# Exponent Laws

① $x^2 \cdot x^3 =$

$2^{18} \cdot 2^{-3} =$

$x^2 \cdot x^{1/3} =$

② $\frac{x^5}{x^3} =$

$\frac{b^8}{b^{-2}} =$

③ $(x^5)^2 =$

$(y^8)^{1/2} =$

④ $\sqrt{x} =$

$\sqrt[3]{x} =$

$\sqrt[4]{x} =$

$\sqrt[5]{x^2} =$

$\sqrt[7]{x^6} =$

⑤ $3^2 =$

⑥ $3^{-2} =$

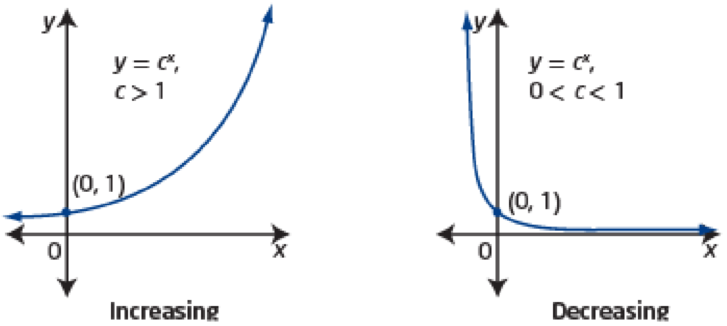
$\left(\frac{2}{5}\right)^{-3} =$

# Exponential Functions

The graph of an **exponential function**, such as $y = c^x$, is increasing for $c > 1$, decreasing for $0 < c < 1$, and neither increasing nor decreasing for $c = 1$. From the graph, you can determine characteristics such as domain and range, any intercepts, and any asymptotes.

**exponential function**

- a function of the form $y = c^x$, where $c$ is a constant ($c > 0$) and $x$ is a variable



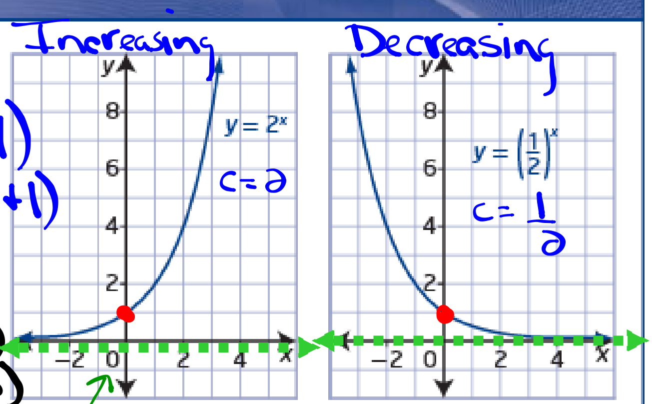

Why is the definition of an exponential function restricted to positive values of $c$?

**Did You Know?**

Any letter can be used to represent the base in an exponential function. Some other common forms are $y = a^x$ and $y = b^x$. In this chapter, you will use the letter $c$. This is to avoid any confusion with the transformation parameters, $a$, $b$, $h$, and $k$, that you will apply in Section 7.2.

## Key Ideas

- An exponential function of the form $y = c^x$, $c > 0$,
  - is increasing for $c > 1$ (Base is greater 1)
  - is decreasing for $0 < c < 1$ (Between 0 + 1)
  - is neither increasing nor decreasing for $c = 1$ (Base equals 1)
  - has a domain of $\{x \mid x \in R\}$ or $(-\infty, \infty)$
  - has a range of $\{y \mid y > 0, y \in R\}$ or $(0, \infty)$
  - has a $y$-intercept of 1 (0,1)
  - has no $x$-intercept
  - has a horizontal asymptote at $y = 0$




↳ $y = k$

## Example 1

### Analyse the Graph of an Exponential Function

Graph each exponential function. Then identify the following:

- the domain and range
- the $x$-intercept and $y$-intercept, if they exist
- whether the graph represents an increasing or a decreasing function
- the equation of the horizontal asymptote

**a)** $y = 4^x$ $c = 4$ (Increasing)

**b)** $f(x) = \left(\frac{1}{2}\right)^x$ $c = \frac{1}{2}$ (Decreasing)

## Solution

**a) Method 1: Use Paper and Pencil**

Use a table of values to graph the function.

Select integral values of $x$ that make it easy to calculate the corresponding values of $y$ for $y = 4^x$. $c=4$ (Increasing)

$y = 4^x$

| $x$ | $y$ |
|---|---|
| −2 | $\frac{1}{16}$ |
| −1 | $\frac{1}{4}$ |
| 0 | 1 |
| 1 | 4 |
| 2 | 16 |

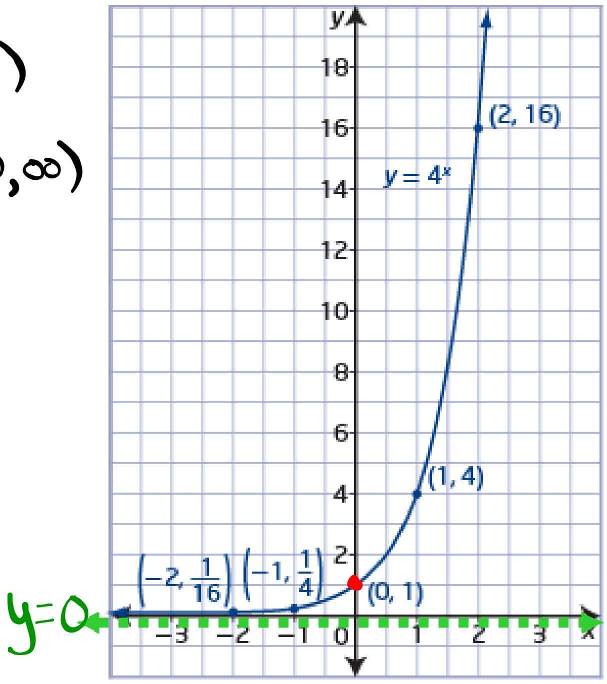
D: $\{x \mid x \in \mathbb{R}\}$ or $(-\infty, \infty)$

R: $\{y \mid y > 0, y \in \mathbb{R}\}$ or $(0, \infty)$

x int: none

y int: $(0, 1)$

HA: $y = 0$



**b) Method 1: Use Paper and Pencil**

Use a table of values to graph the function.

Select integral values of $x$ that make it easy to calculate the corresponding values of $y$ for $f(x) = \left(\frac{1}{2}\right)^x$. $c = \frac{1}{2}$ (Decreasing)

$y = \left(\frac{1}{2}\right)^x$

| $x$ | $f(x)$ |
|---|---|
| −3 | 8 |
| −2 | 4 |
| −1 | 2 |
| 0 | 1 |
| 1 | $\frac{1}{2}$ |
| 2 | $\frac{1}{4}$ |

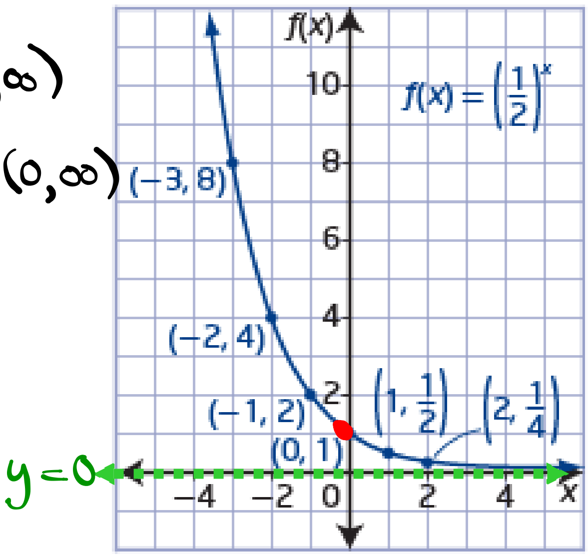
D: $\{x \mid x \in R\}$ or $(-\infty, \infty)$

R: $\{y \mid y > 0, y \in R\}$ or $(0, \infty)$

x int: none

y int: $(0, 1)$

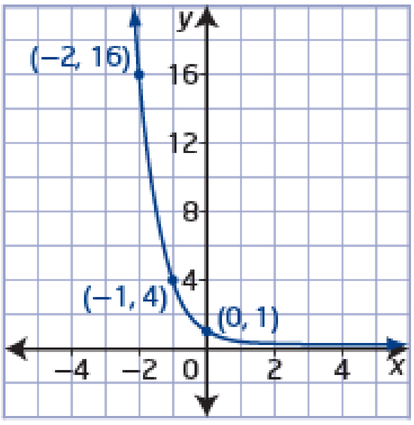
HA: $y = 0$



## Example 2

### Write the Exponential Function Given Its Graph

What function of the form $y = c^x$ can be used to describe the graph shown?




Decreasing $0 < c < 1$

- Base is between 0 & 1

### Solution

Look for a pattern in the ordered pairs from the graph.

| x | y |
|---|---|
| −2 | 16 |
| −1 | 4 |
| 0 | 1 |

$\frac{1}{4}$ $\frac{1}{4}$

As x increases by 1 the y values decrease by a factor of 4

$c = \frac{1}{4}$

(0,1) is on all exponential functions of the form $y = c^x$

Choose a point other than (0, 1) to substitute into the function $y = \left(\frac{1}{4}\right)^x$ to verify that the function is correct. Try the point (−2, 16). $x = -2$ $y = 16$

Why should you not use the point (0, 1) to verify that the function is correct?

Check:

| Left Side | Right Side |
|---|---|
| $y$ | $\left(\frac{1}{4}\right)^x$ |
| $16$ ✓ | $\left(\frac{1}{4}\right)^{-2}$ |
| | $(4)^2$ |
| | $16$ ✓ |

## Example 3

### Application of an Exponential Function

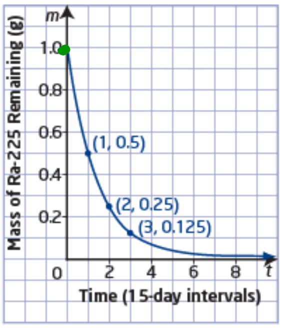
A radioactive sample of radium (Ra-225) has a **half-life** of 15 days. The mass, $m$, in grams, of Ra-225 remaining over time, $t$, in 15-day intervals, can be modelled using the exponential graph shown.

**a)** What is the initial mass of Ra-225 in the sample? What value does the mass of Ra-225 remaining approach as time passes?

**b)** What are the domain and range of this function?

**c)** Write the exponential decay model that relates the mass of Ra-225 remaining to time, in 15-day intervals.

**d)** Estimate how many days it would take for Ra-225 to decay to $\frac{1}{30}$ of its original mass.




a) Initial Amount = 1g

As time passes the radium approaches a mass of 0g.

b) D: $\{x \mid x \geq 0, x \in R\}$ or $[0, \infty)$

R: $\{y \mid 0 < y \leq 1, y \in R\}$ or $(0, 1]$

c) $m = (\text{Initial Amount})(\text{Base})^{\frac{t}{\text{time it takes to..}}}$ $= 15$

Base $= \frac{1}{2}$ (Half life)

$m = (1)\left(\frac{1}{2}\right)^{\frac{t}{15}}$

d) $\frac{\frac{1}{30}}{1} = \frac{\cancel{(1)}\left(\frac{1}{2}\right)^{\frac{t}{15}}}{\cancel{1}}$ (Divide both sides by Initial Amount)

$\frac{1}{30} = \left(\frac{1}{2}\right)^{t/15}$ (Get a common base)

↳ $\frac{\log(1/30)}{\log(1/2)} = 4.91$

$\left(\cancel{\frac{1}{2}}\right)^{4.91} = \left(\cancel{\frac{1}{2}}\right)^{t/15}$

$15 \cdot 4.91 = \frac{t}{15} \cdot 15$ (Multiply both sides by 15)

$\boxed{73.6 = t}$

73.6 days to reach $\frac{1}{30}$ of its initial amount

# Homework

## #1-8 on page 343

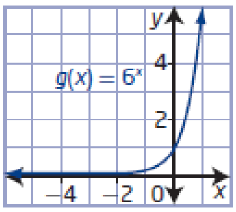
## 7.1 Characteristics of Exponential Functions, pages 342 to 345

**1. a)** No, the variable is not the exponent.
**b)** Yes, the base is greater than 0 and the variable is the exponent.
**c)** No, the variable is not the exponent.
**d)** Yes, the base is greater than 0 and the variable is the exponent.

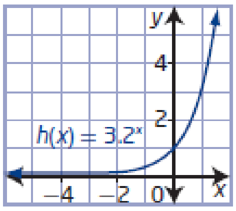
**2. a)** $f(x) = 4^x$ **b)** $g(x) = \left(\frac{1}{4}\right)^x$
**c)** $x = 0$, which is the $y$-intercept

**3. a)** B **b)** C **c)** A

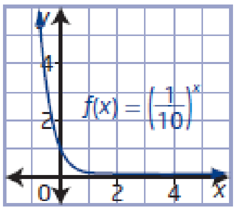
**4. a)** $f(x) = 3^x$ **b)** $f(x) = \left(\frac{1}{5}\right)^x$

**5. a)**


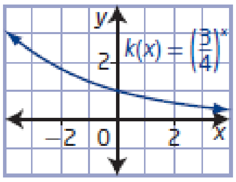

domain $\{x \mid x \in R\}$, range $\{y \mid y > 0, y \in R\}$, $y$-intercept 1, function increasing, horizontal asymptote $y = 0$

**b)**



domain $\{x \mid x \in R\}$, range $\{y \mid y > 0, y \in R\}$, $y$-intercept 1, function increasing, horizontal asymptote $y = 0$

**c)**


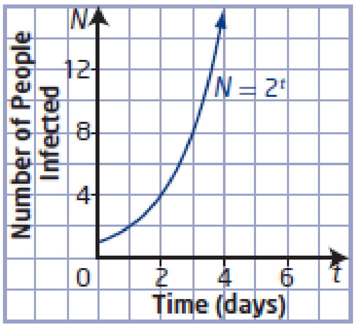

domain $\{x \mid x \in R\}$, range $\{y \mid y > 0, y \in R\}$, $y$-intercept 1, function decreasing, horizontal asymptote $y = 0$

**d)**



domain $\{x \mid x \in R\}$, range $\{y \mid y > 0, y \in R\}$, $y$-intercept 1, function decreasing, horizontal asymptote $y = 0$

**6. a)** $c > 1$; number of bacteria increases over time
**b)** $0 < c < 1$; amount of actinium-225 decreases over time
**c)** $0 < c < 1$; amount of light decreases with depth
**d)** $c > 1$; number of insects increases over time

**7. a)**

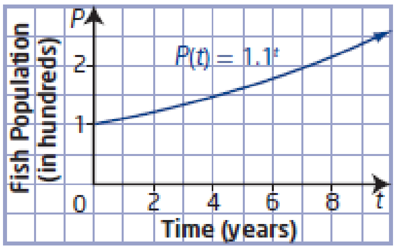



The function $N = 2^t$ is exponential since the base is greater than zero and the variable $t$ is an exponent.

**b)** **i)** 1 person **ii)** 2 people
**iii)** 16 people **iv)** 1024 people

**8. a)** If the population increases by 10% each year, the population becomes 110% of the previous year's population. So, the growth rate is 110% or 1.1 written as a decimal.

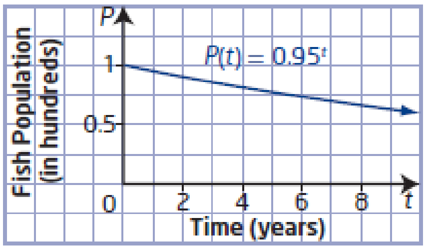
**b)**




domain $\{t \mid t \geq 0, t \in \mathbb{R}\}$ and range $\{P \mid P \geq 100, P \in \mathbb{R}\}$

**c)** The base of the exponent would become $100\% - 5\%$ or 95%, written as 0.95 in decimal form.

**d)**




domain $\{t \mid t \geq 0, t \in \mathbb{R}\}$ and range $\{P \mid 0 < P \leq 100, P \in \mathbb{R}\}$