- 1 T books on a shelf (Order Maters)
 - a) All books are different $7! = 5040 \quad \text{or} \quad {}_{1}P_{1} = \frac{7!}{(1-1)!} = 5040$
 - b) $\frac{3}{2}$ books are identical $\frac{7!}{3!} = \frac{5040}{3} = 3530$
 - c) Books are different and math book is on end.

 $\frac{1}{1} \times \frac{6}{5} \times \frac{5}{4} \times \frac{4}{3} \times \frac{3}{4} \times \frac{1}{4} = 730$

 $\frac{6}{5} \times \frac{4}{5} \times \frac{3}{5} \times \frac{3}{5} \times \frac{1}{5} \times \frac{1}{5} = 720$ Total: 720 + 720 = 1440.

d, Books are different and 4 must be together

- · 4 books together can be considered one object
- 4 books can be arranged 4!
- . 4!4! = 576

Combinations

Focus on...

- explaining the differences between a permutation and a combination
- determining the number of ways to select r elements from n different elements
- solving problems using the number of combinations of n different elements taken r at a time
- solving an equation that involves _C, notation

(Order does not matter)

Sometimes you must consider the order in which the elements of a set are arranged. In other situations, the order is not important. For example, when addressing an envelope, it is important to write the six-character postal code in the correct order. In contrast, addressing an envelope, affixing a stamp, and inserting the contents can be completed in any order.

In this section, you will learn about counting outcomes when order does not matter.

If of combinations is always less than the It of permutations.

Investigate Making Selections When Order Is Not Important

Problem solving, reasoning, and decision-making are highly prized skills in today's workforce. Here is your opportunity to demonstrate those skills.

1. From a group of four students, three are to be elected to an executive

committee with a specific position. The positions are as follows:

1st position

2nd position

President

Vice President

Tressurer

Tressurer 3rd position

- a) Does the order in which the students are elected matter? Why?
- b) In how many ways can the positions be filled from this group?
- 2. Now suppose that the three students are to be selected to serve on a committee.

a) Is the order in which the three students are selected still important? Why or why not? No -> all have same responsibility

- b) How many committees from the group of four students are now possible? 4 3 = 4 c) How does your answer in part b) relate to the answer in step 1b)?

4 is one sixth of 24

combination

- a selection of objects without regard to order
- all of the three-letter combinations of P, Q, R, and S are PQR, PQS, PRS, and QRS (arrangements such as PQR and RPQ are the same combination)

Determining the Number of Possible Combinations

When counting with *Permutations*, the order the objects are chosen is important. When the order of choosing does not have to be considered, we refer to *Combinations*. A <u>combination</u> is a subset of the number of **permutations** and as such, the number of **combinations** for a particular situation is always less than the number of **permutations**.

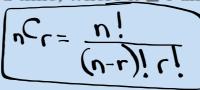
The expression for evaluating combinations is as follows:

The notation ${}_{n}C_{r}$, or $\binom{n}{r}$, represents the number of combinations of n items taken r at a time, where $n \ge r$ and $r \ge 0$.

$${}_{n}C_{r} = \frac{P_{r}}{r!}$$

$$= \frac{n!}{(n-r)!}$$

$$= \frac{n!}{(n-r)!r!}$$



Why must $n \ge r \ge 0$?

A combination is a selection of a group of objects, taken from a larger group, for which the kinds of objects selected is important, but not the order in which they are selected.

There are several ways to find the number of possible combinations. One is to use reasoning. Use the fundamental counting principle and divide by the number of ways that the objects can be arranged among themselves. For example, calculate the number of combinations of three digits made from the digits 1, 2, 3, 4, and 5 without repetitions:

Number of choices	Number of choices	Number of choices
for the first digit	for the second digit	for the third digit
5	4	3

There are $5 \times 4 \times 3$ or 60 ways to arrange 3 items from 5. However, 3 digits can be arranged in 3! ways among themselves, and in a combination these are considered to be the same selection.

So, number of combinations = $\frac{\text{number of permutations}}{3!}$ $= \frac{60}{3!}$ $= \frac{60}{6}$ = 10
What does 3! represent?
the # of ways the digits can be a granged among themsels.

Example 1

A baseball team with 12 players is allowed to send four players to a weekend batting clinic. In how many ways can the group be chosen? (Order does not matter)

Solution

Since order is not important, the group is a **combination**. You are choosing a **combination** of from a group of

$$_{n}C_{r} = \frac{n!}{r!(n-r)!}$$
 $_{12}C_{4} = \frac{12!}{4!(12-4)!}$

$$_{12}\mathbf{C}_{4} = \frac{12!}{4! \ 8!}$$

$$_{12}C_4 = 495$$

Example 2

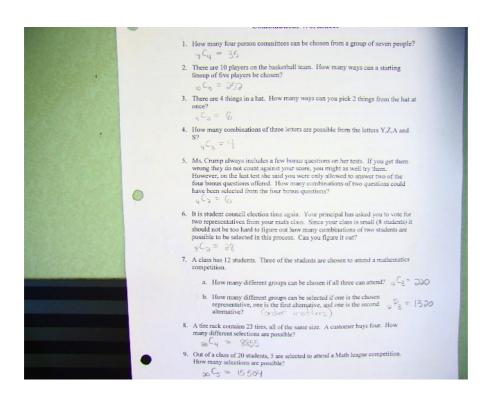
A committee of size 4 and a committee of size 3 are to be assigned from a group of 10 people. How many ways can this be done if no person is assigned to both committees?

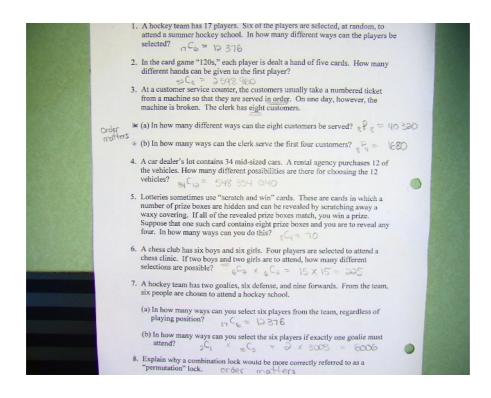
Solution

 $\begin{array}{lll} \underline{1st \ Committee} & \underline{2nd \ Committee} & \underline{Committee} & \underline{Committee} & \underline{Committee} & \underline{AND \ Committee} & \underline{Size \ 4} & \underline{AND \ Committee} & \underline{AND \ Committee} & \underline{Size \ 3} & \underline{Committee} & \underline{Committee}$

person is assigned to both committees.

Questions from Homework





Rianna is writing a geography exam. The instructions say that she must answer a specified number of questions from each section. How many different selections of questions are possible if

- a) she must answer two of the four questions in part A and three of the five questions in part B?
- b) she must answer two of the four questions in part A and at least four of the five questions in part B?

a)
$${}_{4}^{C_{3}}$$
 and ${}_{5}^{C_{3}}$
 ${}_{4}^{C_{3}} = \frac{4!}{(4-3)! \cdot 3!}$
 $= \frac{34}{4}$
 $= 6$
 $\times 10 = 6$

(ombinations)

$$= 4^{C_3} \times 5^{C_4}$$

= 6 x 5
= 30

Case 2: (5 of 5 from Part B)

$$30 + 6 = 36$$
 combinations

Example 3

Simplifying Expressions and Solving Equations With Combinations

- a) Express as factorials and simplify $\frac{{}_{n}C_{5}}{{}_{n-1}C_{3}}$.
- **b)** Solve for *n* if $2({}_{n}C_{2}) = {}_{n+1}C_{3}$.

Homework

Page 534 #1, 4, 5, 6, 8, 11, 14, 15, 16, 17, 20

11.2 Combinations, pages 534 to 536

- 1. a) Combination, because the order that you shake hands is not important.
 - b) Permutation, because the order of digits is important.
 - Combination, since the order that the cars are purchased is not important.
 - Combination, because the order that players are selected to ride in the van is not important.
- 2. $_{5}P_{3}$ is a permutation representing the number of ways of arranging 3 objects taken from a group of 5 objects. $_5C_3$ is a combination representing the number of ways of choosing any 3 objects from a group of 5 objects. $_{5}P_{3} = 60 \text{ and } _{5}C_{3} = 10.$
- 3. a) ${}_{6}P_{4} = 360^{\circ}$ c) ${}_{5}C_{2} = 10^{\circ}$
- b) $_{7}C_{3} = 35$ d) $_{10}C_{7} = 120$

- 4. a) 210
- 5. a) AB, AC, AD, BC, BD, CD b) AB, BA, AC, CA, AD, DA, BC, CB, BD, DB, CD, DC
 - The number of permutations is 2! times the number of combinations.
- **6.** a) n = 10 b) n = 7c) n = 4**d)** n = 5
- 7. a) Case 1: one-digit numbers, Case 2: two-digit numbers, Case 3: three-digit numbers
 - Cases of grouping the 4 members of the 5-member team from either grade: Case 1: four grade 12s, Case 2: three grade 12s and one grade 11, Case 3: two grade 12s and two grade 11s, Case 4: one grade 12 and three grade 11s, Case 5: four grade 11s
- Right Side = $_{11}C_{8}$ **8.** Left Side = $_{11}C_3$ $= \frac{\frac{11!}{(11-8)!8!}}{\frac{11!}{2!6!}}$ $= \frac{11!}{(11-3)!3!}$ $= \frac{11!}{8!3!}$ Left Side = Right Side

- 9. a) ${}_5C_5=1$ b) ${}_5C_0=1$; there is only one way to choose 5 objects from a group of 5 objects and only one way to choose 0 objects from a group of 5 objects.
- 10. a) 4
- **b)** 10
- 11. a) 15 12. Left Side
- b) 22
- $= {_{\scriptscriptstyle n}}C_{\scriptscriptstyle r-1} + {_{\scriptscriptstyle n}}C_{\scriptscriptstyle r}$ $= \frac{n!}{(n-(r-1))!(r-1)!} + \frac{n!}{(n-r)!r!}$ $= \frac{n!}{(n-r+1)!(r-1)!} + \frac{n!}{(n-r)!r!}$ $= \frac{[n!(n-r)!r!] + [n!(n-r+1)!(r-1)!]}{[n!(n-r+1)!(r-1)!]}$ (n-r+1)!(r-1)!(n-r)!r!
- $= \frac{(n-r+1)!(r-1)!(n-r)!r!}{(n-r)!(r-1)!(r-r+1)!}$
- $= \frac{(n-r+1)!(r-1)!(n-r)!r!}{(n-r+1)!(r-1)!(n+1)}$ $= \frac{n!(n-r)!(r-1)!(n+1)}{(n-r+1)!(r-1)!(n-r)!r!}$
- $=\frac{1}{(n-r+1)!r!}$ (n + 1)!
- $=\frac{(n-r+1)!r!}{(n-r+1)!r!}$

Right Side =
$$_{n+1}C_r$$

= $\frac{(n+1)!}{(n+1-r)!r!}$

- Left Side = Right Side
- 13. 20 different burgers; this is a combination because the order the ingredients is put on the burger is not important.
- 14. a) 210
 - combination, because the order of toppings on a pizza is not important
- Method 1: Use a diagram. Method 2: Use combinations. $_{5}C_{2}=10$, the same as the number of combinations of 5 people shaking hands.



- b) 10
- c) The number of triangles is given by ${}_{10}C_3 = \frac{10!}{(10-3)!3!} = \frac{10!}{7!3!}$. The number of lines is given by $C = \frac{10!}{10!} = \frac{10!}{10!}$ of lines is given by ${}_{10}C_2 = \frac{10!}{(10-2)!2!} = \frac{10!}{8!2!}$ The number of triangles is determined by the number of selections with choosing 3 points from 10 non-collinear points, whereas the number of lines is determined by the number of selections with choosing 2 points from the 10 non-collinear points.

16. Left Side =
$${}_{n}C_{r}$$

= $\frac{n!}{(n-r)!r!}$
Right Side = ${}_{n}C_{n-r}$
= $\frac{n!}{(n-(n-r))!(n-r)!}$
= $\frac{n!}{(n-n+r)!(n-r)!}$
= $\frac{n!}{r!(n-r)!}$

 $Left\ Side = Right\ Side$

- **17.** a) 125 970 b) 44 352 c) 1945 **18.** a) 2 598 960 b) 211 926 c) 388 700

- 19. a) 525 b) 576 20. a) $\frac{40!}{20!20!}$ b) 116 280 **20.** a) $\frac{40!}{20!20!}$