

Warm-Up

horizontal
↓ (change sign)

8. Copy and complete the table.

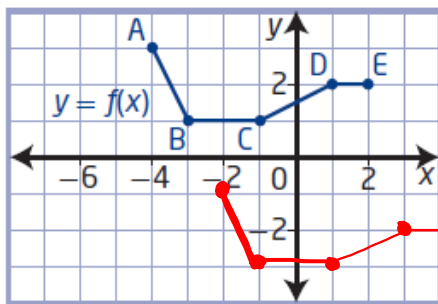
$$y = f(x-h) + k$$

↑ vertical

Translation	Transformed Function	Transformation of Points	
vertical	$y = f(x) + \underline{5}$	$(x, y) \rightarrow (x, y + 5)$	$k = 5$ (Up)
horizontal	$y = f(x \underline{+} 7)$	$(x, y) \rightarrow (x - 7, y)$	$h = -7$ (Left)
horizontal	$y = f(x \underline{-} 3)$	$(x, y) \rightarrow (x + 3, y)$	$h = 3$ (Right)
vertical	$y = f(x) \underline{-} 6$	$(x, y) \rightarrow (x, y - 6)$	$k = -6$ (Down)
horizontal and vertical	$y = f(x \underline{+} 4) \underline{-} 9$ $y + 9 = f(x + 4)$	$(x, y) \rightarrow (x - 4, y - 9)$	$h = -4$ Left $k = -9$ Down
horizontal and vertical	$y = f(x \underline{-} 4) \underline{-} 6$	$(x, y) \rightarrow (x \underline{+} 4, y \underline{-} 6)$	$h = 4$ Right $k = -6$ Down
$h + v$	$y = f(x \underline{+} 2) + 3$	$(x, y) \rightarrow (x \underline{-} 2, y \underline{+} 3)$	$h = -2$ $k = 3$
horizontal and vertical	$y = f(x \underline{=} h) \underline{\pm} k$	$(x, y) \rightarrow (x \underline{+} h, y \underline{+} k)$	

Questions from Homework

4.



$$b) y = f(x - \underline{2}) - \underline{4}$$

$$h = 2 \quad k = -4$$

$$(x, y) \rightarrow (x + 2, y - 4)$$

$$A(-4, 3) \rightarrow (-2, -1)$$

$$B(-3, 1) \rightarrow (-1, -3)$$

$$C(-1, 1) \rightarrow (1, -3)$$

$$D(1, 2) \rightarrow (3, -2)$$

$$E(2, 2) \rightarrow (4, -2)$$

Transformations:

New Functions From Old Functions

Translations

Stretches

✓ Reflections

Reflections and Stretches

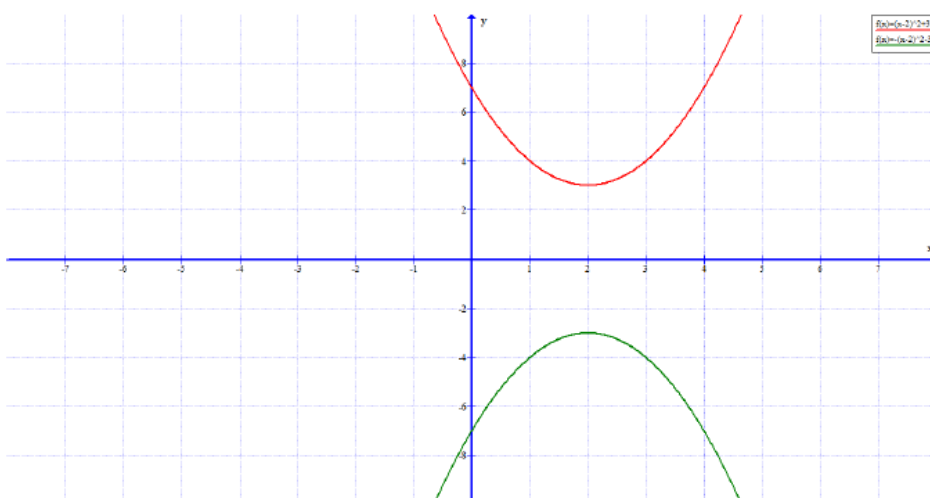
Focus on...

- developing an understanding of the effects of reflections on the graphs of functions and their related equations
- developing an understanding of the effects of vertical and horizontal stretches on the graphs of functions and their related equations

A **reflection** of a graph creates a mirror image in a line called the line of reflection. Reflections, like translations, do not change the shape of the graph. However, unlike translations, reflections may change the orientation of the graph.

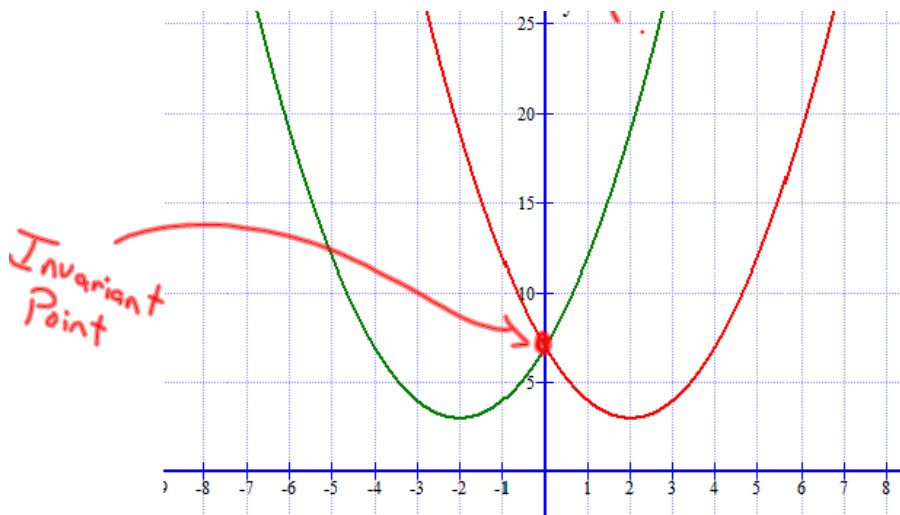
Vertical reflection $(x, y) \rightarrow (x, -y)$

- When the output of a function $y = f(x)$ is multiplied by -1 , the result, $y = -f(x)$, is a reflection of the graph in the x-axis.



Horizontal Reflection $(x, y) \rightarrow (-x, y)$

- When the input of a function $y = f(x)$ is multiplied by -1 , the result, $y = f(-x)$, is a reflection of the graph in the y-axis.

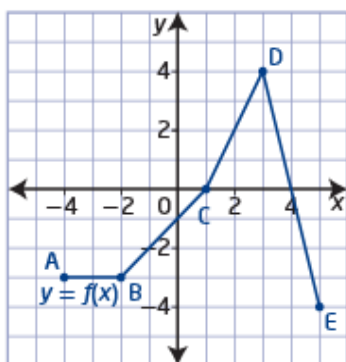


invariant point

- a point on a graph that remains unchanged after a transformation is applied to it
- any point on a curve that lies on the line of reflection is an invariant point

Example 1**Compare the Graphs of $y = f(x)$, $y = -f(x)$, and $y = f(-x)$**

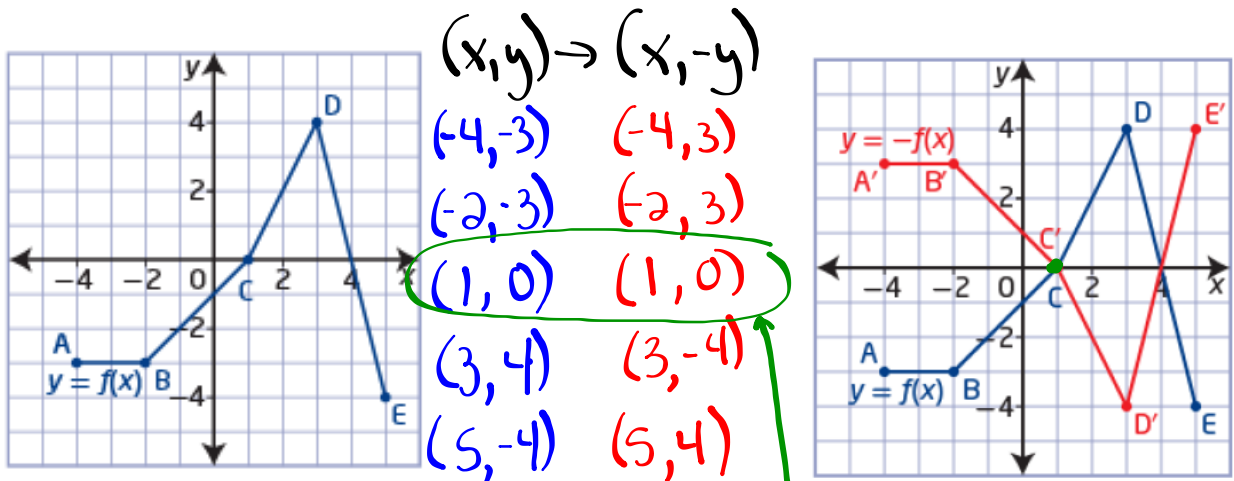
- a) Given the graph of $y = f(x)$, graph the functions $y = -f(x)$ and $y = f(-x)$.
- b) How are the graphs of $y = -f(x)$ and $y = f(-x)$ related to the graph of $y = f(x)$?



Remember...

- When the output of a function $y = f(x)$ is multiplied by -1 , the result, $y = -f(x)$, is a reflection of the graph in the x-axis.

- Sketch $y = -f(x)$ on the axis below (Vertical Reflection)

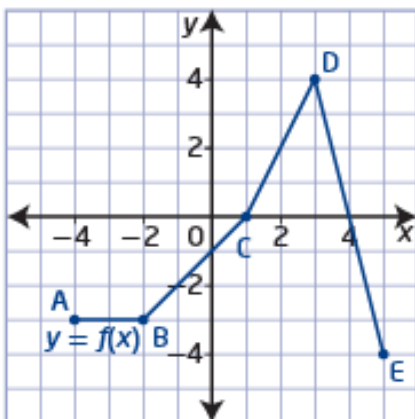


Invariant Point

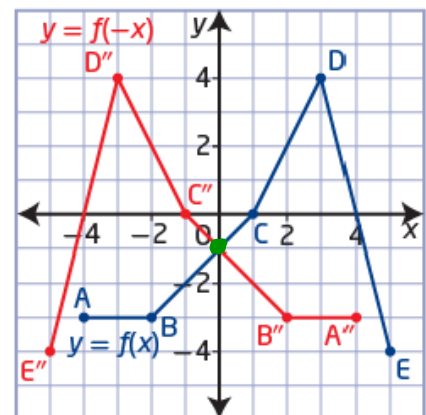
Remember...

- When the input of a function $y = f(x)$ is multiplied by -1 , the result, $y = f(-x)$, is a reflection of the graph in the y -axis.

- Sketch $y = f(-x)$ on the axis below Horizontal reflection



$(x, y) \rightarrow (-x, y)$
 $(-4, -3) \rightarrow (4, -3)$
 $(-2, -3) \rightarrow (2, -3)$
 $(1, 0) \rightarrow (-1, 0)$
 $(3, 4) \rightarrow (-3, 4)$
 $(5, -4) \rightarrow (-5, -4)$



Homework

$$\begin{aligned} *f(-4) &= 2(-4)+1 \\ &= -8+1 \\ &= -7 \end{aligned}$$

$$f(x) = 2x+1$$

x	y
* -4	-7
-2	-3
0	1
2	5
4	9

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Vertical

$$g(x) = -f(x)$$

x	y
-4	7
-2	3
0	-1
2	-5
4	-9

Horizontal

$$h(x) = f(-x)$$

x	y
4	-7
2	-3
0	1
-2	5
-4	9

stretch

- a transformation in which the distance of each x -coordinate or y -coordinate from the line of reflection is multiplied by some scale factor
- scale factors between 0 and 1 result in the point moving closer to the line of reflection; scale factors greater than 1 result in the point moving farther away from the line of reflection

Vertical and Horizontal Stretches

A **stretch**, unlike a translation or a reflection, changes the shape of the graph. However, like translations, stretches do not change the orientation of the graph.

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a , the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x -axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x -axis.
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b , the result, $y = f(bx)$, is a horizontal stretch of the graph about the y -axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y -axis.

Vertical Stretch or Compression...

- When the output of a function $y = f(x)$ is multiplied by a non-zero constant a , the result, $y = af(x)$ or $\frac{y}{a} = f(x)$, is a vertical stretch of the graph about the x -axis by a factor of $|a|$. If $a < 0$, then the graph is also reflected in the x -axis.

Example 2

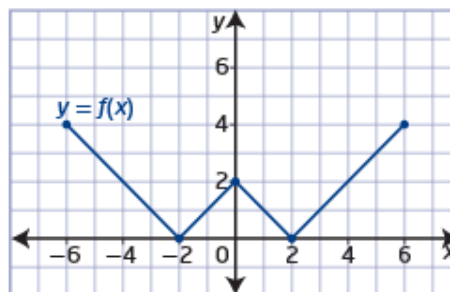
Graph $y = af(x)$

Given the graph of $y = f(x)$,

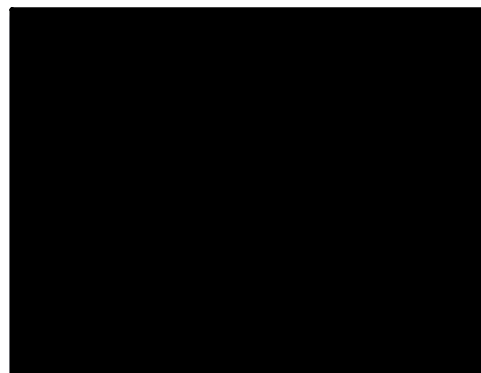
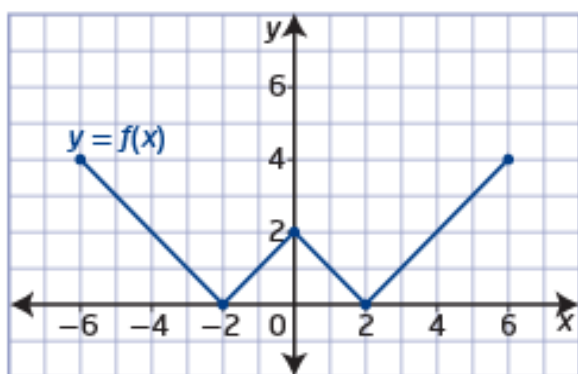
- transform the graph of $f(x)$ to sketch the graph of $g(x)$
- describe the transformation
- state any invariant points
- state the domain and range of the functions

a) $g(x) = 2f(x)$

b) $g(x) = \frac{1}{2}f(x)$



a) $g(x) = 2f(x)$



The invariant points are _____ and _____

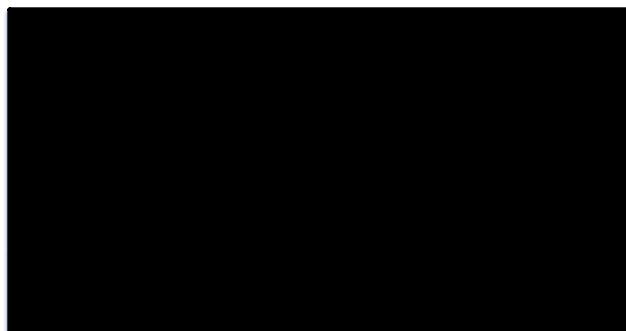
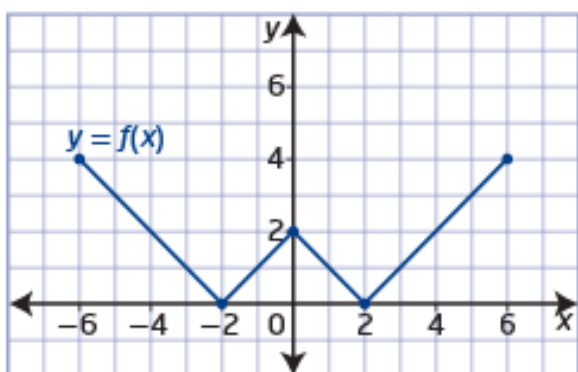
For $f(x)$, the domain is _____

and the range is _____

For $g(x)$, the domain is _____

and the range is _____

$$\text{b) } g(x) = \frac{1}{2}f(x)$$



The invariant points are _____ and _____

For $f(x)$, the domain is _____

and the range is _____

For $g(x)$, the domain is _____

and the range is _____

Horizontal Stretch or Compression...

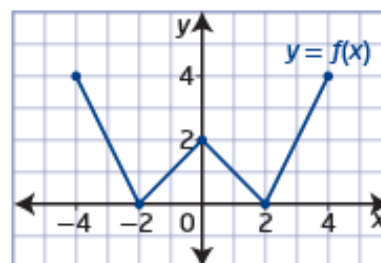
- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b , the result, $y = f(bx)$, is a horizontal stretch of the graph about the y -axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y -axis.

Example 3

Graph $y = f(bx)$

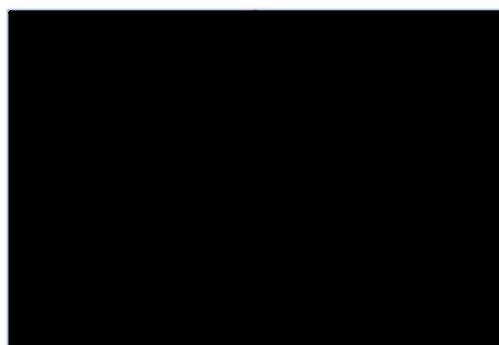
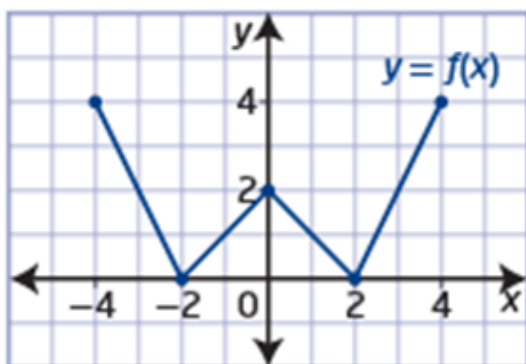
Given the graph of $y = f(x)$,

- transform the graph of $f(x)$ to sketch the graph of $g(x)$
- describe the transformation
- state any invariant points
- state the domain and range of the functions



- $g(x) = f(2x)$
- $g(x) = f\left(\frac{1}{2}x\right)$

a) $g(x) = f(2x)$



The invariant point is

For $f(x)$, the domain is

or and the range is

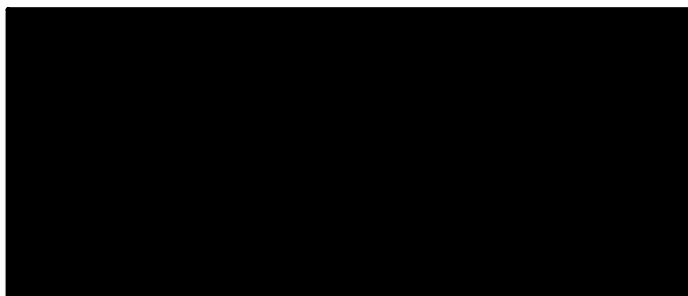
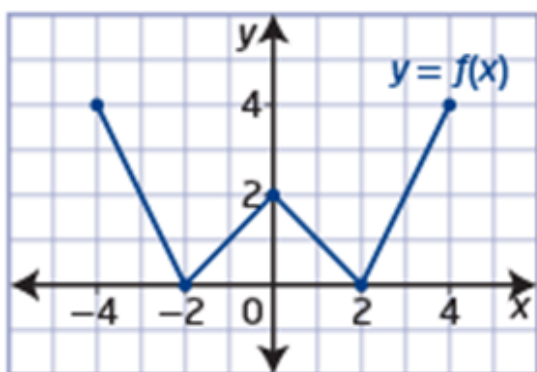
or

For $g(x)$, the domain is

or and the range is

or

$$\text{b) } g(x) = f\left(\frac{1}{2}x\right)$$

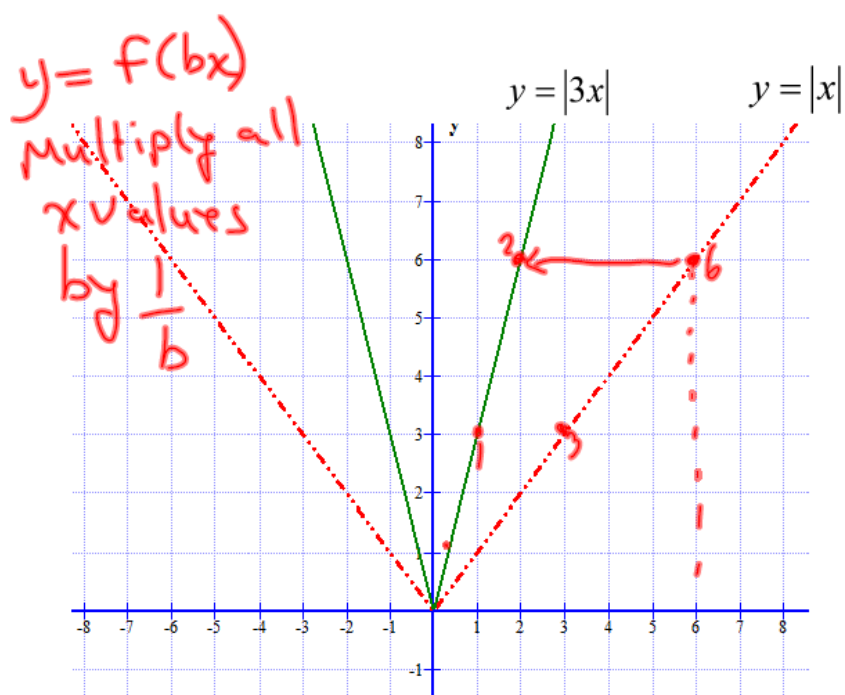


The invariant point is

For $f(x)$, the domain is
and the range is

For $g(x)$, the domain is
and the range is

Horizontal Stretch or Compression...



Horizontal Stretch or Compression...

- When the input of a function $y = f(x)$ is multiplied by a non-zero constant b , the result, $y = f(bx)$, is a horizontal stretch of the graph about the y -axis by a factor of $\frac{1}{|b|}$. If $b < 0$, then the graph is also reflected in the y -axis.

$$y = -3f(-2x) + 7$$

Homework

Determine the Equation of a Translated Function:

