Radical Functions and Transformations

Focus on...

- investigating the function $y = \sqrt{x}$ using a table of values and a graph
- graphing radical functions using transformations
- identifying the domain and range of radical functions

radical function

- a function that involves a radical with a variable in the radicand
- $y = \sqrt{3x}$ and $y = 4\sqrt[3]{5 + x}$ are radical functions.

Example 1

Graph Radical Functions Using Tables of Values

Use a table of values to sketch the graph of each function. Then, state the domain and range of each function.

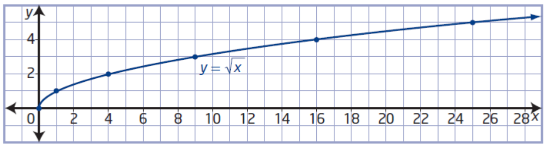
a) $y = \sqrt{x}$ **b)** $y = \sqrt{x - 2}$ **c)** $y = \sqrt{x} - 3$

a) For the function $y = \sqrt{x}$, the radicand x must be greater than or equal to zero, $x \geq 0$.

D: $x \geq 0$

x	y
0	0
1	1
4	2
9	3
16	4
25	5

How can you choose values of x that allow you to complete the table without using a calculator?



The graph has an endpoint at (0, 0) and continues up and to the right. The domain is $\{x \mid x \geq 0, x \in R\}$. The range is $\{y \mid y \geq 0, y \in R\}$.

$[0, \infty)$

b) For the function $y = \sqrt{x - 2}$, the value of the radicand must be greater than or equal to zero.

$h = 2$

D: $x - 2 \geq 0$

$x \geq 2$

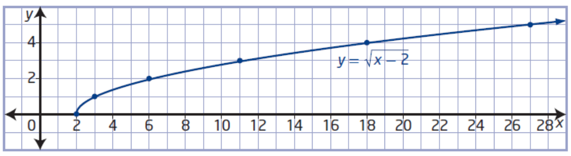
$(x, y) \rightarrow (x + 2, y)$

x	y
2	0
3	1
6	2
11	3
18	4
27	5

How is this table related to the table for $y = \sqrt{x}$ in part a)?

translated 2 units right

How does the graph of $y = \sqrt{x - 2}$ compare to the graph of $y = \sqrt{x}$?



The domain is $\{x \mid x \geq 2, x \in R\}$. The range is $\{y \mid y \geq 0, y \in R\}$.

$[2, \infty)$ $[0, \infty)$

c) The radicand of $y = \sqrt{x} - 3$ must be non-negative.

$k = -3$

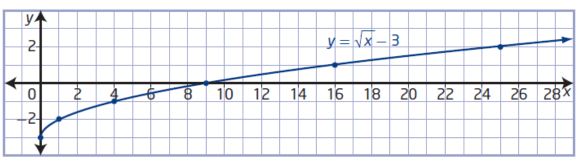
D: $x \geq 0$

$(x, y) \rightarrow (x, y - 3)$

x	y
0	−3
1	−2
4	−1
9	0
16	1
25	2

translated 3 units down

How does the graph of $y = \sqrt{x} - 3$ compare to the graph of $y = \sqrt{x}$?



The domain is $\{x \mid x \geq 0, x \in R\}$ and the range is $\{y \mid y \geq -3, y \in R\}$.

$[0, \infty)$ $[-3, \infty)$

Graphing Radical Functions Using Transformations

You can graph a radical function of the form $y = a\sqrt{b(x - h)} + k$ by transforming the graph of $y = \sqrt{x}$ based on the values of a, b, h, and k. The effects of changing parameters in radical functions are the same as the effects of changing parameters in other types of functions.

- Parameter a results in a vertical stretch of the graph of $y = \sqrt{x}$ by a factor of $|a|$. If $a < 0$, the graph of $y = \sqrt{x}$ is reflected in the x-axis.
- Parameter b results in a horizontal stretch of the graph of $y = \sqrt{x}$ by a factor of $\frac{1}{|b|}$. If $b < 0$, the graph of $y = \sqrt{x}$ is reflected in the y-axis.
- Parameter h determines the horizontal translation. If $h > 0$, the graph of $y = \sqrt{x}$ is translated to the right h units. If $h < 0$, the graph is translated to the left $|h|$ units.
- Parameter k determines the vertical translation. If $k > 0$, the graph of $y = \sqrt{x}$ is translated up k units. If $k < 0$, the graph is translated down $|k|$ units.

$$y = a\sqrt{b(x-h)} + k$$

$$(x, y) \longrightarrow \left(\frac{1}{b}x + h,\ ay + k\right)$$

Example 2

Graph Radical Functions Using Transformations

Sketch the graph of each function using transformations. Compare the domain and range to those of $y = \sqrt{x}$ and identify any changes.

a) $y = 3\sqrt{-(x - 1)}$ **b)** $y - 3 = -\sqrt{2x}$

a) $y = 3\sqrt{-(x-1)}$

$y = a\sqrt{b(x-1)} + k$

a=3 → vertical stretch by a factor of 3.

b=-1 → no horizontal stretch but there is a horizontal reflection in the y-axis

h=1 → translate 1 unit to the right.

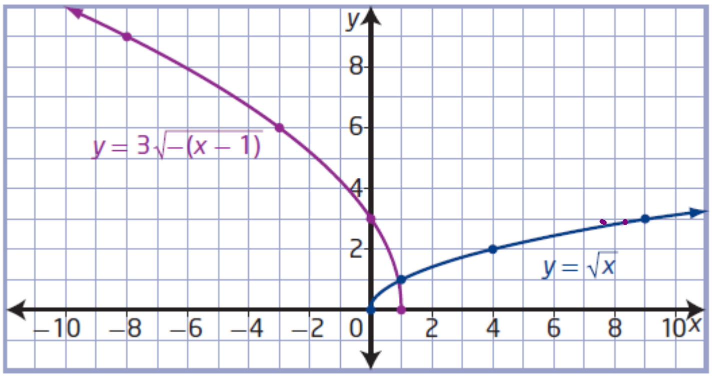
k=0 → no vertical translation

$y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3
16	4
25	5

$(x, y) \longrightarrow \left(\frac{1}{-1}x + 1, 3y + 0\right)$

x	y
1	0
0	3
-3	6
-8	9
-15	12
-24	15



D: $-(x-1) \geq 0$

$-x + 1 \geq 0$

$-x \geq -1$

$x \leq 1$

$\{x \mid x \leq -1, x \in R\}$

$(-\infty, 1]$

R: $\{y \mid y \geq 0, y \in R\}$

$[0, \infty)$

b) $y - 3 = -\sqrt{2x}$

$y = -\sqrt{2x} + 3$

$a = -1 \rightarrow$ no vertical stretch but vertical reflection in x-axis

$b = 2 \rightarrow$ horizontal stretch by a factor of $\frac{1}{2}$.

$h = 0 \rightarrow$ no horizontal translation

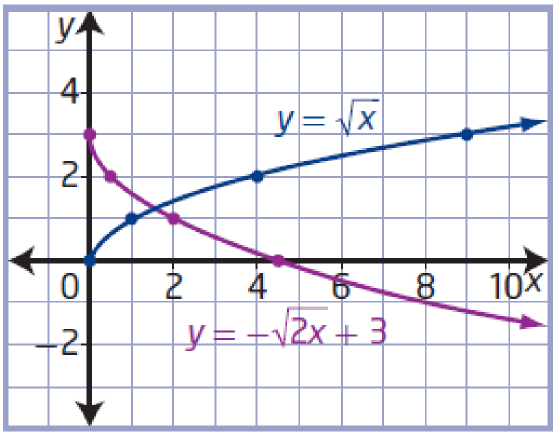
$k = 3 \rightarrow$ translated 3 units up.

$y = \sqrt{x}$

x	y
0	0
1	1
4	2
9	3
16	4
25	5

$(x, y) \rightarrow \left(\frac{1}{2}x + 0, -1y + 3\right)$

x	y
0	3
0.5	2
2	1
4.5	0
8	-1
12.5	-2



D: $2x \geq 0$

$x \geq 0$

$\{x \mid x \geq 0, x \in R\}$

$[0, \infty)$

R: $\{y \mid y \leq 3, y \in R\}$

$(-\infty, 3]$

Homework

#2-5 on page 72-73

5. Sketch the graph of each function using transformations. State the domain and range of each function.

a) $f(x) = \sqrt{-x} - 3$

b) $r(x) = 3\sqrt{x + 1}$

c) $p(x) = -\sqrt{x - 2}$

d) $y - 1 = -\sqrt{-4(x - 2)}$

e) $m(x) = \sqrt{\frac{1}{2}x} + 4$

f) $y + 1 = \frac{1}{3}\sqrt{-(x + 2)}$

$$y = \frac{1}{3}\sqrt{-(x+2)} - 1 \qquad y = a\sqrt{b(x-h)} + k$$

$a = \frac{1}{3} \rightarrow$ vertical stretch by a factor of $\frac{1}{3}$

$b = -1 \rightarrow$ no horizontal stretch but there is a horizontal reflection in the y-axis

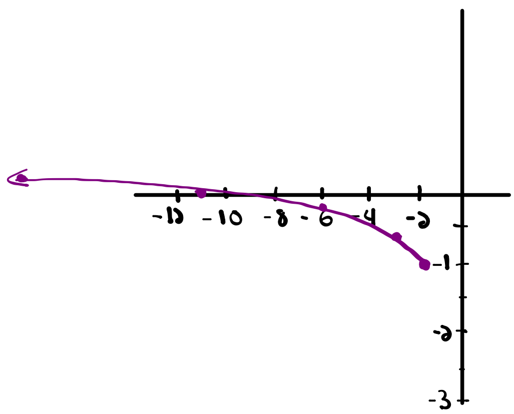
$h = -2 \rightarrow$ horizontal translation 2 units left.

$k = -1 \rightarrow$ vertical translation 1 unit down

$y = \sqrt{x}$ $\qquad (x, y) \longrightarrow \left[\frac{1}{-1}x - 2, \frac{1}{3}y - 1\right]$

x	y
0	0
1	1
4	2
9	3
16	4
25	5

x	y
-2	-1
-3	$-\frac{2}{3}$ or $-0.\overline{6}$
-6	$-\frac{1}{3}$ or $-0.\overline{3}$
-11	0
-18	$\frac{1}{3}$ or $0.\overline{3}$
-27	$\frac{2}{3}$ or $0.\overline{6}$



D: $\{x \mid x \le -2, x \in R\}$ or $(-\infty, -2]$

R: $\{y \mid y \ge -1, y \in R\}$ or $[-1, \infty)$

5. Sketch the graph of each function using transformations. State the domain and range of each function.

a) $f(x) = \sqrt{-x} - 3$

b) $r(x) = 3\sqrt{x + 1}$

c) $p(x) = -\sqrt{x - 2}$

d) $y - 1 = -\sqrt{-4(x - 2)}$

e) $m(x) = \sqrt{\frac{1}{2}x} + 4$

f) $y + 1 = \frac{1}{3}\sqrt{-(x + 2)}$

d) $y - 1 = -\sqrt{-4(x-2)}$ $\qquad y = a\sqrt{b(x-h)} + k$

$y = -\sqrt{\underline{-4}(x-\underline{2})} + \underline{1}$

$a = -1$ → no vertical stretch but there is a vertical reflection in the x-axis.

$b = -4$ → a horizontal stretch by a factor of $\frac{1}{4}$ and a horizontal reflection in the y-axis

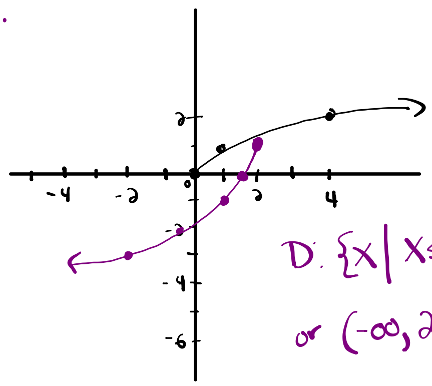
$h = 2$ → a horizontal translation 2 units right

$k = 1$ → a vertical translation 1 unit up

$y = \sqrt{x}$ $\qquad (x, y) \to \left(\frac{1}{-4}x + 2, -1y + 1\right)$

x	y
0	0
1	1
4	2
9	3
16	4

	x	y
	2	1
(1.75)	7/4	0
	1	-1
(-0.25)	-1/4	-2
	-2	-3



D: $\{x \mid x \leq 2, x \in R\}$

or $(-\infty, 2]$

R: $\{y \mid y \leq 1, y \in R\}$

or $(-\infty, 1]$

$$y - 4 = -2\sqrt{-3x-9} + 4$$

$$y = -2\sqrt{-3x-9} + 8$$

$$y = -2\sqrt{-3(x+3)} + 8$$

$a = -2$

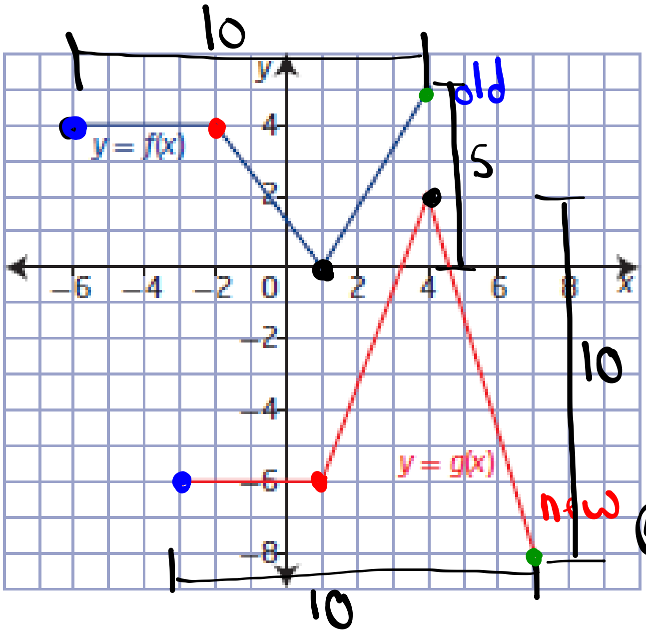
$b = -3$

$h = -3$

$k = 8$

10. The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$.

b)



① Reflection: vertical in x-axis $(a<0)$

② VSF: $\frac{10}{5} = 2$ $\boxed{a=-2}$

③ HSF: $\frac{10}{10} = 1$ $\boxed{b=\frac{1}{1}}$

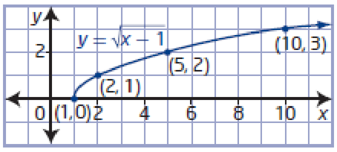
④ VT: $(1,0) \rightarrow (4,2)$ $\boxed{k=2}$

⑤ HT: $(-6,4) \rightarrow (-3,-6)$ $\boxed{h=3}$

⑥ $y = -2f(1(x-3)) + 2$

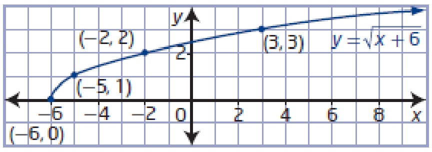
2.1 Radical Functions and Transformations, pages 72 to 77

1. a)



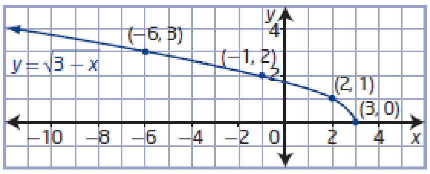
domain $\{x \mid x \geq 1, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

b)



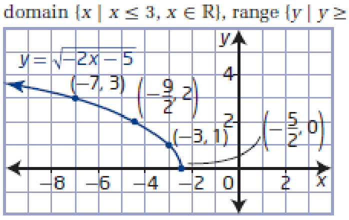
domain $\{x \mid x \geq -6, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

c)



domain $\{x \mid x \leq 3, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

d)



domain $\left\{x \mid x \leq -\frac{5}{2}, x \in \mathrm{R}\right\}$,
range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

2. a) $a = 7 \rightarrow$ vertical stretch by a factor of 7
$h = 9 \rightarrow$ horizontal translation 9 units right
domain $\{x \mid x \geq 9, x \in \mathrm{R}\}$, range $\{y \mid y \geq 0, y \in \mathrm{R}\}$

b) $b = -1 \rightarrow$ reflected in y-axis
$k = 8 \rightarrow$ vertical translation up 8 units
domain $\{x \mid x \leq 0, x \in \mathrm{R}\}$, range $\{y \mid y \geq 8, y \in \mathrm{R}\}$

c) $a = -1 \rightarrow$ reflected in x-axis
$b = \frac{1}{5} \rightarrow$ horizontal stretch factor of 5
domain $\{x \mid x \geq 0, x \in \mathrm{R}\}$, range $\{y \mid y \leq 0, y \in \mathrm{R}\}$

d) $a = \frac{1}{3} \rightarrow$ vertical stretch factor of $\frac{1}{3}$
$h = -6 \rightarrow$ horizontal translation 6 units left
$k = -4 \rightarrow$ vertical translation 4 units down
domain $\{x \mid x \geq -6, x \in \mathrm{R}\}$,
range $\{y \mid y \geq -4, y \in \mathrm{R}\}$

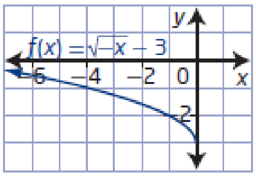
3. a) B **b)** A **c)** D **d)** C

4. a) $y = 4\sqrt{x + 6}$ **b)** $y = \sqrt{8x} - 5$

c) $y = \sqrt{-(x - 4)} + 11$ or $y = \sqrt{-x + 4} + 11$

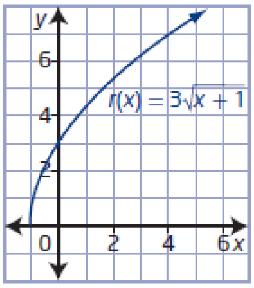
d) $y = -0.25\sqrt{0.1x}$ or $y = -\frac{1}{4}\sqrt{\frac{1}{10}x}$

5. a)



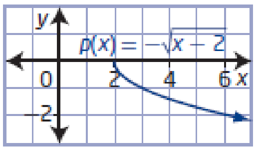
domain
$\{x \mid x \le 0, x \in \mathbb{R}\}$,
range
$\{y \mid y \ge -3, y \in \mathbb{R}\}$

b)



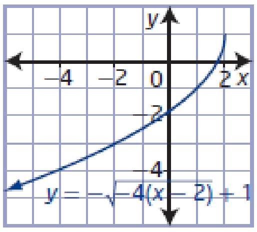
domain
$\{x \mid x \ge -1, x \in \mathbb{R}\}$,
range
$\{y \mid y \ge 0, y \in \mathbb{R}\}$

c)



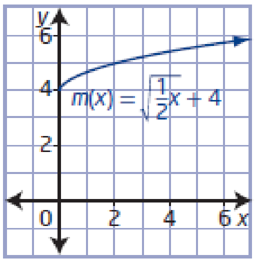
domain
$\{x \mid x \ge 2, x \in \mathbb{R}\}$,
range
$\{y \mid y \le 0, y \in \mathbb{R}\}$

d)



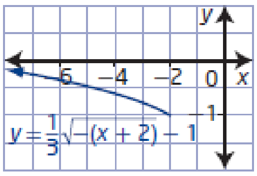
domain
$\{x \mid x \le 2, x \in \mathbb{R}\}$,
range
$\{y \mid y \le 1, y \in \mathbb{R}\}$

e)



domain
$\{x \mid x \ge 0, x \in \mathbb{R}\}$,
range
$\{y \mid y \ge 4, y \in \mathbb{R}\}$

f)

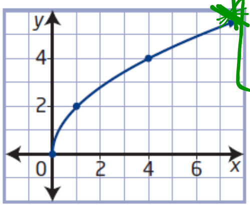


domain
$\{x \mid x \le -2, x \in \mathbb{R}\}$
range
$\{y \mid y \ge -1, y \in \mathbb{R}\}$

Example 3

Determine a Radical Function From a Graph

Mayleen is designing a symmetrical pattern. She sketches the curve shown and wants to determine its equation and the equation of its reflection in each quadrant. The graph is a transformation of the graph of $y = \sqrt{x}$. What are the equations of the four functions Mayleen needs to work with?



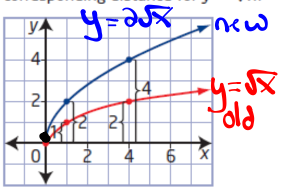
A radical function that involves a stretch can be obtained from either a vertical stretch or a horizontal stretch. Use an equation of the form $y = a\sqrt{x}$ or $y = \sqrt{bx}$ to represent the image function for each type of stretch.

Method 1: Compare Vertical or Horizontal Distances

Superimpose the graph of $y = \sqrt{x}$ and compare corresponding distances to determine the factor by which the function has been stretched.

View as a Vertical Stretch ($y = a\sqrt{x}$)

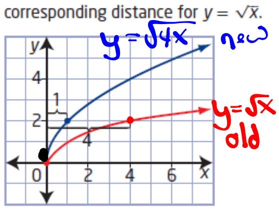
Each vertical distance is 2 times the corresponding distance for $y = \sqrt{x}$.



This represents a vertical stretch by a factor of 2, which means $a = 2$. The equation $y = 2\sqrt{x}$ represents the function.

View as a Horizontal Stretch ($y = \sqrt{bx}$)

Each horizontal distance is $\frac{1}{4}$ the corresponding distance for $y = \sqrt{x}$.



This represents a horizontal stretch by a factor of $\frac{1}{4}$, which means $b = 4$. The equation $y = \sqrt{4x}$ represents the function.

Express the equation of the function as either $y = 2\sqrt{x}$ or $y = \sqrt{4x}$.

① reflections: None
② VSF: $\frac{2}{1} = 2$ $\boxed{a=2}$
③ HSF: Skip if $y = \sqrt{x}$
④ HT: $(0,0) \to (0,0)$ $h=0$
⑤ VT: $(0,0) \to (0,0)$ $k=0$
⑥ $y = 2\sqrt{1(x-0)} + 0$
$y = 2\sqrt{x}$

① reflections: none
② HSF: $\frac{1}{4}$ $\boxed{b=4}$
③ VSF: skip if $y = \sqrt{x}$
④ HT: $(0,0) \to (0,0)$ $h=0$
⑤ VT: $(0,0) \to (0,0)$ $k=0$
⑥ $y = 1\sqrt{4(x-0)} + 0$
$y = \sqrt{4x}$

$y = 2\sqrt{x}$ (Quad 1)
$y = 2\sqrt{-x}$ (Quad 2)
$y = -2\sqrt{-x}$ (Quad 3)
$y = -2\sqrt{x}$ (Quad 4)

2	1
3	4

Homework
#6, 9, 10
(Page 73)