4.1 Exploring the Primary Trigonometric Ratios of Obtuse Angles

(sin, cos, tan)

(greater than 90°)

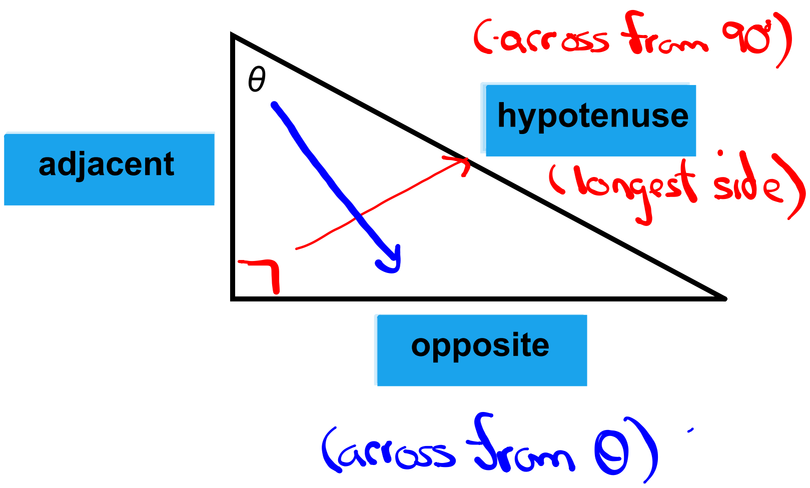
GOAL

Determine the relationships between the primary trigonometric ratios of acute and obtuse angles.

(less than 90°)

Recall the primary trigonometric ratios.
Place the labels in the correct boxes around the right triangle, relative to θ, and in the ratios.

adjacent | opposite | hypotenuse



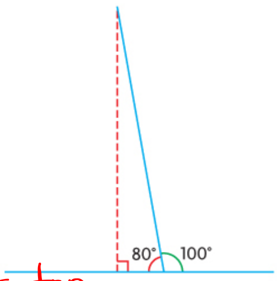
$$\sin\theta = \frac{\text{opposite}}{\text{hypotenuse}}$$

$$\cos\theta = \frac{\text{adjacent}}{\text{hypotenuse}}$$

$$\tan\theta = \frac{\text{opposite}}{\text{adjacent}}$$

EXPLORE the Math

Until now, you have used the primary trigonometric ratios only with **acute angles**. For example, you have used these ratios to determine the side lengths and angle measures in right triangles, and you have used the sine and cosine laws to determine the side lengths and angle measures in acute **oblique triangles**.

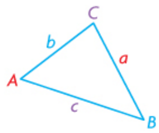


sin, cos, tan

Joe investigated the values of the primary trigonometric ratios for **obtuse angles**. Using a calculator, he determined that the value of sin 100° is 0.9848… .

He knew that he could not create a right triangle with a 100° angle. However, he knew that he could create a triangle using the supplement of 100°, which is 80°. Out of curiosity, he evaluated sin 80° and determined that it has the same value, 0.9848… .

Joe decided to broaden his investigation. He created a table like the one below.



sine law
$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}$$

cosine law
$$a^2 = b^2 + c^2 - 2bc \cos A$$

oblique triangle
A triangle that does not contain a 90° angle.

θ	sin θ	cos θ	tan θ	(180° – θ)	sin (180° – θ)	cos (180° – θ)	tan (180° – θ)
100°	0.9848	-0.1736	-5.6713	80°	0.9848	0.1736	5.6713
110°	0.9397	-0.3420	-2.7475	70°	0.9397	[illegible]	2.7475
120°				60°			
130°				50°			
140°				40°			
150°				30°			
160°				20°			
170°				10°			
180°				0°			

What relationships do you observe when comparing the trigonometric ratios for obtuse angles with the trigonometric ratios for the related supplementary acute angles?

Answer

θ	sin θ	cos θ	tan θ	(180° – θ)	sin (180° – θ)	cos (180° – θ)	tan (180° – θ)
100°	0.9848	−0.1736	−5.6713	80°	0.9848	0.1736	5.6713
110°	0.9397	−0.3420	−2.7475	70°	0.9397	0.3420	2.7475
120°	0.8660	−0.5000	−1.7321	60°	0.8660	0.5000	1.7321
130°	0.7660	−0.6428	−1.1918	50°	0.7660	0.6428	1.1918
140°	0.6428	−0.7660	−0.8391	40°	0.6428	0.7660	0.8391
150°	0.5000	−0.8660	−0.5774	30°	0.5000	0.8660	0.5774
160°	0.3420	−0.9397	−0.3640	20°	0.3420	0.9397	0.3640
170°	0.1736	−0.9848	−0.1763	10°	0.1736	0.9848	0.1763
180°	0	−1	0	0°	0	1	0

(add to 180°)

The sine ratios for supplementary angles are equal.

The cosine and tangent ratios for supplementary angles are opposites.

(add to 180°)

Ex: $\theta = 115°$ $\quad \sin(180° - 115°)$

$\sin 115° = 0.9063$ $\quad \sin(65°) = 0.9063$

$\cos 115° = -0.4226$ $\quad \cos 65° = 0.4226$

$\tan 115° = -2.1445$ $\quad \tan 65° = 2.1445$

Reflecting

A. Compare your observations with a classmate's observations. How are they different? How are they alike?

B. Describe any patterns you observed as the measure of the obtuse angle increased.

Answers

A. I noticed that the sine ratios are the same for an angle and its supplementary angle. My partner noticed that although the decimal expansions are equal, the cosine and tangent ratios are opposites for supplementary angles.

B. I noticed that both the sine and tangent ratios approached zero as the obtuse angle increased. Also, as the obtuse angle increased, the cosine for an angle and its supplementary angle approached -1 and 1, respectively.

In Summary

Key Idea

- There are relationships between the value of a primary trigonometric ratio for an acute angle and the value of the same primary trigonometric ratio for the supplement of the acute angle.

Need to Know

- For any angle θ,
  $\sin \theta = \sin (180° - \theta)$
  $\cos \theta = -\cos (180° - \theta)$
  $\tan \theta = -\tan (180° - \theta)$

Assignment: page 163 (1, 2, 3)

SOLUTIONS ⇒ 4.1 Exploring the Primary Trigonometric Ratios of Obtuse Angles

1. Which of the following equations are valid? Give reasons for your choices.

a) $\sin 25° = \sin 65°$

This is not valid.
$\sin 25° = \sin(180° - 25°)$
$\sin 25° = \sin 155°$

b) $\cos 70° = -\cos 110°$

This is valid.
* $\cos \theta = -\cos(180° - \theta)$

c) $\tan 46° = \tan 134°$

This is not valid.
$\tan 46° = -\tan(180° - 46°)$
$\tan 46° = -\tan(134°)$

d) $\sin 122° = \sin 58°$

This is valid

* $\sin\theta = \sin(180° - \theta)$

e) $\cos 135° = \cos 45°$

This is not valid.

$\cos 135° = -\cos(180° - 135°)$

$\cos 135° = -\cos 45°$

f) $\tan 175° = -\tan 5°$

This is valid

* $\tan\theta = -\tan(180° - \theta)$

2. Calculate each ratio to four decimal places. Predict another angle that will have an equal or opposite trigonometric ratio. Check your prediction.

a) $\sin 15°$

$\sin 15° = 0.2588$

$\sin\theta = \sin(180° - \theta)$
$\sin 15° = \sin(180° - 15°)$
$\sin 15° = \sin 165°$

Check: $\sin 165° = 0.2588$ ✓

b) $\cos 62°$

$\cos 62° = 0.4695$

$\cos\theta = -\cos(180° - \theta)$
$\cos 62° = -\cos(180° - 62°)$
$\cos 62° = -\cos(118°)$

Check: $-\cos 118° = 0.4695$ ✓

c) $\tan 35°$

$$\boxed{\tan 35° = 0.7002}$$

$\tan \theta = -\tan(180° - \theta)$
$\tan 35° = -\tan(180° - 35°)$
$\tan 35° = -\tan 145°$

Check: $-\tan 145° = 0.7002$ ✓

d) $\sin 170°$

$$\boxed{\sin 170° = 0.1736}$$

$\sin \theta = \sin(180° - \theta)$
$\sin 170° = \sin(180° - 170°)$
$\sin 170° = \sin 10°$

Check: $\sin 10° = 0.1736.$ ✓

3. Determine two angles between 0° and 180° that have each sine ratio.

a) 0.64

$\sin \theta = 0.64$
$\theta = \sin^{-1}(0.64)$
$\theta = 40°$

$\sin\theta = \sin(180° - \theta)$
$\sin 40° = \sin(180° - 40°)$
$\sin 40° = \sin 140°$

The two angles are 40° and 140°

b) $\frac{1}{3}$

$\sin \theta = 0.3333$
$\theta = \sin^{-1}(0.3333)$
$\theta = 19°$

$\sin\theta = \sin(180° - \theta)$
$\sin 19° = \sin(180° - 19°)$
$\sin 19° = \sin 161°$

The two angles are 19° and 161°

c) 0.95.

$\sin\theta = 0.95$
$\theta = \sin^{-1}(0.95)$
$\theta = 72°$

$\sin\theta = \sin(180° - \theta)$
$\sin 72° = \sin(180° - 72°)$
$\sin 72° = \sin 108°$

The two angles are 72° and 108°.

d) $\frac{7}{23}$

$\sin\theta = 0.3043$
$\theta = \sin^{-1}(0.3043)$
$\theta = 18°$

$\sin\theta = \sin(180° - \theta)$
$\sin 18° = \sin(180° - 18°)$
$\sin 18° = \sin 162°$

The two angles are 18° and 162°.

Attachments

FM11-4s1.gsp