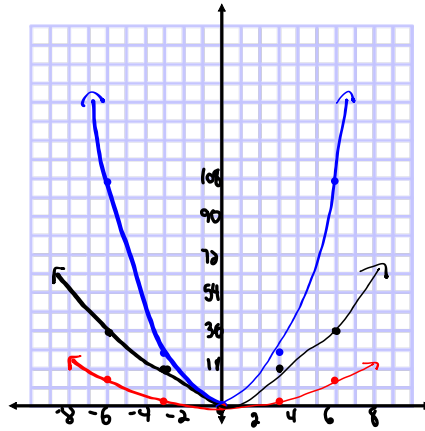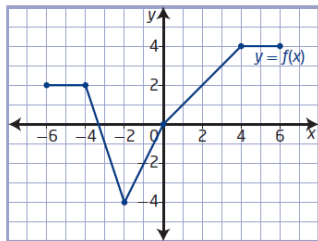# Questions from Homework

2. a) Copy and complete the table of values
for the given functions.

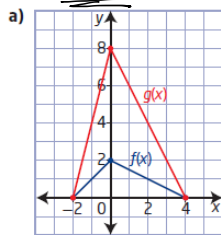| x	$f(x) = x^2$	$g(x) = 3f(x)$	$h(x) = \frac{1}{3}f(x)$
−6	36	108	12
−3	9	27	3
0	0	0	0
3	9	27	3
6	36	108	12

6. The graph of the function $y = f(x)$ is
<u>vertically stretched about the x-axis by
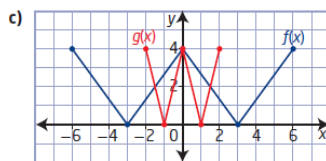a factor of 2.</u> $a = 2$

$(x,y) \rightarrow (x, 2y)$

$f(x)$ $g(x)$

$D: [-6,6]$ $D: [-6,6]$

$R: [-4,4]$ $R: [-8,8]$

7. Describe the transformation that
must be applied to the graph of $f(x)$
to obtain the graph of $g(x)$. Then,
determine the equation of $g(x)$ in the
form $y = af(bx)$.

a)

$(x,y) \rightarrow (x, 4y)$ A vertical stretch
by a factor of 4

$f(x)$ $g(x)$ $a = 4$

$(-2,0)$ $(-2,0)$

$(0,2)$ $(0,8)$ $\boxed{y = 4f(x)}$

$(4,0)$ $(4,0)$

c)

$(x,y) \rightarrow (\frac{1}{3}x, y)$ A horizontal
compression by
a factor of $\frac{1}{3}$

$f(x)$ $g(x)$

$(-6,4)$ $(-2,4)$

$(-3,0)$ $(-1,0)$ $b = 3$

$(0,4)$ $(0,4)$

$(3,0)$ $(1,0)$ $\boxed{y = f(3x)}$

$(6,4)$ $(2,4)$

1

Warm-Up...

Given that (-2 , 5) is a point on the graph of $y = f(x)$, determine the coordinates of this point once the following transformations are applied...

(1) $y = 3f(x)$

$a = 3 \rightarrow$ vertical stretch by a factor of 3

$(x, y) \rightarrow (x, 3y)$

$(-2, 5) \rightarrow (-2, 15)$

(2) $y = f\left(-\frac{1}{3}x\right)$

$b = -\frac{1}{3} \rightarrow$ horizontal stretch by a factor of 3 & a horizontal reflection in the y-axis

$(x, y) \rightarrow (-3x, y)$

$(-2, 5) \quad (6, 5)$

(3) $y = 4f\left[\left(\frac{1}{2}\right)(x + 5)\right] - 3$

$a = 4$ vertical stretch by a factor of 4

$b = \frac{1}{2}$ horizontal stretch by a factor of 2

$h = -5$ translated 5 units left

$K = -3$ translated 3 units down

$(x, y) \rightarrow (2x - 5, 4y - 3)$

$(-2, 5) \rightarrow (-9, 17)$

(4) $y - 5 = -2f(-2x + 6)$

factor

$y = -2f(-2x + 6) + 5$

$y = -2f[-2(x - 3)] + 5$

$a = -2$ vertical stretch by a factor of 2 and a vertical reflection in the x-axis

$b = -2$ horizontal stretch by a factor of $\frac{1}{2}$ and a horizontal reflection in the y-axis

$h = 3$ translated 3 units right

$K = 5$ translated 5 units up

$(x, y) \rightarrow \left(-\frac{1}{2}x + 3, -2y + 5\right)$

$(-2, 5) \rightarrow (4, -5)$

2

Transformations:

2. The function $y = f(x)$ is transformed to the function $g(x) = -3f(4x - 16) - 10$. Copy and complete the following statements by filling in the blanks.

The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the **a** by a factor of **b**. It is vertically stretched about the **c** by a factor of **d**. It is reflected in the **e**, and then translated **f** units to the right and **g** units down.

factor

$$g(x) = -3f(4x-16)-10$$

$$g(x) = -3f[4(x-4)]-10$$

$$a = -3 \qquad b = 4 \qquad h = 4 \qquad k = -10$$

a) y-axis

b) $\frac{1}{4}$

c) _____

d) _____

e) _____

f) _____

g) _____

Summary of Transformations...

Transformations of the graphs of functions	
$f(x) + k$	shift $f(x)$ up k units
$f(x) - k$	shift $f(x)$ down k units
$f(x + h)$	shift $f(x)$ left h units
$f(x - h)$	shift $f(x)$ right h units
$f(-x)$	reflect $f(x)$ about the y-axis
$-f(x)$	reflect $f(x)$ about the x-axis
$af(x)$	When $0 < a < 1$ – vertical shrinking of $f(x)$ When $a > 1$ – vertical stretching of $f(x)$ **Multiply the y values by a**
$f(bx)$	When $0 < b < 1$ – horizontal stretching of $f(x)$ When $b > 1$ – horizontal shrinking of $f(x)$ **Divide the x values by b**

Transformations:

$$y = f(x) \longrightarrow y = af\big(b(x - h)\big) + k$$
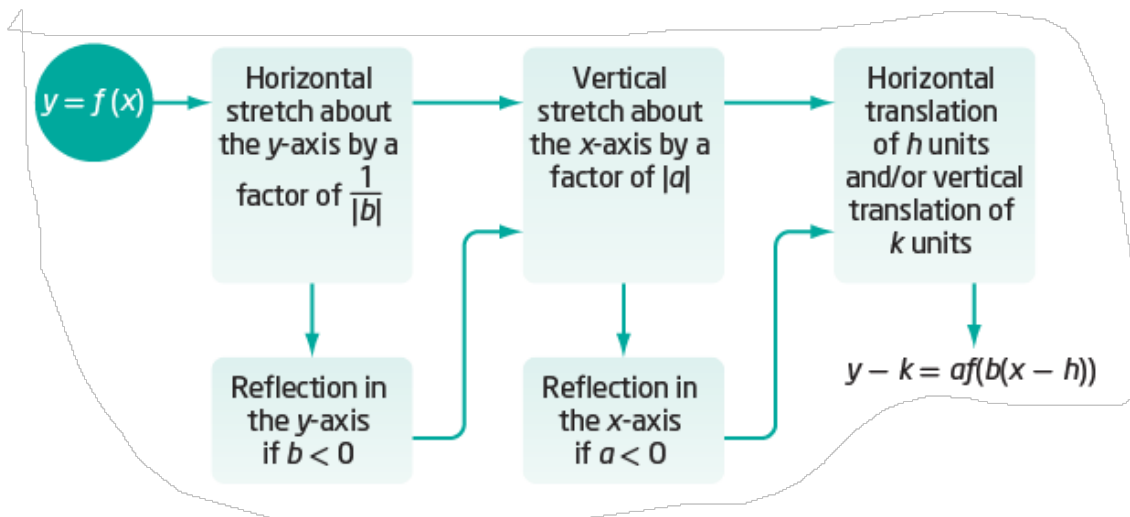
Mapping Rule: $(x, y) \rightarrow \left(\dfrac{1}{b}x + h, \, ay + k \right)$

Important note for sketching...

Transformations should be applied in following order:
1. Reflections
2. Stretches
3. Translations

Remember....RST

$y = f(x)$	Horizontal stretch about the y-axis by a factor of $\frac{1}{|b|}$	Vertical stretch about the x-axis by a factor of $|a|$	Horizontal translation of h units and/or vertical translation of k units
	Reflection in the y-axis if $b < 0$	Reflection in the x-axis if $a < 0$	$y - k = af(b(x - h))$

Example 1
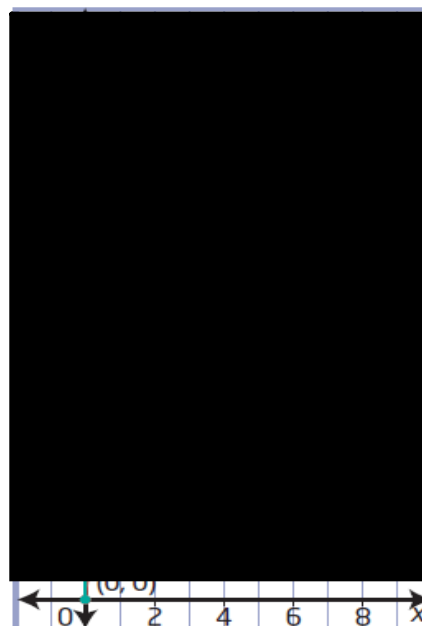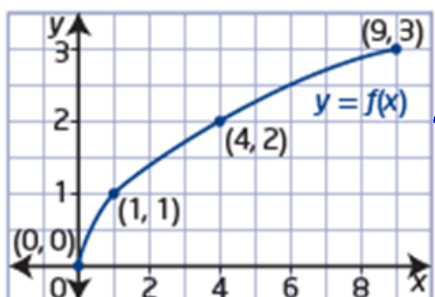
Graph a Transformed Function

Describe the combination of transformations that must be applied to the function $y = f(x)$ to obtain the transformed function. Sketch the graph, showing each step of the transformation.

a) $y = 3f(2x)$
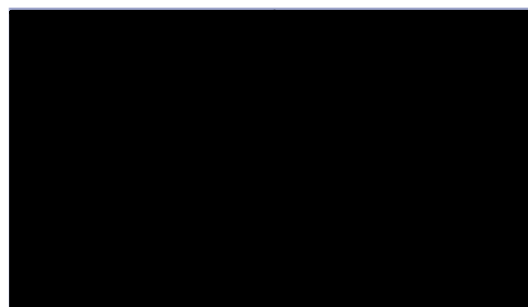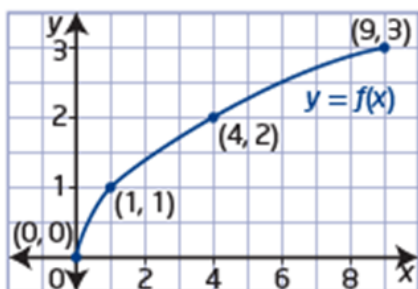
b) $y = f(3x + 6)$
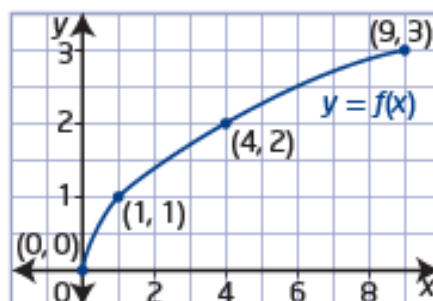
a) $y = 3f(2x)$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{2}$ and then vertically stretched about the x-axis by a factor of 3.

b) $y = f(3x + 6)$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{3}$ and then horizontally translated 2 units to the left.
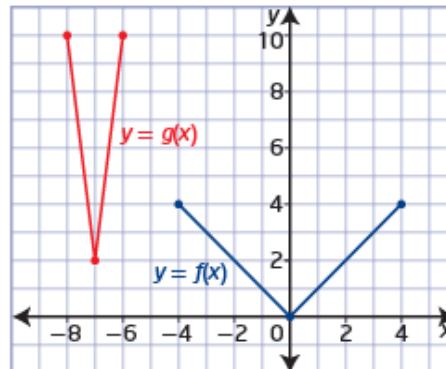
Homework

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Example 3

Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.
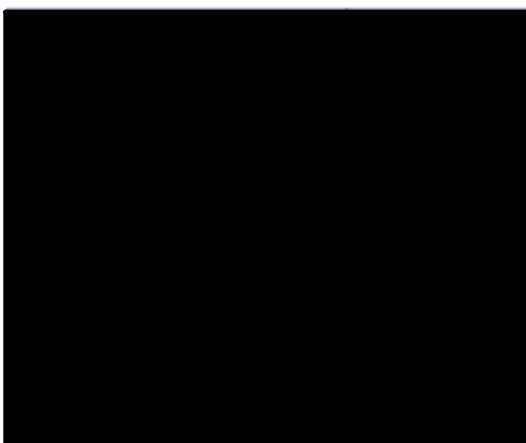

Solution

Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.

$(-4, 4) \rightarrow (-8, 10)$

$(0, 0) \rightarrow (-7, 2)$

$(4, 4) \rightarrow (-6, 10)$

The equation of the transformed function is ██████████████

██████████████████

How could you use the mapping

$$(x, y) \rightarrow \left(\frac{1}{b}x + h, \, ay + k\right) \text{ to}$$

verify this equation?

17. The graph of the function $y = 2x^2 + x + 1$ is stretched vertically about the x-axis by a factor of 2, stretched horizontally about the y-axis by a factor of $\frac{1}{3}$, and translated 2 units to the right and 4 units down. Write the equation of the transformed function.