3.2 Proving and Applying the Sine Law

You now know how to solve for unknown angles and side lengths in a right-angled triangle.

How do we obtain missing measurements in oblique (non-right) triangles?

Option #1 LAW OF SINES

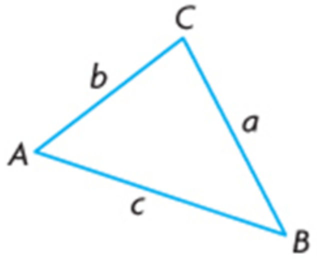
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

Missing Side ☺

OR

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

Missing Angle ☺



When will you use the Law of sines?

You will use the Law of Sines when:

A) you are given two angles and a non-included side (AAS).

B) you are given two angles and an included side (ASA).

C) you are given two sides and an angle opposite to one of them (SSA).

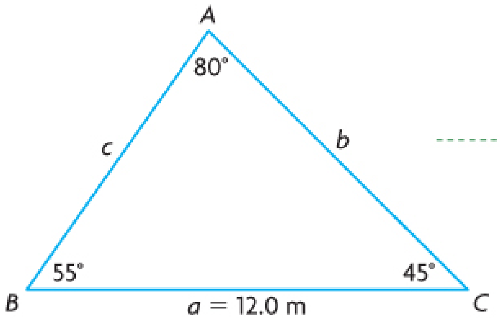
In other words...when you have 2 "matching pairs"

APPLY the Math

EXAMPLE 1 Using reasoning to determine the length of a side

A triangle has angles measuring 80° and 55°. The side opposite the 80° angle is 12.0 m in length. Determine the length of the side opposite the 55° angle to the nearest tenth of a metre.

Elizabeth's Solution



I named the triangle *ABC* and decided that the 80° angle was ∠*A*. Then I sketched the triangle, including all of the information available.

I knew that the third angle, ∠*C*, had to measure 45°, because the angles of a triangle add to 180°. I needed to determine *b*.

Since the triangle does not contain a right angle, I couldn't use the primary trigonometric ratios.

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{12.0}{\sin 80°} = \frac{b}{\sin 55°}$$

$$12.0 \sin 55° = b \sin 80°$$

$$\frac{12.0 \sin 55°}{\sin 80°} = \frac{b \cancel{\sin 80°}}{\cancel{\sin 80°}}$$

$$9.981... = b$$

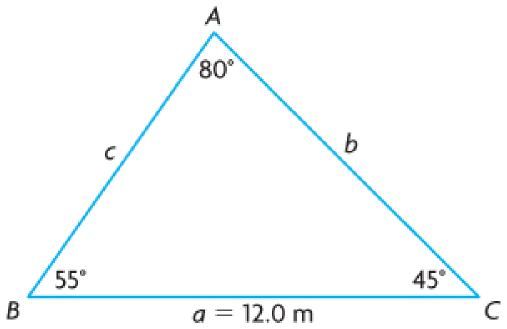
I could use the sine law if I knew an opposite side–angle pair, plus one more side or angle in the triangle. I knew *a* and ∠*A* and I wanted to know *b*, so I related *a*, *b*, sin *A*, and sin *B* using $\frac{a}{\sin A} = \frac{b}{\sin B}$. Since *b* was in the numerator, I could multiply both sides by sin 55° to solve for *b*.

The length of *AC* is 10.0 m.

I rounded to the nearest tenth. It made sense that the length of *AC* is shorter than the length of *BC*, since the measure of ∠*B* is less than the measure of ∠*A*.

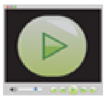
EXAMPLE 1 Using reasoning to determine the length of a side

A triangle has angles measuring 80° and 55°. The side opposite the 80° angle is 12.0 m in length. Determine the length of the side opposite the 55° angle to the nearest tenth of a metre.

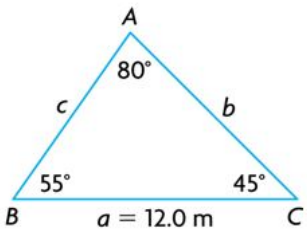


Your Turn

Using $\triangle ABC$ above, determine the length of AB to the nearest tenth of a metre.



Answer



$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{12.0}{\sin 80°} = \frac{c}{\sin 45°}$$

$$12.0 \sin 45° = c \sin 80°$$

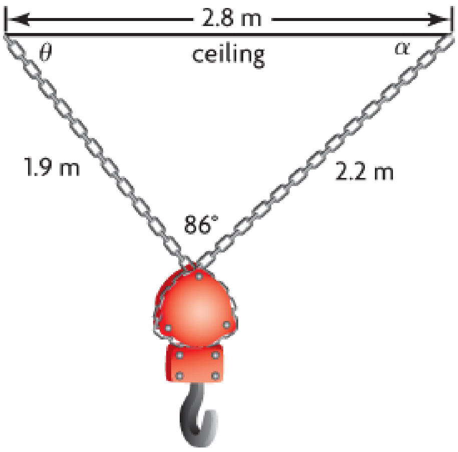
$$\frac{12.0 \sin 45°}{\sin 80°} = \frac{c \cancel{\sin 80°}}{\cancel{\sin 80°}}$$

$$8.616... = c$$

The length of AB is 8.6 m.

EXAMPLE 2 Solving a problem using the sine law

Toby uses chains attached to hooks on the ceiling and a winch to lift engines at his father's garage. The chains, the winch, and the ceiling are arranged as shown. Toby solved the triangle using the sine law to determine the angle that each chain makes with the ceiling to the nearest degree. He claims that $\theta = 40°$ and $\alpha = 54°$. Is he correct? Explain, and make any necessary corrections.



Communication *Tip*

Greek letters are often used as variables to represent the measures of unknown angles. The most commonly used letters are θ (theta), α (alpha), β (beta), and γ (gamma).

Sanjay's Solution

I know Toby's calculations are incorrect, since α must be the smallest angle in the triangle.

In any triangle, the shortest side is across from the smallest angle. Since 1.9 m is the shortest side, $\alpha < \theta$. Toby's values do not meet this condition.

$$\frac{\sin \alpha}{1.9} = \frac{\sin 86°}{2.8}$$

To correct the error, I used the sine law to determine α.

2.8 sin α = 1.9 sin 86°

$$\frac{\cancel{2.8} \sin \alpha}{\cancel{2.8}} = \frac{1.9 \sin 86°}{2.8}$$

I multiplied both sides by 1.9 to solve for sin α. Then I evaluated the right side of the equation.

$$\sin \alpha = 0.6769...$$
$$\alpha = \sin^{-1}(0.6769...)$$
$$\alpha = 42.603...°$$

$$\theta = 180° - 86° - 42.603...°$$
$$\theta = 51.396...°$$

I used the fact that angles in a triangle add to 180° to determine θ.

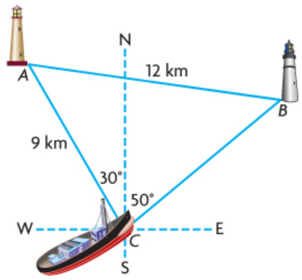
Toby was incorrect. The correct measures of the angles are:

$\alpha \doteq 43°$
and
$\theta \doteq 51°$

My determinations are reasonable, because the shortest side is opposite the smallest angle.

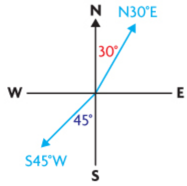
EXAMPLE 3 Using reasoning to determine the measure of an angle

The captain of a small boat is delivering supplies to two lighthouses, as shown. His compass indicates that the lighthouse to his left is located at N30°W and the lighthouse to his right is located at N50°E. Determine the compass direction he must follow when he leaves lighthouse B for lighthouse A.



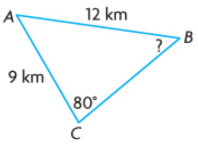
Communication ***Tip***

Directions are often stated in terms of north and south on a compass. For example, N30°E means travelling in a direction 30° east of north. S45°W means travelling in a direction 45° west of south.



Compass Rose Animation

Anthony's Solution



I drew a diagram. I labelled the sides of the triangle I knew and the angle I wanted to determine.

$$\frac{\sin B}{AC} = \frac{\sin C}{AB}$$

I knew AC, AB, and $\angle C$, and I wanted to determine $\angle B$. So I used the sine law that includes these four quantities.

I used the proportion with sin B and sin C in the numerators so the unknown would be in the numerator.

$$\frac{\sin B}{9} = \frac{\sin 80°}{12}$$

I substituted the given information and then solved for sin B.

12.0 sin B = 9 sin 80°

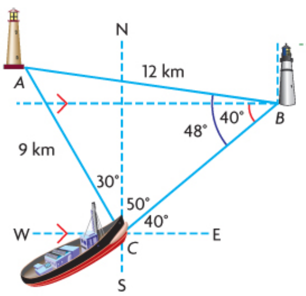
$$\frac{\cancel{12.0} \sin B}{\cancel{12.0}} = \frac{9 \sin 80°}{12.0}$$

$$\sin B = 0.7386...$$

$$\angle B = \sin^{-1}(0.7386...)$$
$$\angle B = 47.612...°$$

The measure of $\angle B$ is 48°.

The answer seems reasonable. $\angle B$ must be less than 80°, because 9 km is less than 12 km.



I drew a diagram and marked the angles I knew. I knew east-west lines are all parallel, so the alternate interior angle at B must be 40°.

The captain must head N82°W from lighthouse B.

The line segment from lighthouse B to lighthouse A makes an 8° angle with west-east. I subtracted this from 90° to determine the direction west of north.

EXAMPLE 3 Using reasoning to determine the measure of an angle

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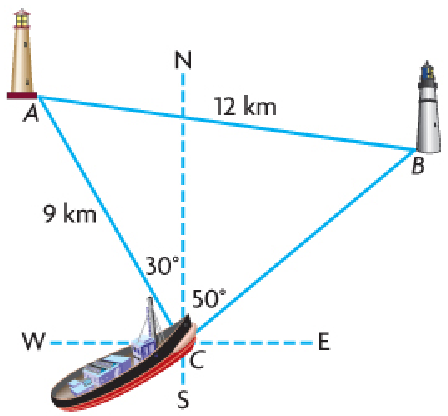


Your Turn

In $\triangle ABC$ above, *CB* is about 9.6 km. Use the sine law to determine $\angle A$. Verify your answer by determining the sum of the angles.

Answer

$$\frac{\sin C}{c} = \frac{\sin A}{a}$$

$$\frac{\sin 80°}{12} = \frac{\sin A}{9.6}$$

$$9.6 \sin 80° = 12 \sin A$$

$$\frac{9.6 \sin 80°}{12} = \frac{\cancel{12} \sin A}{\cancel{12}}$$

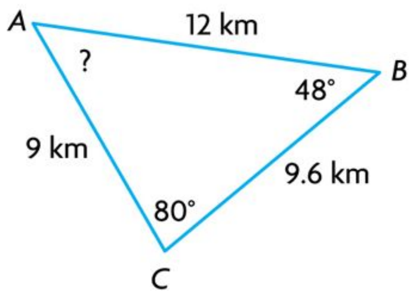
$$0.7878 = \sin A$$

$$\sin^{-1}(0.7878) = A$$

$$51.9804 = A$$

$$\angle A = 52°$$

$$52° + 48° + 80° = 180°$$



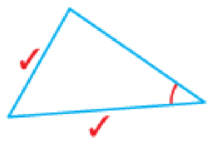
In Summary

Key Idea

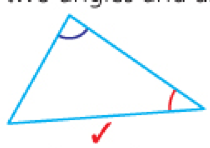
- The sine law can be used to determine unknown side lengths or angle measures in acute triangles.

Need to Know

- You can use the sine law to solve a problem modelled by an acute triangle when you know:
  - two sides and the angle opposite a known side.



  - two angles and any side.



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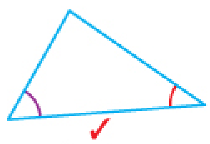


- If you know the measures of two angles in a triangle, you can determine the third angle because the angles must add to 180°.
- When determining side lengths, it is more convenient to use:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

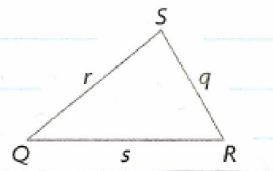
- When determining angles, it is more convenient to use:

$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

<u>Assignment:</u> **pgs. 124 - 127**
1, 2, 3, 4, 5, 7, 15

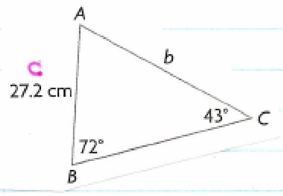
SOLUTIONS ⇒ Proving and Applying the Sine Law

1. Write three equivalent ratios using the sides and angles in the triangle at the right.



$$\frac{q}{\sin Q} = \frac{r}{\sin R} = \frac{s}{\sin S}$$

2 a) Determine length b to the nearest tenth of a centimetre.



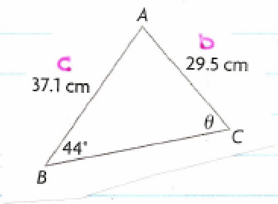
$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{b}{\sin 72^\circ} = \frac{27.2}{\sin 43^\circ}$$

$$\frac{b\cancel{\sin 43^\circ}}{\cancel{\sin 43^\circ}} = \frac{(27.2)(\sin 72^\circ)}{\sin 43^\circ}$$

$$b = 37.9\text{ cm}$$

b) Determine the measure of θ to the nearest degree.



$$\frac{\sin C}{c} = \frac{\sin B}{b}$$

$$\frac{\sin \theta}{37.1} = \frac{\sin 44^\circ}{29.5}$$

$$\frac{\cancel{29.5}\sin\theta}{\cancel{29.5}} = \frac{(37.1)(\sin 44^\circ)}{29.5}$$

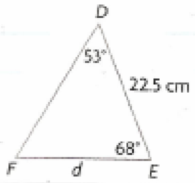
$$\sin\theta = 0.8736$$

$$\theta = \sin^{-1}(0.8736)$$

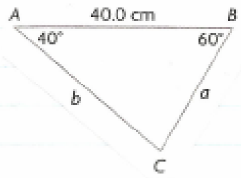
$$\theta = 61^\circ$$

3. Determine the indicated side lengths to the nearest tenth of a unit and the indicated angle measures to the nearest degree.

a)



b)



Need to find ∠F first:

$F = 180° - 68° - 53°$

$F = 59°$

$$\frac{d}{\sin D} = \frac{f}{\sin F}$$

$$\frac{d}{\sin 53°} = \frac{22.5}{\sin 59°}$$

$$\frac{d \cancel{\sin 59°}}{\cancel{\sin 59°}} = \frac{22.5 \sin 53°}{\sin 59°}$$

$d = 21.0$ cm

Need to find ∠C first:

$C = 180° - 40° - 60°$

$C = 80°$

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$

$$\frac{a}{\sin 40°} = \frac{40.0}{\sin 80°}$$

$$\frac{a \cancel{\sin 80°}}{\cancel{\sin 80°}} = \frac{40.0 \sin 40°}{\sin 80°}$$

$a = 26.1$

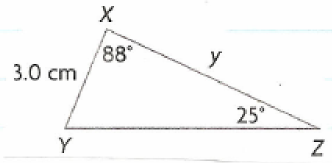
$$\frac{b}{\sin B} = \frac{c}{\sin C}$$

$$\frac{b}{\sin 60°} = \frac{40.0}{\sin 80°}$$

$$\frac{b \cancel{\sin 80°}}{\cancel{\sin 80°}} = \frac{40.0 \sin 60°}{\sin 80°}$$

$b = 35.2$

c)



Need to find <Y first:

$Y = 180° - 88° - 25°$

$Y = 67°$

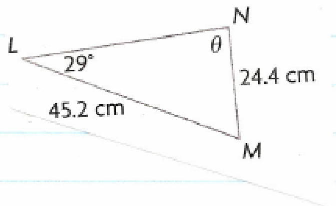
$$\frac{y}{\sin Y} = \frac{z}{\sin Z}$$

$$\frac{y}{\sin 67°} = \frac{3.0}{\sin 25°}$$

$$\frac{y\cancel{\sin 25°}}{\cancel{\sin 25°}} = \frac{3.0 \sin 67°}{\sin 25°}$$

$y = 6.5\text{ cm}$

d)



$$\frac{\sin N}{n} = \frac{\sin L}{l}$$

$$\frac{\sin \theta}{45.2} = \frac{\sin 29°}{24.4\text{ cm}}$$

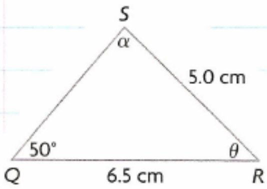
$$\frac{\cancel{24.4}\sin \theta}{\cancel{24.4}} = \frac{45.2 \sin 29°}{24.4}$$

$\sin \theta = 0.8981$

$\theta = \sin^{-1}(0.8981)$

$\theta = 64°$

e)



$$\frac{\sin S}{s} = \frac{\sin Q}{q}$$

$$\frac{\sin \alpha}{6.5} = \frac{\sin 50^\circ}{5.0}$$

$$\frac{\cancel{5.0}\sin \alpha}{\cancel{5.0}} = \frac{6.5 \sin 50^\circ}{5.0}$$

$$\sin \alpha = 0.9959$$

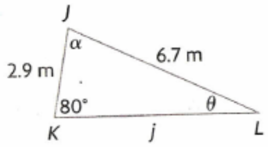
$$\alpha = \sin^{-1}(0.9959)$$

$$\alpha = 85^\circ$$

To find <R:

$$\theta = 180^\circ - 50^\circ - 85^\circ$$
$$= 45^\circ$$

f)



$\frac{\sin L}{l} = \frac{\sin K}{k}$

$\frac{\sin \theta}{2.9} = \frac{\sin 80°}{6.7}$

$\frac{\cancel{6.7}\sin\theta}{\cancel{6.7}} = \frac{2.9 \sin 80°}{6.7}$

$\sin\theta = 0.4263$

$\theta = \sin^{-1}(0.4263)$

$\theta = 25°$

To find < J:

$\alpha = 180° - 80° - 25°$

$\alpha = 75°$

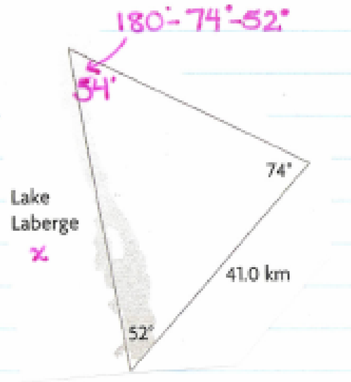
$\frac{j}{\sin J} = \frac{k}{\sin K}$

$\frac{j}{\sin 75°} = \frac{6.7}{\sin 80°}$

$\frac{j\cancel{\sin 80°}}{\cancel{\sin 80°}} = \frac{6.7 \sin 75°}{\sin 80°}$

$j = 6.6\text{m}$

4. Scott is studying the effects of environmental changes on fish populations in his summer job. As part of his research, he needs to know the distance between two points on Lake Laberge, Yukon. Scott makes the measurements shown and uses the sine law to determine the lake's length as 36.0 Km.



a) Agathe, Scott's research partner, says that his answer is incorrect. Explain how she knows.

The length of the lake is opposite the largest angle of the triangle, therefore it must be the longest side. A length of 36 km would not make it the longest side.

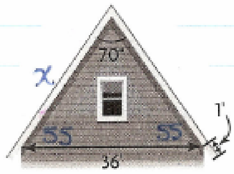
b) Determine the distance between the two points to the nearest tenth of a kilometer.

$$\frac{x}{\sin 74^\circ} = \frac{41.0}{\sin 54^\circ}$$

$$\frac{x \sin 54^\circ}{\sin 54^\circ} = \frac{41.0 \sin 74^\circ}{\sin 54^\circ}$$

$$x = 48.7 \text{ km}$$

5. An architect designed a house and must give more instructions to the builders. The rafters that hold up the roof are equal in length. The rafters extend beyond the supporting wall as shown. How long are the rafters? Express your answer to the nearest inch.



Since the roof is an isosceles triangle, the remaining angles would be:

$$\frac{180° - 70°}{2} = \frac{110°}{2} = 55°$$

$$\frac{x}{\sin 55°} = \frac{36}{\sin 70°}$$

$$\frac{x \sin 70°}{\sin 70°} = \frac{36 \sin 55°}{\sin 70°}$$

$$x = 31.4 \text{ ft}$$

Each rafter would be:

31.4 ft + 1 ft

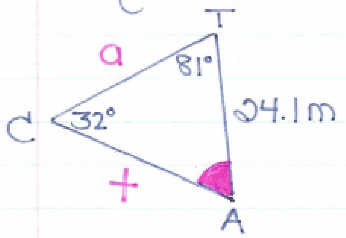
= 32.4 ft

or

32 ft 5 in.

7. In $\triangle CAT$, $\angle C = 32°$, $\angle T = 81°$, and $c = 24.1$ m. Solve the triangle. Round sides to the nearest tenth of a meter.

Sketch:



To find $\angle A$:

$180° - 81° - 32°$
$= 67°$

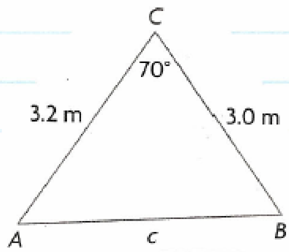
To find a:

$$\frac{a}{\sin A} = \frac{c}{\sin C}$$
$$\frac{a}{\sin 67°} = \frac{24.1}{\sin 32°}$$
$$\frac{a \cancel{\sin 32°}}{\cancel{\sin 32°}} = \frac{24.1 \sin 67°}{\sin 32°}$$
$$a = 41.9 \text{ m}$$

To find t:

$$\frac{t}{\sin T} = \frac{c}{\sin C}$$
$$\frac{t}{\sin 81°} = \frac{24.1}{\sin 32°}$$
$$\frac{t \cancel{\sin 32°}}{\cancel{\sin 32°}} = \frac{24.1 \sin 81°}{\sin 32°}$$
$$t = 44.9 \text{ m}$$

15. Jim says that the sine law cannot be used to determine the length of side c in $\triangle ABC$. Do you agree or disagree? Explain.



I agree with Jim. In order to find "c", you would need to know an angle and its opposite side. In this diagram we do not know "A" or "B".

PM11-3s2.gsp

Compass.html

Compass.swf

3s2e1 final.mp4

3s2e3 final.mp4