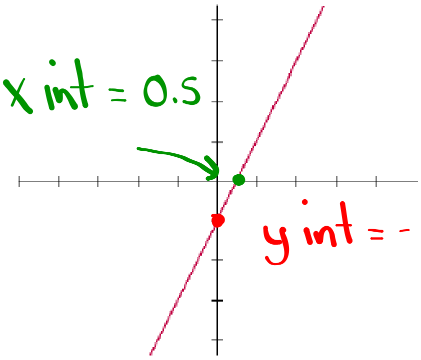
Catalog of Essential Functions

1. Linear

x int = 0.5



y int = -1

Straight Line

highest exponent

Equation will be degree one

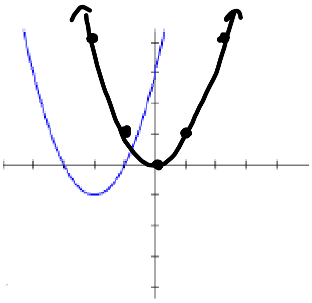
Should be able to identify the ***slope, intercepts, and equation*** from the graph

$m = \frac{rise}{run} = \frac{2}{1} = 2$

$y = mx + b$ (equation)

$y = 2x - 1$

2. Quadratic



degree of 2

Parabola (U-Shaped)

Either *y or x* will be squared (not both!)

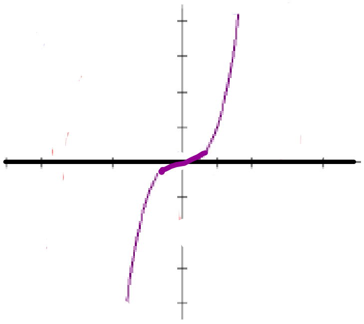
Should know the 4 basic quadratic functions

Should be able to apply transformations to the basic quadratic functions

$y = x^2$

x	y
-2	4
-1	1
0	0
1	1
2	4

3. Cubic



S-Shaped

We will work with functions that are raised to the third power

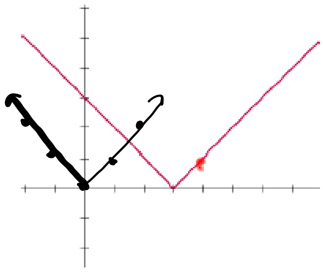
degree of 3

$y = x^3$

x	y
-2	-8
-1	-1
0	0
1	1
2	8

Catalog of Essential Functions

4. Absolute Value



V-Shaped

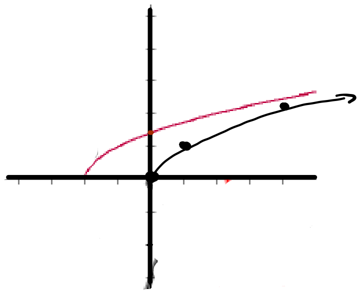
Equation will have a variable within the absolute value bars

Should be able to apply transformations to the basic absolute value function

$y=|x|$

x	y
-2	2
-1	1
0	0
1	1
2	2

5. Square Root /Radical



Half Parabola

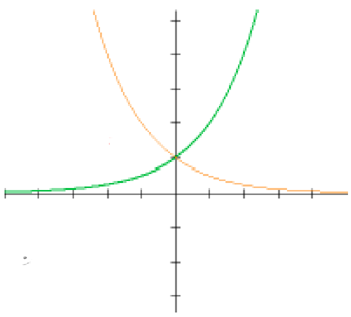
Equation will have a variable under the square root sign

Should be able to apply transformations to the basic square root function

$y=\sqrt{x}$

x	y
0	0
1	1
4	2
9	3

6. Exponential



Steadily increasing or decreasing

Base will be a number and variable will appear in the exponent $y=2^x$ | $y=5^x$ | $y=(\frac{1}{2})^x$

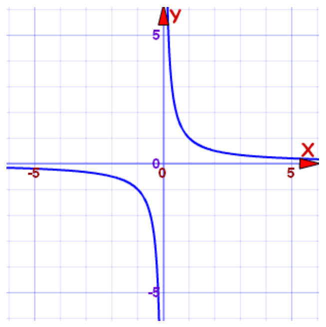
Should be able to identify the ***horizontal asymptote***

$y=2^x$

x	y
-2	1/4
-1	1/2
0	1
1	2
2	4

Catalog of Essential Functions

7. Reciprocal



Will have two branches

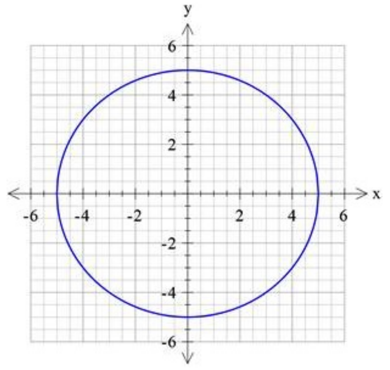
Equation will have a variable within the denominator of a rational expression

Should be able to identify the ***vertical and horizontal asymptotes***

$y = \frac{1}{x}$

x	y
-2	$-\frac{1}{2}$ or -0.5
-1	-1
0	undefined
1	1
2	$\frac{1}{2}$ or 0.5

8. Circle

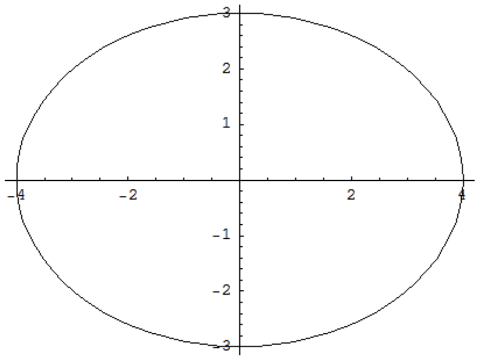


- General form: $(x-h)^2 + (y-k)^2 = r^2$

  * center: (h, k)
  * radius $= r$

- Be able to identify the function that would describe either just the top or bottom of the circle.

9. Ellipse



- General form: $\frac{(x-h)^2}{a^2} + \frac{(y-k)^2}{b^2} = 1$

Where...
- Center: (h, k)
- a > b
- If a is the denominator of the "y" term the ellipse will have a vertical major axis.

Transformations:

New Functions From Old Functions

1. Translations
2. Stretches
3. Reflections

Translations

h = horizontal translation (Shift Left/Right) add to x-values

K = vertical translation (Shift Up/Down) add to y-values

Focus on…

- determining the effects of *h* and *k* in $\boxed{y - k = f(x - h)}$ or $y = f(x-\underline{h})+\underline{k}$ on the graph of $y = f(x)$
- sketching the graph of $y - k = f(x - h)$ for given values of *h* and *k*, given the graph of $y = f(x)$
- writing the equation of a function whose graph is a vertical and/or horizontal translation of the graph of $y = f(x)$

Ex: ① $y = (x\underline{-3})^2 \underline{+2}$

$k = 2 \rightarrow$ Up 2

$h = 3 \rightarrow$ Right 3

② $y - 4 = |x + 3|$

$y = |x\underline{+3}| + \underline{4}$

$k = 4 \rightarrow$ Up 4

$h = -3 \rightarrow$ Left 3

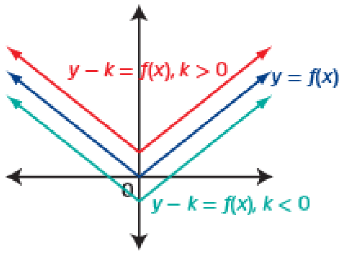
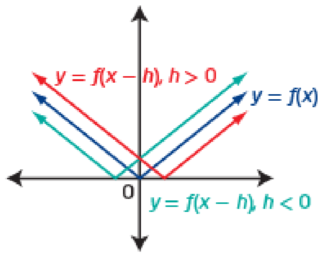
Function notation

③ $g(x) = f(x\underline{+2}) - \underline{1}$

$k = -1 \rightarrow$ down 1

$h = -2 \rightarrow$ Left 2

- Translations are transformations that shift all points on the graph of a function up, down, left, and right without changing the shape or orientation of the graph.
- The table summarizes translations of the function $y = f(x)$.

Function	Transformation from $y = f(x)$	Mapping	Example
$y - k = f(x)$ or $y = f(x) + k$	A vertical translation If $k > 0$, the translation is up. If $k < 0$, the translation is down.	$(x, y) \rightarrow (x, y + k)$	
$y = f(x - h)$	A horizontal translation If $h > 0$, the translation is to the right. If $h < 0$, the translation is to the left.	$(x, y) \rightarrow (x + h, y)$	

* change the sign when you remove from brackets

- A sketch of the graph of $y - k = f(x - h)$, or $y = f(x - h) + k$, can be created by translating key points on the graph of the base function $y = f(x)$.

Translation

- To *translate* or *shift* a graph is to move it up, down, left, or right without changing its shape.
- Translation is summarized by the following table and illustration:

Vertical and Horizontal Shifts Suppose $c > 0$. To obtain the graph of

$y = f(x) + c$, shift the graph of $y = f(x)$ a distance c units upward

$y = f(x) - c$, shift the graph of $y = f(x)$ a distance c units downward

$y = f(x - c)$, shift the graph of $y = f(x)$ a distance c units to the right

$y = f(x + c)$, shift the graph of $y = f(x)$ a distance c units to the left

Translations illustrated...

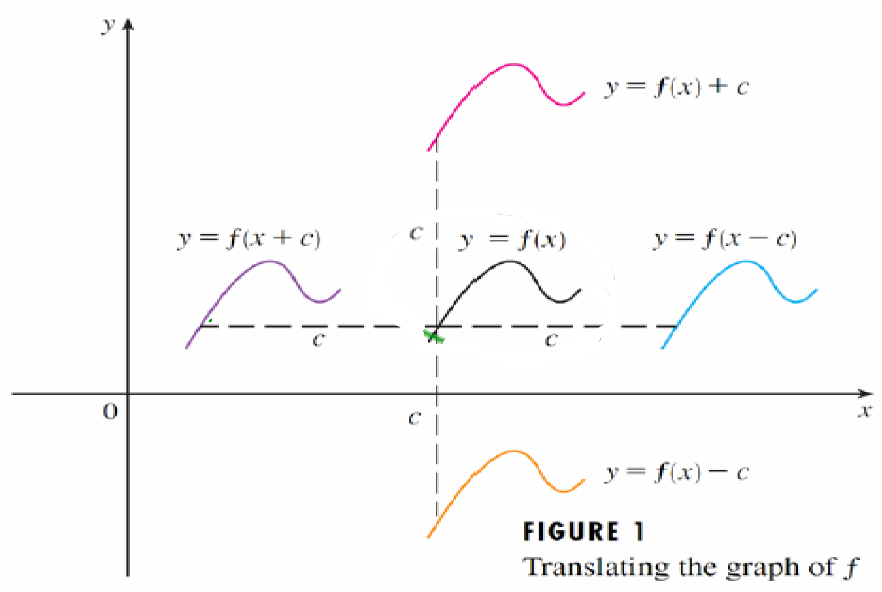


FIGURE 1
Translating the graph of f

Using Mapping Notation to Describe Transformations:

*Think of this as a set of instructions to follow to transform a graph.

$y=x^2$ $\quad k=2 \rightarrow$ up 2 $\quad h=5$ Right 5

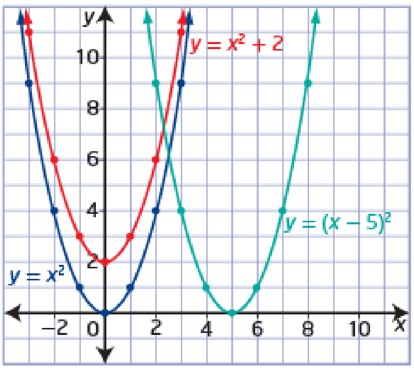
x	$y=x^2$
−3	9
−2	4
−1	1
0	0
1	1
2	4
3	9

x	$y=x^2+\underline{2}$
−3	11
−2	6
−1	3
0	2
1	3
2	6
3	11

$(x,y)\rightarrow(x, y+2)$

x	$y=(x-\underline{5})^2$
2	9
3	4
4	1
5	0
6	1
7	4
8	9

$(x,y)\rightarrow(x+5, y)$



Make table showing $y=\sqrt{x+1}-3$

$h=-1$

$k=-3$

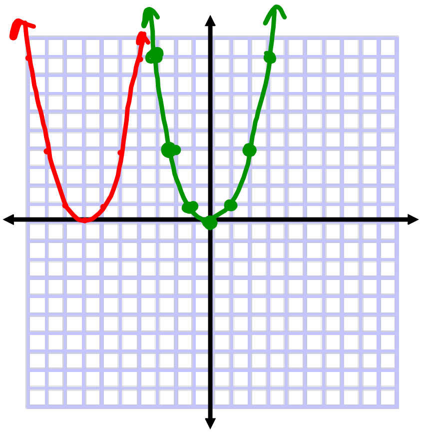
$(x,y)\rightarrow(x-1, y-3)$

base $y=\sqrt{x}$

x	y
0	0
1	1
4	2
9	3

x	y
−1	−3
0	−2
3	−1
8	0

Identify the translations for each of the following and sketch the transformation



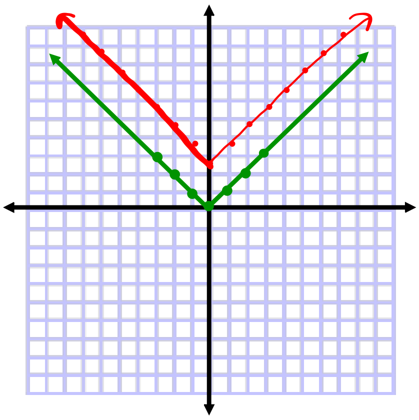
base:
$f(x) = x^2$

x	f(x)
-2	4
-1	1
0	0
1	1
2	4

$h = -7$

$f(x) = (x+7)^2$

x	f(x)
-9	4
-8	1
-7	0
-6	1
-5	4



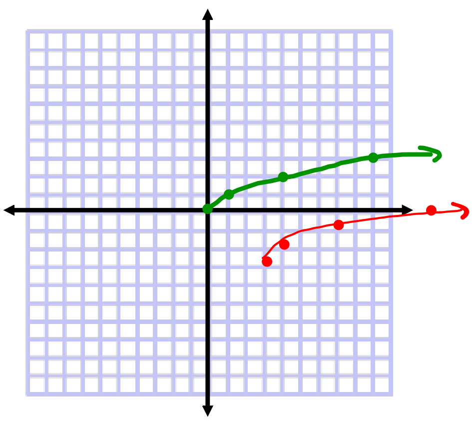
base:
$f(x) = |x|$

x	f(x)
-2	2
-1	1
0	0
1	1
2	2

$k = 3$

$f(x) = |x| + 3$

x	f(x)
-2	5
-1	4
0	3
1	4
2	5



base:
$f(x) = \sqrt{x}$

x	f(x)
0	0
1	1
4	2
9	3

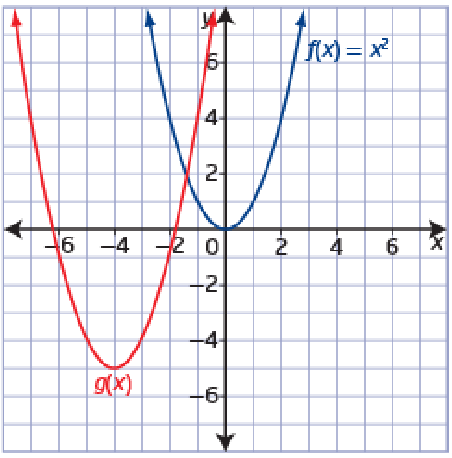
$h = 3$ $k = -2$

$f(x) = \sqrt{x-3} - 2$

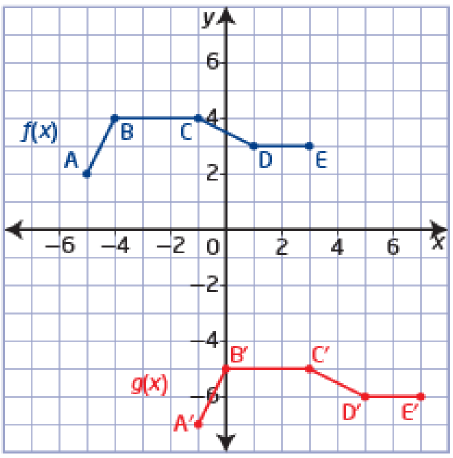
x	f(x)
3	-2
4	-1
7	0
12	1

Determine the Equation of a Translated Function:

a)



b)



Homework