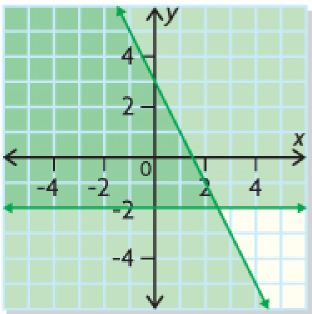
5.3 Graphing to Solve Systems of Linear Inequalities

GOAL

Solve problems by modelling systems of linear inequalities.

EXPLORE...

- What conclusions can you make about the system of linear inequalities graphed below?



SAMPLE ANSWER

Any or all of the following solutions are acceptable:

- It represents a system of two linear inequalities, each with a straight boundary and a solution region.
- One linear inequality is $y \leq -2x + 3$, and the horizontal inequality is $y \geq -2$. I determined $y \leq -2x + 3$ using the slope and y-intercept and the form $y = mx + b$, and I was able to identify $y \geq -2$ because it's a horizontal line through -2 on the y-axis.
- Both inequalities include the possibility of equality because the boundaries are solid.
- The solution set of the system is represented by the overlapping region because it's where the solution regions for the two linear inequalities overlap. The solution set includes points along the boundaries of the overlap.
- The domain and range are from the set of real numbers because the solution region is green and not stippled.
- All four quadrants are included so there are no restrictions on the set of real numbers.

LEARN ABOUT the Math

A company makes two types of boats on different assembly lines: aluminum fishing boats and fibreglass bow riders.

- When both assembly lines are running at full capacity, a maximum of 20 boats can be made in a day.
- The demand for fibreglass boats is greater than the demand for aluminum boats, so the company makes at least 5 more fibreglass boats than aluminum boats each day.

What combinations of boats should the company make each day?

EXAMPLE 1 Solving a problem with discrete whole-number variables using a system of inequalities

Mary's Solution: Using graph paper

Let x represent the number of aluminum fishing boats.
Let y represent the number of fibreglass bow riders.

$x \in W$ and $y \in W$

I knew I could solve this problem by representing the situation algebraically with a system of two linear inequalities and graphing the system.

Since only complete boats are sold, I knew that x and y are whole numbers and the graph would consist of discrete points in the first quadrant.

The relationship between the two types of boats can be represented by this system of inequalities:

$x + y \leq 20$

$x + 5 \leq y$

The two inequalities describe

- a combination of boats to a maximum of 20.
- at least 5 more fibreglass boats than aluminum boats.

$x + y = 20$

y-intercept:	x-intercept:
$0 + y = 20$	$x + 0 = 20$
$y = 20$	$x = 20$
$(0, 20)$	$(20, 0)$

$x + 5 = y$

y-intercept:	x-intercept:
$0 + 5 = y$	$x + 5 = 0$
$y = 5$	$x = -5$
$(0, 5)$	$(-5, 0)$

To graph each linear inequality, I knew I had to graph its boundary as a stippled line, and then shade and stipple the correct half plane.

To graph each boundary, I wrote each linear equation and then determined the x- and y-intercepts so I could plot and join them.

$x + 5 = y$

$(5) + 5 = y$

$10 = y$

(5, 10) is a point on this boundary.

For $x + 5 = y$, I knew $(-5, 0)$ wasn't going to be a point on the boundary, because it's not in the first quadrant, so I chose another point by solving the equation for $x = 5$.

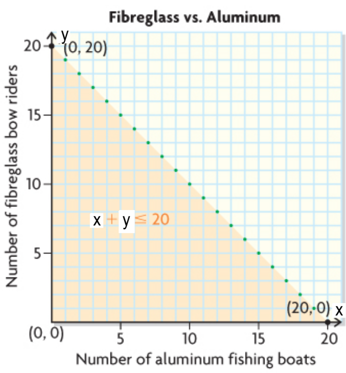
Test (0, 0) in $x + y \leq 20$.

LS	RS
$x + y$	20
$0 + 0$	
0	

True

Since $0 \leq 20$, (0, 0) is in the solution region.

I tested point (0, 0) to determine which half plane to shade for $x + y \leq 20$.



I drew a green stippled boundary connecting (0, 20) and (20, 0) and shaded the half plane below it orange, because the solution region is discrete.

Test (0, 0) in $x + 5 \leq y$

LS	RS
$x + 5$	y
$0 + 5$	0
5	

False

Since 5 is not less than or equal to 0, (0, 0) is not in the solution region.

I tested (0, 0) to determine which half plane to shade for $x + 5 \leq y$

LEARN ABOUT the Math

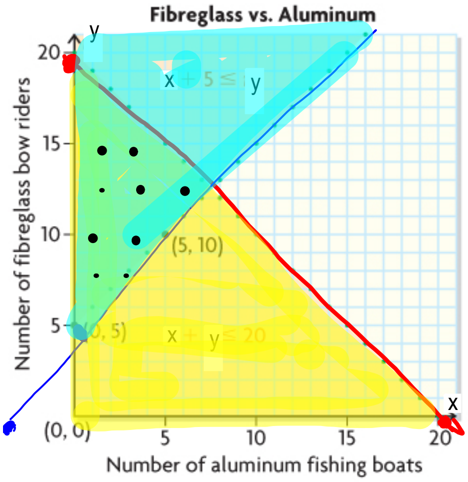
A company makes two types of boats on different assembly lines: aluminum fishing boats and fibreglass bow riders.

- When both assembly lines are running at full capacity, a maximum of 20 boats can be made in a day.
- The demand for fibreglass boats is greater than the demand for aluminum boats, so the company makes at least 5 more fibreglass boats than aluminum boats each day.

❓ What combinations of boats should the company make each day?

EXAMPLE 1 Solving a problem with discrete whole-number variables using a system of inequalities

Mary's Solution: Using graph paper

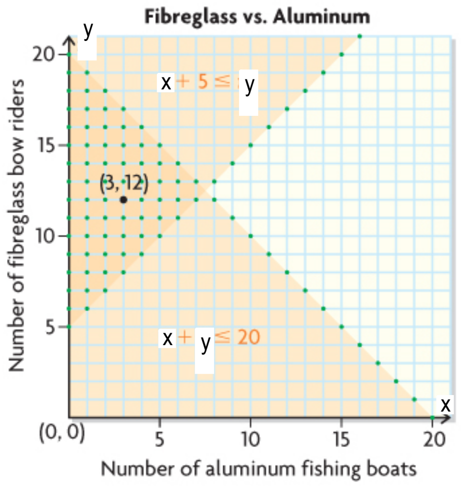


I plotted the points (0, 5) and (5, 10) on the same coordinate plane. I used these points to draw a green stippled boundary for $x + 5 \le y$

I shaded the half plane above the boundary orange, since the test point (0, 0) is not a solution to the linear inequality and the solution region is discrete.

I knew that the solution set for the system of linear inequalities is represented by the intersection or overlap of the solution regions of the two inequalities. This made sense since points in this region satisfy both inequalities.

I knew that the triangular solution region included discrete points along its three boundaries, including the y-axis from $y = 5$ to $y = 20$.



Since the solution set for the system contains only discrete points with whole-number coordinates, I stippled its solution region.

I knew that any whole-number point in the triangular solution region is a possible solution. For example, (3, 12) is a possible solution.

$\{(x, y) \mid x + y \le 20, x \in W, y \in W\}$

$\{(x, y) \mid x + 5 \le y, x \in W, y \in W\}$

Any point with whole-number coordinates in the intersecting or overlapping region is an acceptable combination. For example, 3 aluminum boats and 12 fibreglass boats is an acceptable combination.

I knew that (3, 12) worked because this gives a total of 15 boats with 9 more fibreglass boats than aluminum boats.

Reflecting

A. Is every point on the boundaries of the solution region a possible solution? Explain.

B. Are the three points where the boundaries intersect part of the solution region? Explain.

C. How would the graph change if fewer than 25 boats were made each day?

D. All points with whole-number coordinates in the solution region are valid, but are they all reasonable? Explain.

Answers

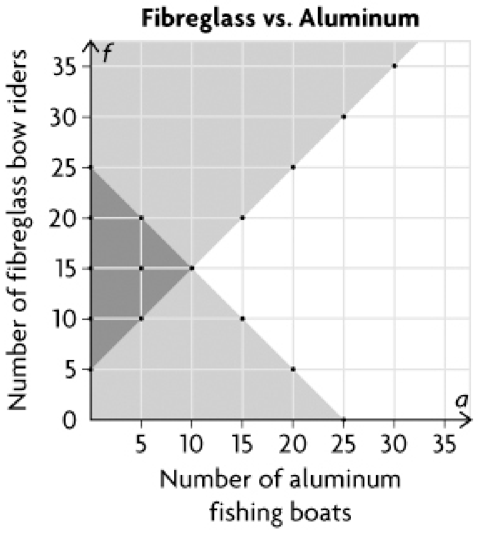
A. No. Only whole-number coordinate points on the boundaries are part of the solution region, because the variables represent numbers of boats and only whole numbers of boats make sense.

B. Yes. Equality is possible for both inequalities, and all of these points have whole-number coordinates: (0, 5), (5, 10), and (0, 20).

C. $x + y \leq 20 \rightarrow x + y \leq 25$

The solution region would be larger, because its boundary would move up.

$x + y \leq 25$
$y \geq x + 5$



D. This would depend on the market. For example, if there was a high demand for boats, then points in the solution region with high coordinates, such as (7, 13), would probably make more sense. If there was a low demand for fishing boats, then points with low x-coordinates, such as (0, 20), would make more sense.

- $3x + 2y > -6 \quad \{x \in R, y \in R\}$ dashed line
- $y \leq 3 \quad \{x \in R, y \in R\}$ solid line

① $3x + 2y = -6$

② x int $(y=0)$ | y int $(x=0)$

$3x + 2(0) = -6$ | $3(0) + 2y = -6$

$3x + 0 = -6$ | $0 + 2y = -6$

$3x = -6$ | $2y = -6$

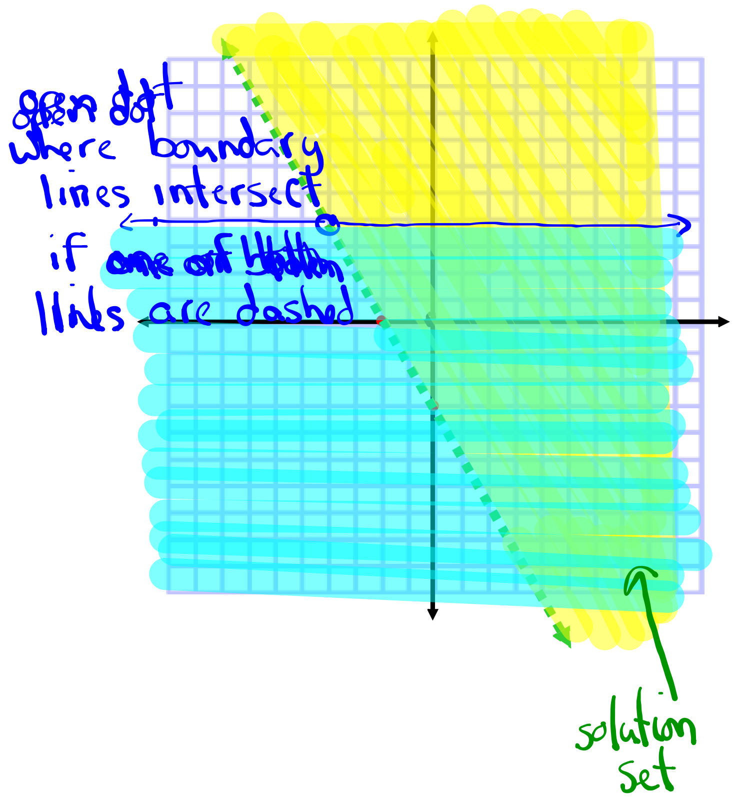
$x = -2$ | $y = -3$

$(-2, 0)$ | $(0, -3)$

③ $3x + 2y > -6$

LS	RS
$3(0) + 2(0)$	-6
$0 + 0$	
0	True

Since $0 > -6$
$(0,0)$ is in the solution set



① $y = 3$

② Skip

③ Skip

APPLY the Math

EXAMPLE 2 Solving graphically a system of two linear inequalities with continuous variables

Graph the solution set for the following system of inequalities. Choose two possible solutions from the set.

$3x + 2y > -6$

$y \leq 3$

Peter's Solution: Using graph paper

$x \in \mathrm{R}, y \in \mathrm{R}$

I assumed both x and y are in the set of real numbers because restrictions on the domain and range were not stated. I knew the graph would have a continuous solution region and could be in all four quadrants.

$3x + 2y > -6$

x-intercept:	y-intercept:
$3x + 2(0) = -6$	$3(0) + 2y = -6$
$\frac{3x}{3} = -\frac{6}{3}$	$\frac{2y}{2} = -\frac{6}{2}$
$x = -2$	$y = -3$
$(-2, 0)$	$(0, -3)$

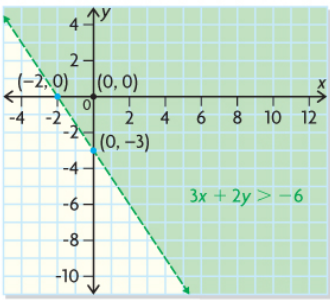
To graph $3x + 2y > -6$, I identified the x- and y-intercepts of the linear equation of the boundary $3x + 2y = -6$.

Test $(0, 0)$ in $3x + 2y > -6$.

LS	RS
$3x + 2y$	-6
$3(0) + 2(0)$	
0	

Since $0 > -6$,
$(0, 0)$ is in the solution region.

I used the test point $(0, 0)$ to determine which region to shade.

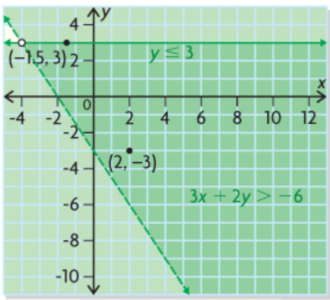


I drew a dashed green line for the boundary since the $>$ sign does not include the possibility of equality and the solution set is continuous.

I shaded the half plane that included $(0, 0)$, since $(0, 0)$ is a solution to the linear inequality. I used green shading to show a continuous solution region.

$y \leq 3$

I knew that I should draw a solid horizontal green boundary because the inequality has one variable, y, the sign is $\leq$ and the solution set is continuous.



I shaded the half plane below the boundary, since all the points in this region have y-coordinates that are less than 3.

Where the solid and dashed boundaries intersect, I drew an open dot to show that this point is not part of the solution region. It made sense that the intersection point is not included because none of the points on the boundary of $3x + 2y > -6$ are included in its solution region.

I knew that all the points in the overlapping solution region, which included points along its solid boundary, represented the solution set, because x and y are in the set of real numbers.

The overlapping solution region represents the solution set of the system of linear inequalities. Therefore, $(2, -3)$ and $(-1.5, 3)$ are two possible solutions.

Any point in the solution region is a possible solution.

EXAMPLE 2 Solving graphically a system of two linear inequalities with continuous variables

Graph the solution set for the following system of inequalities. Choose two possible solutions from the set.

$3x + 2y > -6$

$y \leq 3$

Your Turn

How would the solution region change if $x \in \mathrm{I}$ and $y \in \mathrm{I}$? How would it stay the same?

Answer

The solution region would have the same shape and size, and cover the same four quadrants, but it would include only discrete points with integer coordinates inside the solution region and along the boundary for $y \leq 3$. If graphing by hand, it would be stippled to show this. The stippling would include the boundary of $y \leq 3$.

In Summary

Key Ideas

- When graphing a system of linear inequalities, the boundaries of its solution region may or may not be included, depending on the types of linear inequalities ($\geq$, $\leq$, $<$, or $>$) in the system.
- Most systems of linear inequalities representing real-world situations are restricted to the first quadrant because the values of the variables in the system must be positive.

Need to Know

- Any point in the solution region for a system is a valid solution, but some solutions may make more sense than others depending on the context of the problem.
- You can validate a possible solution from the solution region by checking to see if it satisfies each linear inequality in the system.
  For example, to validate if (2, 2) is a solution to the system:
  $x + y \geq 1$
  $2 > x - 2y$

Validating (2, 2) for $x + y \geq 1$:

LS	RS
$x + y$	1
$2 + 2$	
4	

$4 \geq 1$ valid

Validating (2, 2) for $2 > x - 2y$:

LS	RS
2	$x - 2y$
	$2 - 2(2)$
	-2

$2 > -2$ valid

- Use an open dot to show that an intersection point of a system's boundaries is excluded from the solution set. An intersection point is excluded when a dashed line intersects either a dashed or solid line.
- Use a solid dot to show that an intersection point of a system's boundaries is included in the solution set. This occurs when both boundary lines are solid.

Assignment: **page 235 - 236**
Questions 1a, 3, 4c, 7ab

SOLUTIONS ⇒ 5.3 Graphing to Solve Systems of Linear Inequalities.

1. Graph the solution set for each system of inequalities. Determine a solution. Check its validity.

a) $x+y \geq 2$ ↳ Solid line

$x < 4$. ↳ dashed line

* Assume $x \in R, y \in R$ ↳ Continuous

① Equations of the boundaries:

↳ $x+y=2$

↳ $x=4$

② Two points on each boundary (x-int & y-int):

↳ $x+y=2$

For $x=0$, $0+y=2$, $y=2$, y-int ⇒ 2

For $y=0$, $x+0=2$, $x=2$, x-int ⇒ 2

↳ $x=4$ (O.K)

* Vertical line

③ Test Point (0,0):

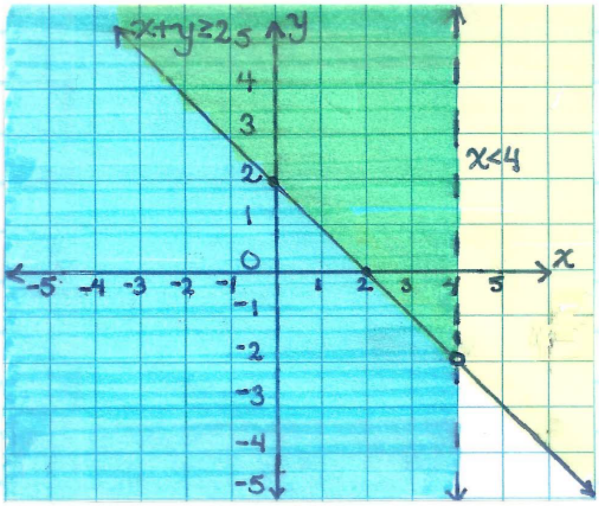
L.S	R.S
$x+y$	2
$0+0$	
0	

Since $0<2$, (0,0) is not located in the solution region

Test point not required!

↳ Shade to the left of the line.

④ Graph:



⑤ Solution:

For example => (2, 3)

⑥ Verifying Solution:

For $x+y \geq 2$:

L.S	R.S
$x+y$	2
$2+3$	
5	

$5 \geq 2$ Valid

For $x<4$:

L.S	R.S
2	4

$2<4$ Valid

3. For each system of linear inequalities, explain whether the boundaries and their points of intersection are part of the solution region.

a) $\{(x,y) \mid y \geq -2x, x \in R, y \in R\}$

$\{(x,y) \mid -3 < x-y, x \in R, y \in R\}$

The boundary line $y \geq -2x$ would be included in the solution region. The boundary line $-3 < x-y$ would not be included in the solution region. Since a solid line would be intersecting a dashed line, the intersection point would not be included in the solution region.

b) $\{(x, y) \mid x + y \leq -2, x \in I, y \in I\}$

$\{(x, y \mid 2y \geq x, x \in I, y \in I\}$

The points with integer coordinates on the boundary $x + y \leq -2$ would be included in the solution region. The points with integer coordinates on the boundary $2y \geq x$ would also be included in the solution region. The point of intersection would only be included in the solution region if it had integer coordinates. (You would need to graph to find out!)

c) $\{(x,y) \mid x+3y \geq 0, x \in I, y \in I\}$

$\{(x,y) \mid x+y > 2, x \in I, y \in I\}$

The points with integer coordinates on the boundary $x+3y \geq 0$ would be included in the solution region. The boundary $x+y > 2$ would not be included in the solution region and as such neither would the point of intersection.

4. Graph each system. Determine a solution for each.

c) $\{(x,y) \mid 3y-2x \leq 6,\ x \in N,\ y \in N\}$ * Discrete (Stippled)

Solid

$\{(x,y) \mid 2y-3x \leq 6,\ x \in N,\ y \in N\}$ * Discrete (Stippled)

Solid.

① Equations of the boundaries:

↳ $3y-2x=6$ ↳ $2y-3x=6$

② Two points on each boundary (x-int & y-int):

↳ $3y-2x=6$

For $x=0$,
$3y-2(0)=6$
$\frac{\cancel{3}y}{\cancel{3}}=\frac{6}{3}$
$y=2.$
y-int ⇒ 2

For $y=0$,
$3(0)-2x=6$
$\frac{-\cancel{2}x}{-\cancel{2}}=\frac{6}{-2}$
$x=-3$
x-int ⇒ -3

↳ $2y-3x=6$

For $x=0$,
$2y-3(0)=6$
$\frac{\cancel{2}y}{\cancel{2}}=\frac{6}{2}$
$y=3$
y-int ⇒ 3

For $y=0$,
$2(0)-3x=6$
$\frac{-\cancel{3}x}{-\cancel{3}}=\frac{6}{-3}$
$x=-2.$
x-int ⇒ -2

③ Test Point (0,0):

L.S	R.S.
$3y-2x$	6
$3(0)-2(0)$	
$0-0$	
0	

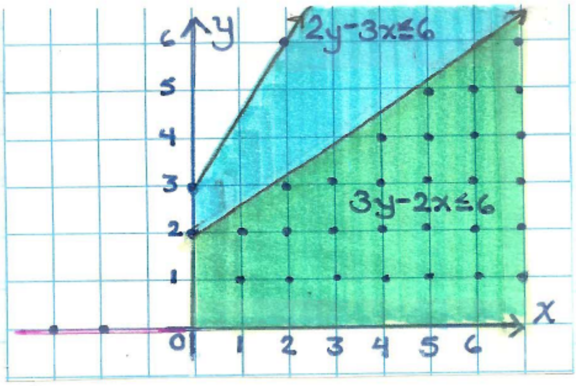
Since $0 \leq 6$, (0,0) is located in the solution region.

Test Point (0,0):

L.S.	R.S.
$2y-3x$	6
$2(0)-3(0)$	
$0-0$	
0	

Since $0 \leq 6$, (0,0) is located in the solution region.

④ Graph:



⑤ Solution:

For example ⇒ (5,3)

7a) Graph the solution set for the following system of inequalities. Determine a solution. Check its validity.

$9x + 18y < 18$ Dashed

$3x - 6y \leq 18$ Solid

*Assume $x \in R, y \in R$ ↳ Continuous

① Equations of the boundaries:

↳ $9x + 18y = 18$

↳ $3x - 6y = 18$

② Two points on each boundary (x-int & y-int):

↳ $9x + 18y = 18$

For $x=0$,	For $y=0$,
$9(0) + 18y = 18$	$9x + 18(0) = 18$
$0 + 18y = 18$	$9x + 0 = 18$
$\frac{\cancel{18}y}{\cancel{18}} = \frac{18}{18}$	$\frac{\cancel{9}x}{\cancel{9}} = \frac{18}{9}$
$y = 1$	$x = 2$
y-int => 1	x-int => 2

↳ $3x - 6y = 18$

For $x=0$,	For $y=0$,
$3(0) - 6y = 18$	$3x - 6(0) = 18$
$0 - 6y = 18$	$3x - 0 = 18$
$\frac{-\cancel{6}y}{-\cancel{6}} = \frac{18}{-6}$	$\frac{\cancel{3}x}{\cancel{3}} = \frac{18}{3}$
$y = -3$	$x = 6$
y-int => -3	x-int => 6

③ Test Point (0,0):

L.S.	R.S.
$9x+18y$	18
$9(0)+18(0)$	
$0+0$	
0	

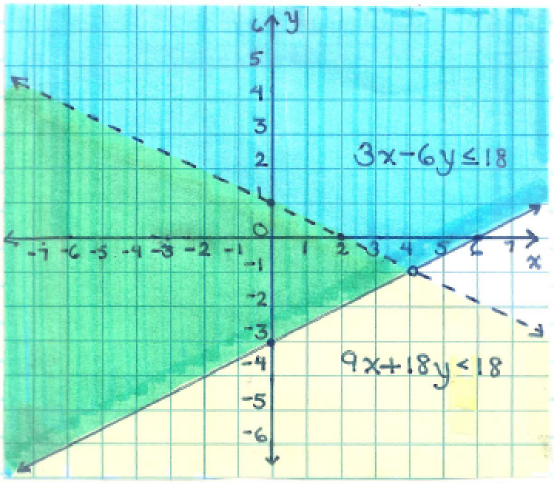
Since $0<18$, $(0,0)$ is located in the solution region.

Test Point (0,0):

L.S	R.S.
$3x-6y$	18
$3(0)-6(0)$	
$0-0$	
0	

Since $0\le 18$, $(0,0)$ is located in the solution region.

④ Graph:



b) Is each point below a possible solution to the system? How do you know?

i) $(4,-1) \Rightarrow$ No, not located in the solution region.

ii) $(-2,2) \Rightarrow$ No, not located in the solution region.

iii) $(-4,-2) \Rightarrow$ Yes, located in the solution region.

iv) $(9,1) \Rightarrow$ No, not located in the solution region.

v) $(-2.5,-1.5) \Rightarrow$ Yes, located in the solution region.

vi) $(2,-2) \Rightarrow$ Yes, located in the solution region.

Attachments

fm6s3-p7.tns

fm6s3-p4.tns

6Ws3e2.mp4

6Ws3e3.mp4