SOLUTIONS => 4.3 The Ambiguous Case of the Sine Law (WORKBOOK)

1. Determine whether each description of a triangle involves the SSA situation.

a) In ΔABC, $\angle A = 13°$, $a = 2cm$, and $b = 6cm$.

This is a SSA situation.

b) In ΔDEF, $\angle D = 89°$, $d = 14cm$, and $f = 11cm$

$\angle A = 89°$, $a = 14cm$, and $b = 11cm$

This is a SSA situation.

c) In ΔPQR, $\angle P = 38°$, $q = 27cm$, and $r = 19cm$.

This is not a SSA situation. (SAS)

3. Calculate the height of each triangle, to the nearest tenth of a centimeter. Determine the number of triangles that are possible. Give your reasons.

a) In ΔRST, $\angle R = 103°$, $r = 16$ cm, and $s = 9$cm.

$\angle A = 103°$, $a = 16$cm, and $b = 9$cm

① This is a SSA situation. (Matching Pair).

② We can skip this step since $\angle A$ is obtuse.

③ Since $\angle A$ is obtuse and $a > b$, one obtuse triangle is possible.

b) In ΔXYZ, $\angle X = 50°$, $x = 5.2$ cm, and $z = 7.1$ cm.

$\angle A = 50°$, $a = 5.2$cm, and $b = 7.1$cm

① This is a SSA situation. (Matching Pair).

② $h = b \sin A$

$h = 7.1 \sin 50°$

$h = 5.4$ cm

③ Since $\angle A$ is acute and $a < h$, no triangle is possible.

c) In ΔABC, $\angle A = 74°$; $a = 28.0$cm, and $b = 28.9$cm

① This is a SSA situation. (Matching Pair).

② $h = b \sin A$

$h = 28.9 \sin 74°$

$h = 27.8$ cm

③ Since $\angle A$ is acute and $h < a < b$, two triangles are possible.

(two possible answers)

4. Given each set of measurements for ΔABC, determine the number of triangles that are possible. Draw a diagram to support your answer.

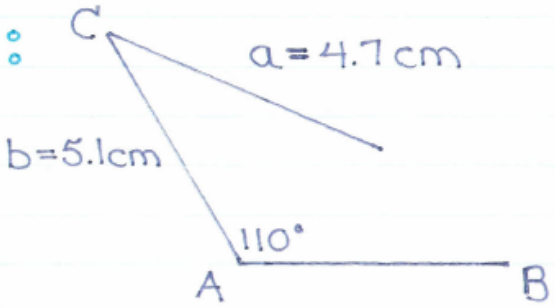
a) $\angle A = 110°$, $a = 4.7$cm, and $b = 5.1$cm.

① This is a SSA situation. (Matching Pair).

② We can skip this step since $\angle A$ is obtuse.

③ Since $\angle A$ is obtuse and $a < b$, no triangle is possible.

Diagram:



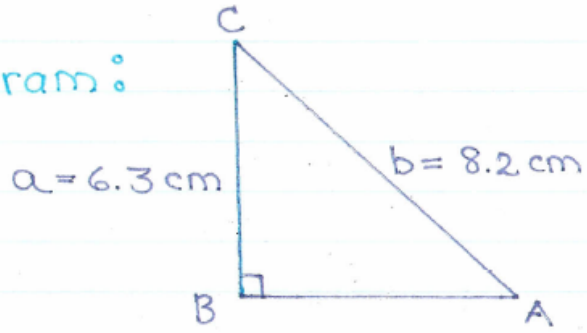
b) $\angle A = 50°$, $a = 6.3\text{cm}$, and $b = 8.2\text{cm}$.

① This is a SSA situation. (Matching Pair)

② $h = b \sin A$
$h = 8.2 \sin 50°$
$h \doteq 6.3\text{ cm}$

③ Since $\angle A$ is acute and $a = h$, one right triangle is possible.

Diagram:



⑤ In $\triangle XYZ$ $\angle X = 67°$, $x = 3.2m$ and $y = 3.4m$ /

$\angle A = 67°$, $a = 3.2m$ and $b = 3.4m$

a) Find $\angle Y$ ($\angle B$)

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin B}{3.4} = \frac{\sin 67°}{3.2}$$

$$\frac{3.2 \sin B}{3.2} = \frac{3.4 \sin 67°}{3.2}$$

$$\sin B = 0.9780$$

$$\angle B = \sin^{-1}(0.9780)$$

$$\boxed{\angle B = 78°}$$

b) State another possible value for $\angle B$

$$\sin\theta = \sin(180° - \theta)$$
$$\sin 78° = \sin(180° - 78°)$$
$$\sin 78° = \sin 102°$$

$\angle B = 78°$ or $102°$

c) $\angle A = 67°$, $a = 3.2m$ and $b = 3.4m$

① STEP 1: SSA

② STEP 2: $h = b \sin A$
$h = 3.4 \sin 67°$
$h = 3.13m$

③ STEP 3: Since $\angle A$ is acute and / /
$h < a < b$ so there are
2 possible triangles so both
answers are correct

5. In ΔXYZ, $\angle X = 67°$, $x = 3.2$m, and $y = 3.4$m

$\angle A = 67°$, $a = 3.2$m, and $b = 3.4$m

a) Determine a possible measure for $\angle Y$, to the nearest degree.

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin B}{3.4} = \frac{\sin 67°}{3.2}$$

$$\frac{3.2 \sin B}{3.2} = \frac{3.4 \sin 67°}{3.2}$$

$$\sin B = 0.9780$$

$$B = \sin^{-1}(0.9780)$$

$$B = 78°$$

b) State another possible value for $\angle B$. Give one reason why this is a possible value.

$\sin B = \sin(180° - B)$
$\sin B = \sin(180° - 78°)$
$\sin B = \sin 102°$

Since $\angle A$ is acute, $\angle B$ could also be obtuse. (102°)

c) Which of these possible values for $\angle B$ corresponds to a possible triangle? Explain.

$h = b \sin A$

$h = 3.4 \sin 67°$
$h = 3.4(0.9205)$
$h = 3.1$ m

Since $\angle A$ is acute and $h < a < b$, two triangles are possible. Therefore both values for $\angle B$ correspond to possible triangles.

MULTIPLE CHOICE

6. In ΔMNP, $n = 4.5$ cm and $\angle M = 35°$. What is the height of the triangle from base p?

b = 4.5 cm <A = 35°

c

$h = b \sin A$ Option => "C"

$h = 4.5 \sin 35°$
$h = 4.5(0.5736)$
$h = 2.6$ cm

7. In ΔPQR, $\angle P = 108°$, $q = 4.9m$, and $p = 4.5m$.

$\angle A = 108°$, $b = 4.9m$, and $a = 4.5m$

Which statement is true for this set of measurements?

Since $\angle A$ is obtuse and $a < b$, no triangle is possible.

Option => "B" This is an SSA situation; no triangle is possible.

8. Which set of measurements can produce two possible triangles?

* We need to check option "A" and option "B". The solution cannot be option "C" since $\angle A$ is obtuse.

"A"

$h = b\sin A$
$h = 4.0\sin 62°$
$h = 4.0(0.8829)$
$h = 3.5$ m

Since $\angle A$ is acute and $a < h$, no triangle is possible.

"B"

$h = b\sin A$
$h = 7.4\sin 52°$
$h = 7.4(0.7880)$
$h = 5.8$ m

Since $\angle A$ is acute and $h < a < b$, two triangles are possible.

Option => "B"

9. In obtuse ΔABC, $\angle B = 24°$, $b = 18$ cm, and $a = 22$ cm. Calculate the measure of $\angle A$, to the nearest degree. Is there more than one possible answer? Explain how you know.

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin A}{22} = \frac{\sin 24°}{18}$$

$$\frac{18 \sin A}{18} = \frac{22 \sin 24°}{18}$$

$$\sin A = 0.4971$$

$$A = \sin^{-1}(0.4971)$$

$$A = 30°$$

$\sin 30° = \sin(180° - 30°)$

$\sin 30° = \sin 150°$

$\angle A$ could be 30° or 150° since there are 2 possible triangles.

$h = b \sin A$

↳ $h = a \sin B$

$h = 22 \sin 24°$

$h = 22(0.4067)$

$h = 8.9$ cm

($h < b < a$; 2 possible triangles)

<A=97°, a=5.3m, and b=4.8m

10. In ΔDEF, $\angle D = 97°$; $d = 5.3cm$, and $e = 4.8cm$. Calculate the measure of $\angle E$, to the nearest degree. Is there more than one possible answer? Explain how you know.

<B

$$\frac{\sin B}{b} = \frac{\sin A}{a}$$

$$\frac{\sin B}{4.8} = \frac{\sin 97°}{5.3}$$

$$\frac{5.3 \sin B}{5.3} = \frac{4.8 \sin 97°}{5.3}$$

$$\sin B = 0.8989$$

$$B = \sin^{-1}(0.8989)$$

$$B = 64°$$

Since $\angle A$ is obtuse, and $a > b$, one obtuse triangle is possible.