5.2 Exploring Graphs of Systems of Linear Inequalities

GOAL

Explore graphs of situations that can be modelled by systems of two linear inequalities in two variables.

EXPLORE the Math

A nursery school serves morning and afternoon snacks to its students. The morning snacks are fruits, vegetables, and juice, and the afternoon snacks are cheese and milk.

- The school can accommodate 50 students or fewer altogether. Students can attend for just the morning or for a full day.
- The morning snack costs \$1 per student per week, and the afternoon snack costs \$2 per student per week.
- The weekly snack budget is \$120 or less.

? What combinations of morning and full-day students can the school accommodate and stay within the weekly snack budget?

$x + 3y \le 120$

- boundary line is solid
- solution is stippled $(x \in W, y \in W)$ can't have half a person

① $x + 3y = 120$

② x-int $(y=0)$

$x + 3(0) = 120$

$x + 0 = 120$

$x = 120$

$(120, 0)$

y-int $(x=0)$

$(0) + 3y = 120$

$\frac{3y}{3} = \frac{120}{3}$

$y = 40$

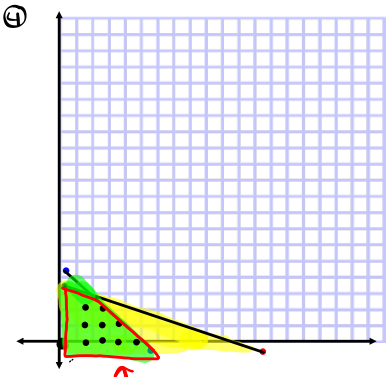
$(0, 40)$

③ $x + 3y \le 120$

LS	RS
$(0) + 3(0)$	120
$0 + 0$	True
0	

since $0 \le 120$ shade on the plane where $(0,0)$ lies

④



$x + y \le 50$

- boundary line is solid
- solution is stippled $(x \in W, y \in W)$ can't have half a person

① $x + y = 50$

② x-int $(y=0)$

$x + (0) = 50$

$x = 50$

$(50, 0)$

y-int $(x=0)$

$(0) + y = 50$

$y = 50$

$(0, 50)$

③ $x + y \le 50$

LS	RS
$(0) + (0)$	50
0	True

Since $0 \le 50$ shade on the plane where $(0,0)$ lies

What combinations of morning and full-day students can the school accommodate and stay within the weekly snack budget?

Sample Solution

First, we represented the two unknowns in the problem using x and y:

- x is the number of morning students.
- y is the number of full-day students.

Then we wrote a linear inequality to represent each part of the problem:

- The total cost of the snacks, as it relates to the number of students, is the sum of the cost of the morning snack multiplied by the number of morning students and the sum of the cost of the afternoon snack multiplied by the number of full-day students. The total cost is \$120 or less:

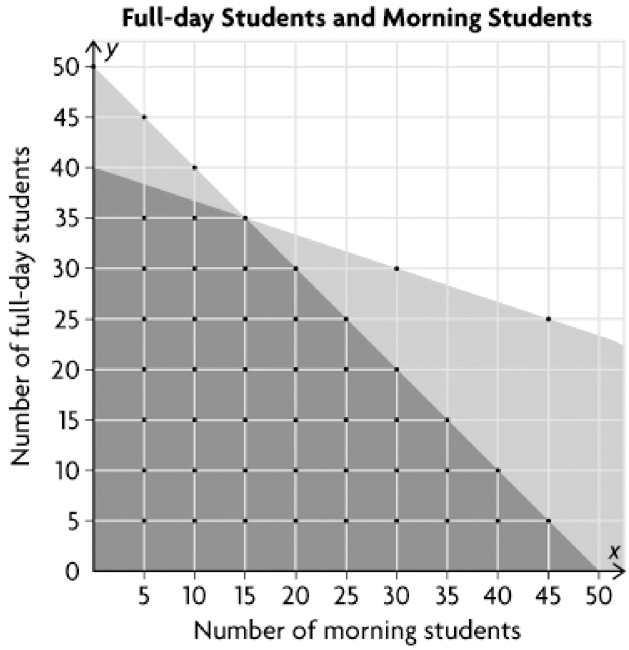
  $x + 3y \leq 120$

- The total number of students can be up to and including 50 students:

  $x + y \leq 50$

Finally, we graphed both linear inequalities on the same coordinate plane.

$x + 3y \leq 120$
$x + y \leq 50$



We knew that the area where the two solution regions intersect or overlap contains points that represent all the possible combinations of morning and full-day students that will work. We also knew that only whole-number points, such as (24, 24) and (8, 36), make sense.

Reflecting

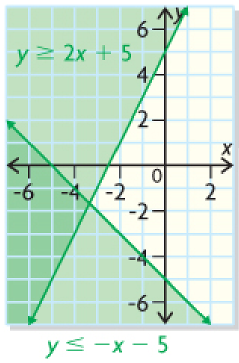
A. In a small group, compare the strategies that could be used to solve the problem. Would each strategy allow you to identify all the possible solutions to the problem? Explain.

B. How could a **system of linear inequalities** represent all the possible solutions?

C. Suggest a possible combination of morning and full-day students that would solve the problem. Explain your choice.

system of linear inequalities

A set of two or more linear inequalities that are graphed on the same coordinate plane; the intersection of their solution regions represents the solution set for the system.



Answers

A. We wrote two inequalities and then tried different values for x and y until we found some that worked for both. This strategy would not identify all possible solutions to the problem. We just found a few solutions that would work because we guessed and tested.

B. The problem has two parts involving the same two variables, and each can be represented by a linear inequality using the same two variables. When you have two or more linear inequalities like this, you have a system of linear inequalities. When you graph the system on one coordinate plane, the area where their solution regions overlap contains all the points that solve both inequalities. You can choose points from this overlapping area as possible solutions to the problem.

C. (30, 20) represents a possible combination of morning and full-day students. I chose this point because it is in the solution region and has whole-number coordinates. As well, it makes sense to have the most students possible, given the conditions of the problem.

Reflecting

D. How would the graph of the system of inequalities change in each situation?

i) The snack budget is \$126 per week.

ii) The school accommodates 30 or fewer students.

iii) The school has 50 or more students, and the weekly snack budget is no more than \$48.

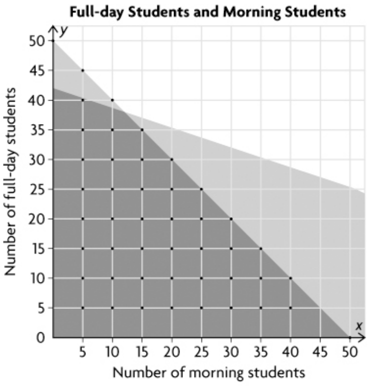
Answers

D. i) $x + 3y \leq 120 \rightarrow x + 3y \leq 126$

The other inequality would stay the same.

The overlapping region would be slightly larger, because the boundary for the cost inequality would move up.

$x + 3y \leq 126$
$x + y \leq 50$

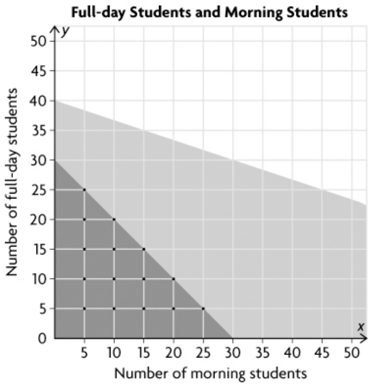


ii) $x + y \leq 50 \rightarrow x + y \leq 30$

The other inequality would stay the same.

The overlapping region would get much smaller, because the boundary for the total students would move down quite a bit.

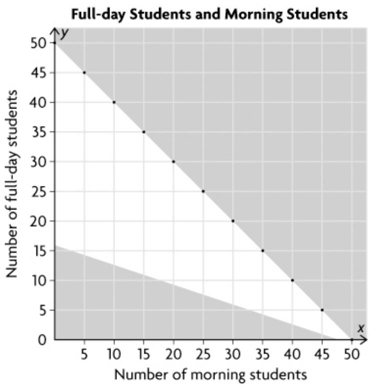
$x + 3y \leq 120$
$x + y \leq 30$



iii) The system of linear inequalities would become $x + 3y \leq 48$ and $x + y \geq 50$.

There would be no overlapping region, so there would be no solution.

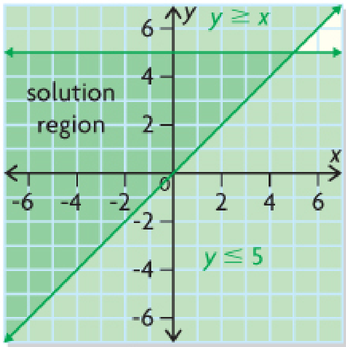
$x + 3y \leq 48$
$x + y \geq 50$



In Summary

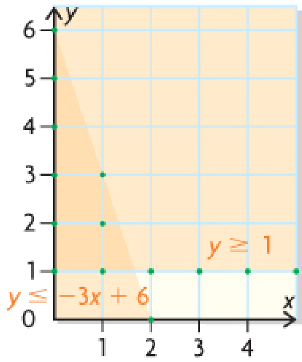
Key Ideas

- Some contextual situations can be modelled by a system of two or more linear inequalities.
- All of the inequalities in a system of linear inequalities are graphed on the same coordinate plane. The region where their solution regions intersect or overlap represents the solution set to the system. For example, this graph shows the solution region to this system:
  $\{(x, y) \mid y \geq x, x \in \mathrm{R}, y \in \mathrm{R}\}$
  $\{(x, y) \mid y \leq 5, x \in \mathrm{R}, y \in \mathrm{R}\}$



Need to Know

- As with the solution region for a single linear inequality, the solution region for a system of linear inequalities can be discrete or continuous and can be restricted to certain quadrants. For example, the graph to the right shows the system described below:
  $\{(x, y) \mid y \geq 1, x \in \mathrm{W}, y \in \mathrm{W}\}$
  $\{(x, y) \mid y \leq -3x + 6, x \in \mathrm{W}, y \in \mathrm{W}\}$
  Its solution region is restricted to discrete points with whole-number coordinates in the first quadrant.
- If the solution regions for the linear inequalities in the system do not overlap, there is no solution.



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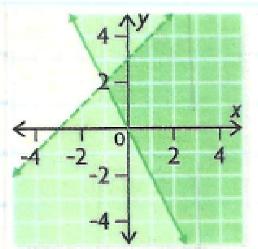
Questions 1 and 2

SOLUTIONS ⇒ 5.2 Exploring Graphs of Systems of Linear Inequalities

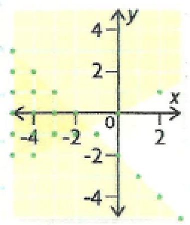
1. Three systems of linear inequalities have been graphed below. For each system, describe what you can infer from the graph about the restrictions on the domain and range.

a) $y \geq -2x$
$-3 < x - y$



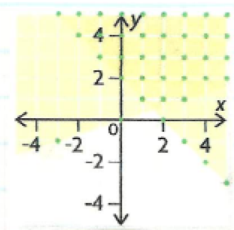
You can infer from this graph that for this system of inequalities ⇒ $x \in R$, $y \in R$.

b) $x+3y \geq 0$
$x+y \geq 2$



You can infer from this graph that for this system of inequalities $\Rightarrow x \in I, y \in I$

c) $x+y \leq -2$
$2y \geq x$



You can infer from this graph that for this system of inequalities $\Rightarrow x \in I, y \in I$.

2. Graph each system of linear inequalities. Justify your representation of the solution set.

a) $\{(x, y) \mid -x + 2y \geq -4, x \in R, y \in R\}$

Solid Line → * Continuous

$\{(x, y) \mid y \geq x, x \in R, y \in R\}$

Solid Line → * Continuous

① Equations of the boundaries:

↳ $-x + 2y = -4$

↳ $y = x$

② Two points on each boundary (x-int & y-int):

↳ $-x+2y=-4$

For $x=0$,
$0+2y=-4$
$\frac{\cancel{2}y}{\cancel{2}}=\frac{-4}{2}$
$y=-2$
y-int => -2

For $y=0$,
$-x+2(0)=-4$
$\frac{-x}{-1}=\frac{-4}{-1}$
$x=4$
x-int => 4.

↳ $y=x$ (O.K)
* Diagonal line which passes through $(-1,-1)$, $(0,0)$, $(1,1)$ etc.

*Since (0,0) is on the line

③ Test Point (0,0):

L.S.	R.S
$-x+2y$	-4
$0+2(0)$	
$0+0$	
0	

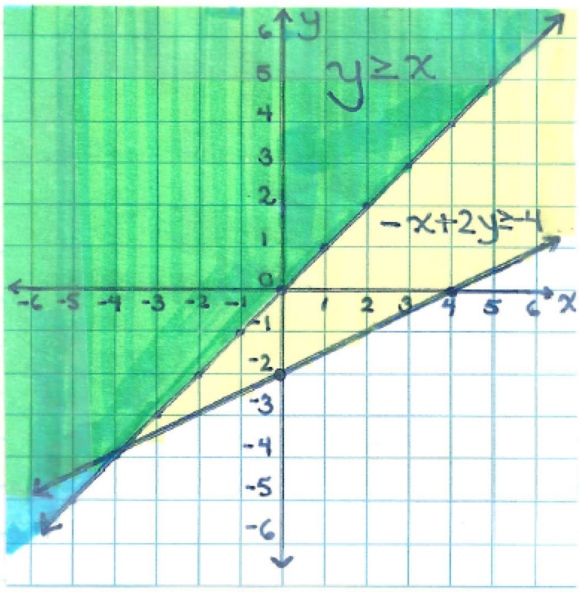
Since $0 \geq 4$, (0,0) is located in the solution region.

Test Point (0,1):

L.S	R.S
y	x
1	0

Since $1>0$, (0,1) is located in the solution region

④ Graph:



b) $\{(x,y) \mid 2x+3y \leq 9, x \in I, y \in I\}$ * Discrete (Stippled)

Solid line

$\{(x,y) \mid y-6x \geq 1, x \in I, y \in I\}$ * Discrete (Stippled)

Solid line

① Equations of the boundaries:

↳ $2x+3y=9$ ↳ $y-6x=1$

② Two points on each boundary (x-int & y-int):

↳ $2x+3y=9$

For $x=0$,
$2(0)+3y=9$
$0+3y=9$
$\frac{\cancel{3}y}{\cancel{3}}=\frac{9}{3}$
$y=3$
y-int => 3

For $y=0$,
$2x+3(0)=9$
$\frac{\cancel{2}x}{\cancel{2}}=\frac{9}{2}$
$x=4.5$
x-int => 4.5

↳ $y-6x=1$

For $x=0$,
$y-6(0)=1$
$y-0=1$
$y=1$
y-int => 1

For $y=0$,
$0-6x=1$
$\frac{\cancel{-6}x}{\cancel{-6}}=\frac{1}{-6}$
$x=-0.17$
x-int => -0.17

③ Test Point (0,0):

L.S	R.S
$2x+3y$	9
$2(0)+3(0)$	
$0+0$	
0	

Since $0 \leq 9$, (0,0) is located in the solution region.

Test Point (0,0):

L.S	R.S.
$y-6x$	1
$0-6(0)$	
$0-0$	
0	

$0 < 1$, therefore (0,0) is not located in the solution region.

④ Graph: