0 < 6$, (0,0) is in the solution region.

④ Graph:



6. Graph the solution set for each linear inequality.

b) $\{(x,y) \mid x+6y-14<0,\ x\in I,\ y\in I\}$ * Discrete (Stippled)

Dashed Line

① Equation of boundary: $x+6y-14=0$

② Boundary's x-int and y-int:

For $y=0$:
$x+6(0)-14=0$
$x+0-14=0$
$x-14=0$
$x=14$
x-int => (14, 0)

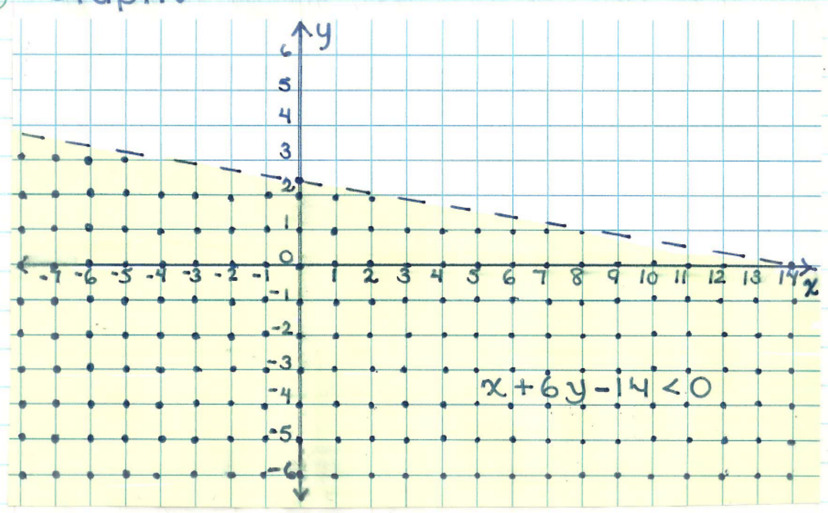
For $x=0$:
$0+6y-14=0$
$6y-14=0$
$\frac{6y}{6}=\frac{14}{6}$
$y=\frac{14}{6}$
$y=\frac{7}{3}$ or 2.3.
y-int => (0, 2.3)

③ Test Point (0,0):

L.S.	R.S.
$x+6y-14$	0
$0+6(0)-14$	
$0+0-14$	
-14	

Since $-14<0$, (0,0) is in the solution region.

④ Graph:



d) $\{(x,y) \mid 2x+2 \le 5+x,\ x \in I,\ y \in I\}$ * Discrete (Stippled)

Solid line

① Equation of boundary: $2x+2=5+x$

$2x-x=5-2$

$x=3$

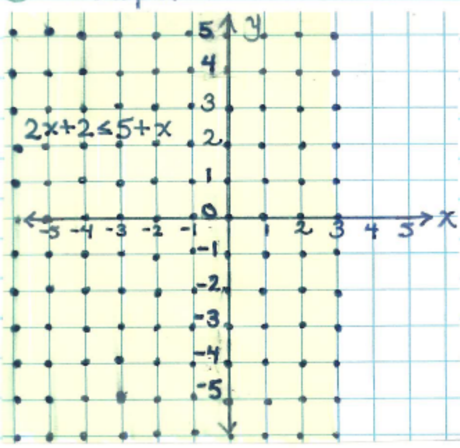
② Skip => Vertical Line

③ Test Point (0,0):

L.S	R.S.
$2x+2$	$5+x$
$2(0)+2$	$5+0$
$0+2$	5
2	

Since $2 \le 5$, (0,0) is located in the solution region.

④ Graph:



fm6s1-p5.tns

6Ws1e1.mp4

6Ws1e2.mp4

6Ws1e3.mp4

fm6s1-p9.tns