

Square Root of a Function

Focus on...

- sketching the graph of $y = \sqrt{f(x)}$ given the graph of $y = f(x)$
- explaining strategies for graphing $y = \sqrt{f(x)}$ given the graph of $y = f(x)$
- comparing the domains and ranges of the functions $y = f(x)$ and $y = \sqrt{f(x)}$, and explaining any differences

square root of a function

- the function $y = \sqrt{f(x)}$ is the square root of the function $y = f(x)$
- $y = \sqrt{f(x)}$ is only defined for $f(x) \geq 0$

The function $y = \sqrt{2x + 1}$ represents the square root of the function $y = 2x + 1$.

x	$y = 2x + 1$	$y = \sqrt{2x + 1}$
<u>0</u>	1	$\sqrt{1} = 1$
4	9	$\sqrt{9} = 3$
12	25	$\sqrt{25} = 5$
24	49	$\sqrt{49} = 7$
\vdots	\vdots	\vdots

when $x = \underline{0}$

$$y = 2x + 1$$

$$y = 2(0) + 1$$

$$y = 0 + 1$$

$$y = 1$$

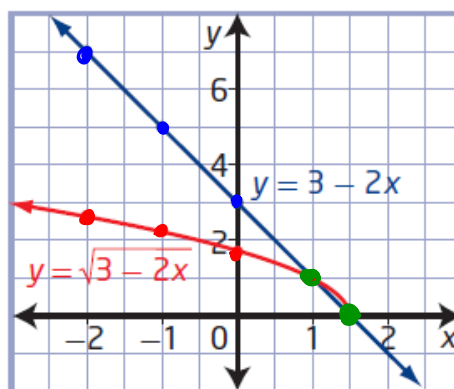
Example 1

Compare Graphs of a Linear Function and the Square Root of the Function

- a) Given $f(x) = 3 - 2x$, graph the functions $y = f(x)$ and $y = \sqrt{f(x)}$.
 b) Compare the two functions.

Use a table of values to graph $y = 3 - 2x$ and $y = \sqrt{3 - 2x}$.

x	$y = 3 - 2x$	$y = \sqrt{3 - 2x}$
-2	7	$\sqrt{7} = 2.7$
-1	5	$\sqrt{5} = 2.2$
0	3	$\sqrt{3} = 1.7$
1	1	$\sqrt{1} = 1$
1.5	0	$\sqrt{0} = 0$



Invariant points are
 $(1, 1)$ and $(1.5, 0)$

$$y = 3 - 2x:$$

$$D: \{x | x \in \mathbb{R}\} \text{ or } (-\infty, \infty)$$

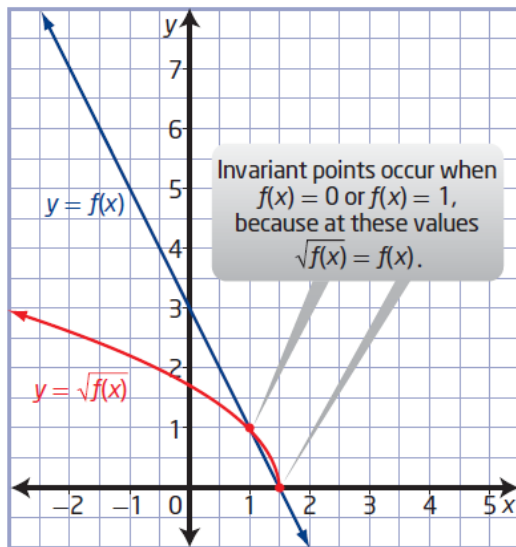
$$R: \{y | y \in \mathbb{R}\} \text{ or } (-\infty, \infty)$$

$$y = \sqrt{3 - 2x}$$

$$D: \{x | x \leq 1.5, x \in \mathbb{R}\} \text{ or } (-\infty, 1.5]$$

$$R: \{y | y \geq 0, y \in \mathbb{R}\} \text{ or } [0, \infty)$$

b) Compare the graphs.



Why is the graph of $y = \sqrt{f(x)}$ above the graph of $y = f(x)$ for values of y between 0 and 1? Will this always be true?

For $y = f(x)$, the domain is $\{x \mid x \in \mathbb{R}\}$ and the range is $\{y \mid y \in \mathbb{R}\}$.

For $y = \sqrt{f(x)}$, the domain is $\{x \mid x \leq 1.5, x \in \mathbb{R}\}$ and the range is $\{y \mid y \geq 0, y \in \mathbb{R}\}$.

Invariant points occur at $(1, 1)$ and $(1.5, 0)$.

How does the domain of the graph of $y = \sqrt{f(x)}$ relate to the restrictions on the variable in the radicand? How could you determine the domain algebraically?

Relative Locations of $y = f(x)$ and $y = \sqrt{f(x)}$

The domain of $y = \sqrt{f(x)}$ consists only of the values in the domain of $f(x)$ for which $f(x) \geq 0$.

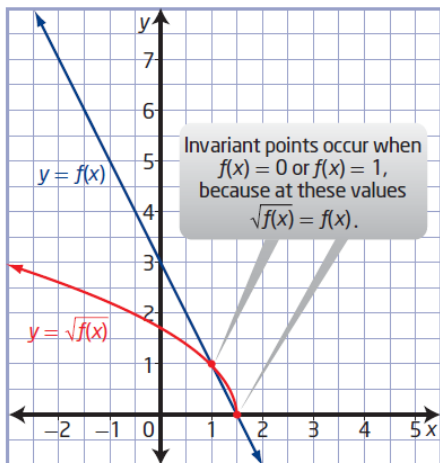
The range of $y = \sqrt{f(x)}$ consists of the square roots of the values in the range of $y = f(x)$ for which $\sqrt{f(x)}$ is defined.

The graph of $y = \sqrt{f(x)}$ exists only where $f(x) \geq 0$. You can predict the location of $y = \sqrt{f(x)}$ relative to $y = f(x)$ using the values of $f(x)$.

Value of $f(x)$	$f(x) < 0$	$f(x) = 0$	$0 < f(x) < 1$	$f(x) = 1$	$f(x) > 1$
Relative Location of Graph of $y = \sqrt{f(x)}$	The graph of $y = \sqrt{f(x)}$ is undefined.	The graphs of $y = \sqrt{f(x)}$ and $y = f(x)$ intersect on the x-axis.	The graph of $y = \sqrt{f(x)}$ is above the graph of $y = f(x)$.	The graph of $y = \sqrt{f(x)}$ intersects the graph of $y = f(x)$.	The graph of $y = \sqrt{f(x)}$ is below the graph of $y = f(x)$.

Handwritten notes: undefined I.P. Above f(x) I.P. Below f(x)

b) Compare the graphs.



Why is the graph of $y = \sqrt{f(x)}$ above the graph of $y = f(x)$ for values of y between 0 and 1? Will this always be true?

Your Turn

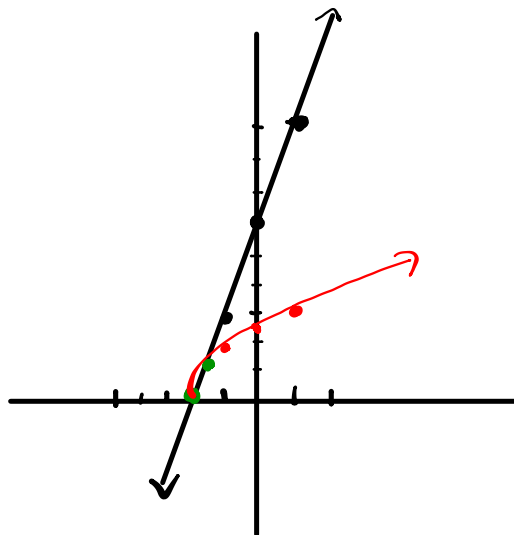
- a) Given $g(x) = 3x + 6$, graph the functions $y = g(x)$ and $y = \sqrt{g(x)}$.
 b) Identify the domain and range of each function and any invariant points.

$$y = 3x + 6$$

x	y
-2	0
-1	3
0	6
1	9
2	12

$$y = \sqrt{3x+6}$$

x	y
-2	0
-1	1.7
0	2.4
1	3
2	3.5



$$D: \{x | x \in \mathbb{R}\}$$

$$R: \{y | y \in \mathbb{R}\}$$

$$D: \{x | x \geq -2, x \in \mathbb{R}\}$$

$$R: \{y | y \geq 0, y \in \mathbb{R}\}$$

Invariant Points: $(-2, 0)$ + $(-\frac{5}{3}, 1)$

$$3x + 6 = 1$$

$$3x = -5$$

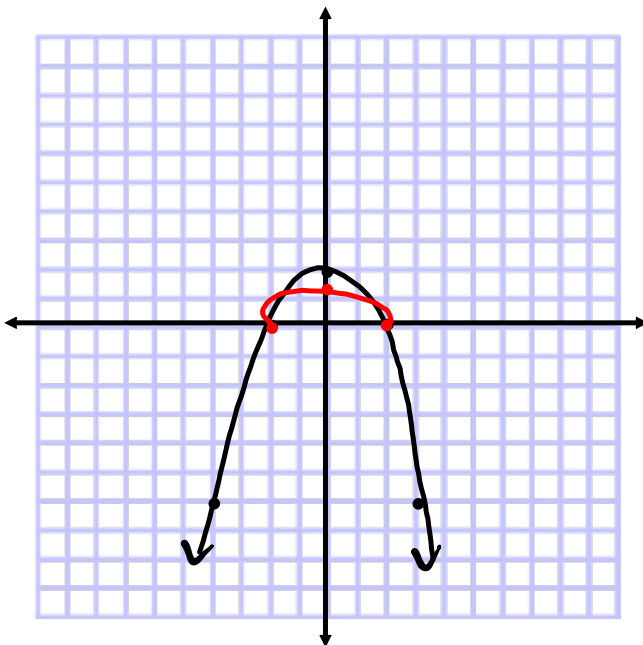
$$x = -\frac{5}{3}$$

Example 2

Compare the Domains and Ranges of $y = f(x)$ and $y = \sqrt{f(x)}$

Identify and compare the domains and ranges of the functions in each pair.

a) $y = 2 - 0.5x^2$ and $y = \sqrt{2 - 0.5x^2}$



$$y = 2 - 0.5x^2$$

x	y
-4	-6
-2	0
0	2
2	0
4	-6

$$D: \{x | x \in \mathbb{R}\}$$

$$R: \{y | y \leq 2, y \in \mathbb{R}\}$$

$$y = \sqrt{2 - 0.5x^2}$$

x	y
-4	und.
-2	0
0	1.41
2	0
4	und.

$$D: \{x | -2 \leq x \leq 2, x \in \mathbb{R}\}$$

$$R: \{y | 0 \leq y \leq 1.41, y \in \mathbb{R}\}$$

Example 3

Graph the Square Root of a Function From the Graph of the Function

Step 1: Locate invariant points

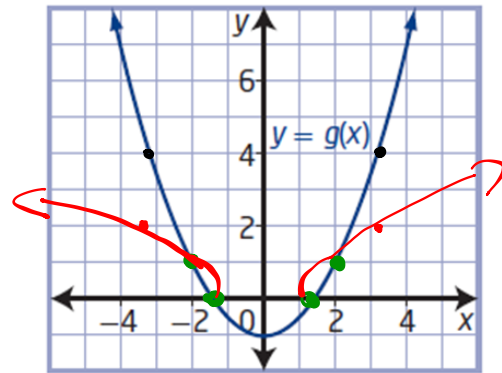
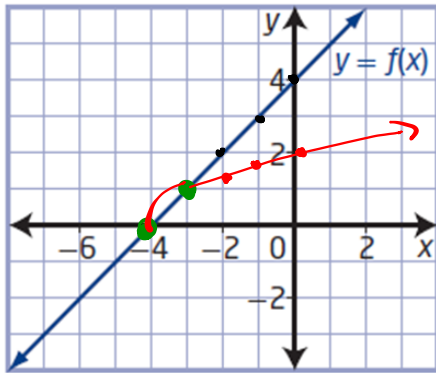
Step 2: Draw the portion of each graph between the invariant points

Step 3: Locate other key points on $y = f(x)$ and $y = g(x)$ where the values are greater than 1. Transform these points to locate image points on the graphs of $y = \sqrt{f(x)}$ and $y = \sqrt{g(x)}$.

$\sqrt{f(x)}$
(above $f(x)$)

$\sqrt{g(x)}$ below $f(x)$

Using the graphs of $y = f(x)$ and $y = g(x)$, sketch the graphs of $y = \sqrt{f(x)}$ and $y = \sqrt{g(x)}$.



$y = f(x)$
 D: $\{x | x \in \mathbb{R}\}$ or $(-\infty, \infty)$
 R: $\{y | y \in \mathbb{R}\}$ or $(-\infty, \infty)$

$y = \sqrt{f(x)}$
 D: $\{x | x \geq -4, x \in \mathbb{R}\}$ or $[-4, \infty)$
 R: $\{y | y \geq 0, y \in \mathbb{R}\}$ or $[0, \infty)$

$y = g(x)$
 D: $\{x | x \in \mathbb{R}\}$ or $(-\infty, \infty)$
 R: $\{y | y \geq -1, y \in \mathbb{R}\}$ or $[-1, \infty)$

$y = \sqrt{g(x)}$
 D: $\{x | x \leq -1.5, x \geq 1.5, x \in \mathbb{R}\}$
 or $(-\infty, -1.5] + [1.5, \infty)$
 R: $\{y | y \geq 0, y \in \mathbb{R}\}$ or $[0, \infty)$

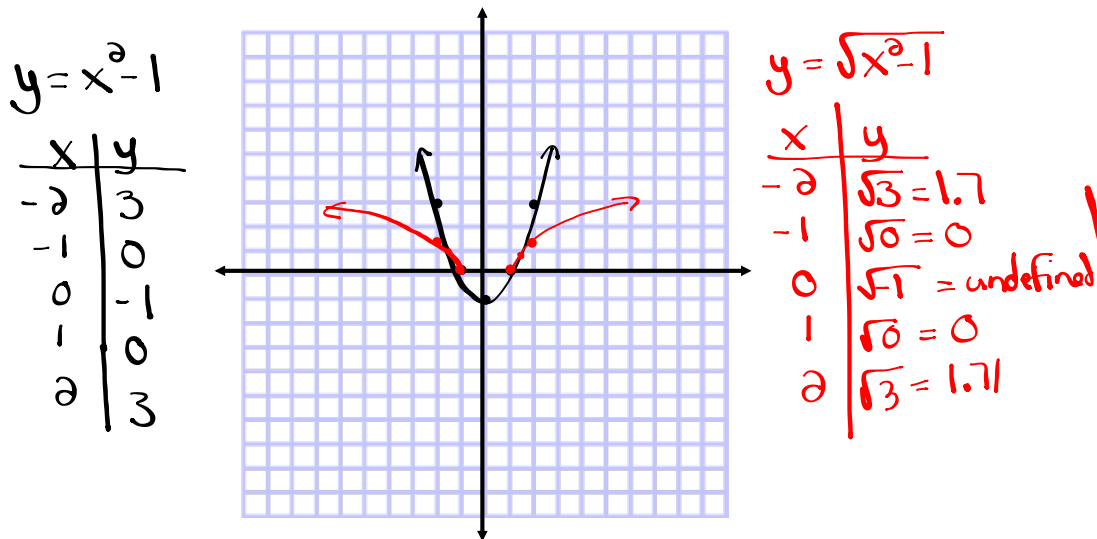
Key Ideas

- You can use values of $f(x)$ to predict values of $\sqrt{f(x)}$ and to sketch the graph of $y = \sqrt{f(x)}$.
- The key values to consider are $f(x) = 0$ and $f(x) = 1$.
- The domain of $y = \sqrt{f(x)}$ consists of all values in the domain of $f(x)$ for which $f(x) \geq 0$.
- The range of $y = \sqrt{f(x)}$ consists of the square roots of all values in the range of $f(x)$ for which $f(x)$ is defined.
- The y -coordinates of the points on the graph of $y = \sqrt{f(x)}$ are the square roots of the y -coordinates of the corresponding points on the original function $y = f(x)$.

What do you know about the graph of $y = \sqrt{f(x)}$ at $f(x) = 0$ and $f(x) = 1$? How do the graphs of $y = f(x)$ and $y = \sqrt{f(x)}$ compare on either side of these locations?

Your Turn

- 1) Identify and compare the domains and ranges of the functions $y = x^2 - 1$ and $y = \sqrt{x^2 - 1}$. Verify your answers.

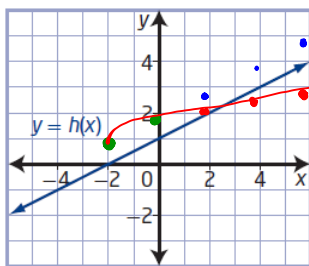


D: $\{x | x \in \mathbb{R}\}$ or $(-\infty, \infty)$
 R: $\{y | y \geq -1, y \in \mathbb{R}\}$ or $[-1, \infty)$

D: $\{x | x \leq -1 \text{ and } x \geq 1, x \in \mathbb{R}\}$
 $(-\infty, -1]$ and $[1, \infty)$

R: $\{y | y \geq 0, y \in \mathbb{R}\}$ or $[0, \infty)$

- 2) Using the graph of $y = h(x)$, sketch the graph of $y = \sqrt{h(x)}$.



- (i) Locate invariant points $(-2, 0)$ and $(0, 1)$
- (ii) draw the portion of square root curve between invariant points. (above the original)
- (iii) Locate key points on original and transform them (take square root of y-values)

$y = h(x)$
 D: $\{x | x \in \mathbb{R}\}$ or $(-\infty, \infty)$
 R: $\{y | y \in \mathbb{R}\}$ or $(-\infty, \infty)$

$y = \sqrt{h(x)}$
 D: $\{x | x \geq -2, x \in \mathbb{R}\}$ or $[-2, \infty)$
 R: $\{y | y \geq 0, y \in \mathbb{R}\}$ or $[0, \infty)$

Page 86-87
#1-6

