Warm-Up...

Given that (-2 , 5) is a point on the graph of $y = f(x)$, determine the coordinates of this point once the following transformations are applied...

(1) $y = 3f(x)$

a = 3 → vertical stretch by a factor of 3

$(x,y) \to (x, 3y)$

$(-2,5) \to \boxed{(-2,15)}$

(2) $y = f\left(\left(-\frac{1}{3}\right)x\right)$

$b = -\frac{1}{3}$ → horizontal stretch by a factor of 3 & a horizontal reflection in the y-axis

$(x,y) \to (-3x, y)$

$(-2,5)$ $\boxed{(6,5)}$

(3) $y = 4f\left[\left(\frac{1}{2}\right)(x+5)\right] - 3$

a = 4 vertical stretch by a factor of 4

$b = \frac{1}{2}$ horizontal stretch by a factor of 2

h = -5 translated 5 units left

K = -3 translated 3 units down

$(x,y) \to (2x-5, 4y-3)$

$(-2,5) \to \boxed{(-9,17)}$

(4) $y - 5 = -2f(-2x+6)$

factor

$y = -2f(-2x+6) + 5$

$y = -2f[-2(x-3)] + 5$

a = -2 vertical stretch by a factor of 2 and a vertical reflection in the x-axis

b = -2 horizontal stretch by a factor of $\frac{1}{2}$ and a horizontal reflection in the y-axis

h = 3 translated 3 units right

K = 5 translated 5 units up

$(x,y) \to \left(-\frac{1}{2}x + 3, -2y + 5\right)$

$(-2,5) \to \boxed{(4,-5)}$

Transformations:

2. The function $y = f(x)$ is transformed to the function $g(x) = -3f(4x - 16) - 10$. Copy and complete the following statements by filling in the blanks.

The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the a by a factor of b. It is vertically stretched about the c by a factor of d. It is reflected in the e, and then translated f units to the right and g units down.

factor

$$g(x) = -3f(4x-16)-10$$

$$g(x) = -3f[4(x-4)]-10$$

$a=-3$ $b=4$ $h=4$ $k=-10$

a) y-axis

b) 1/4

c) x-axis

d) 3

e) x-axis

f) 4

g) 10

Summary of Transformations...

Transformations of the graphs of functions			
$f(x)+k$	shift $f(x)$ up k units	vertical trans.	$(x,y)\to(x,y+k)$
$f(x)-k$	shift $f(x)$ down k units	" "	$(x,y)\to(x,y-k)$
$f(x+h)$	shift $f(x)$ left h units	horizontal trans	$(x,y)\to(x-h,y)$
$f(x-h)$	shift $f(x)$ right h units	" "	$(x,y)\to(x+h,y)$
$f(-x)$	reflect $f(x)$ about the y-axis	horizontal ref.	$(x,y)\to(-x,y)$
$-f(x)$	reflect $f(x)$ about the x-axis	vertical ref.	$(x,y)\to(x,-y)$
$af(x)$	When $0 < a < 1$ – vertical shrinking of $f(x)$ When $a > 1$ – vertical stretching of $f(x)$ **Multiply the y values by a**		$(x,y)\to(x,ay)$
$f(bx)$	When $0 < b < 1$ – horizontal stretching of $f(x)$ When $b > 1$ – horizontal shrinking of $f(x)$ **Divide the x values by b**		$(x,y)\to(\frac{1}{b}x,y)$

Transformations:

$$y = f(x) \longrightarrow y = \underline{a}f\left(\underline{b}(x - \underline{h})\right) + \underline{k}$$

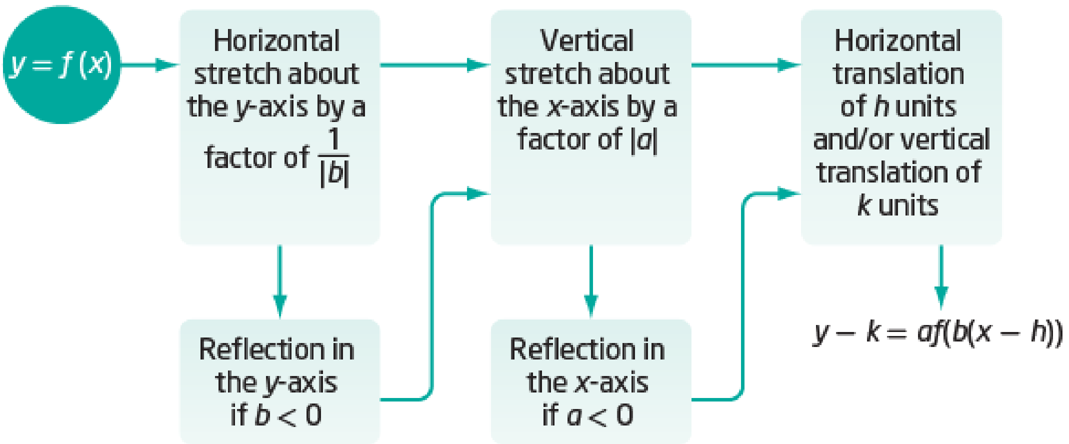
Mapping Rule: $(x, y) \rightarrow \left(\frac{1}{b}x + h, ay + k\right)$

Important note for sketching...

Transformations should be applied in following order:

1. Reflections
2. Stretches
3. Translations

Remember....RST



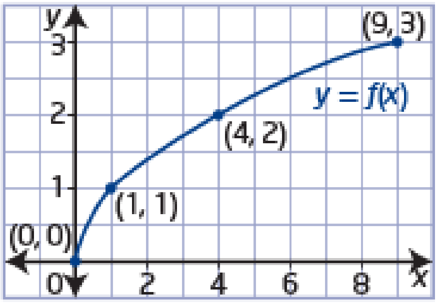
Example 1

Graph a Transformed Function

Describe the combination of transformations that must be applied to the function $y = f(x)$ to obtain the transformed function. Sketch the graph, showing each step of the transformation.

a) $y = 3f(2x)$

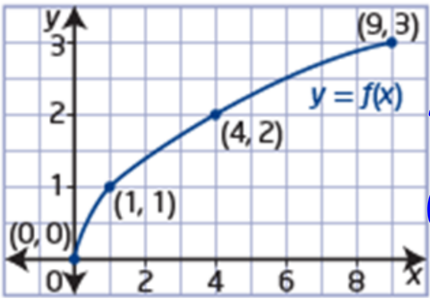
b) $y = f(3x + 6)$



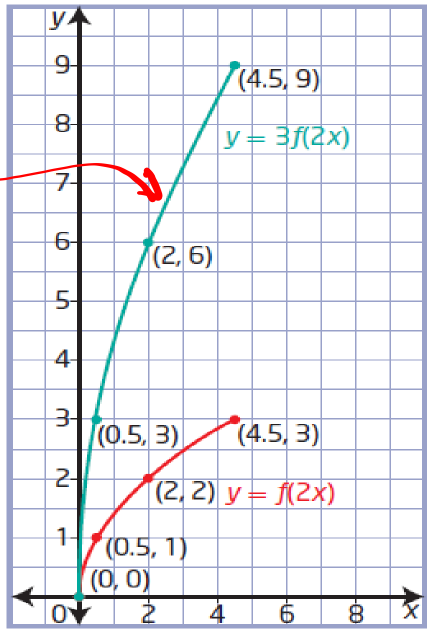
a) $y = 3f(2x)$ $a = 3$ $b = 2$ $h = 0$ $k = 0$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{2}$ and then vertically stretched about the x-axis by a factor of 3.

$(x, y) \to \left(\frac{1}{2}x + 0, 3y + 0\right)$



(0, 0)	(0, 0)
(1, 1)	(1/2, 3)
(4, 2)	(2, 6)
(9, 3)	(9/2, 9)



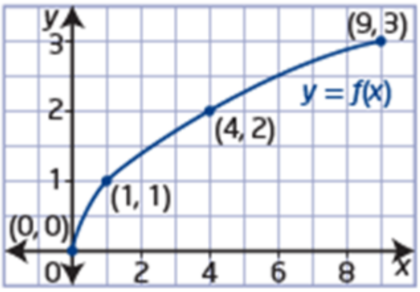
factor

b) $y = f(3x + 6)$ $a=1$ $b=3$ $h=-2$ $k=0$

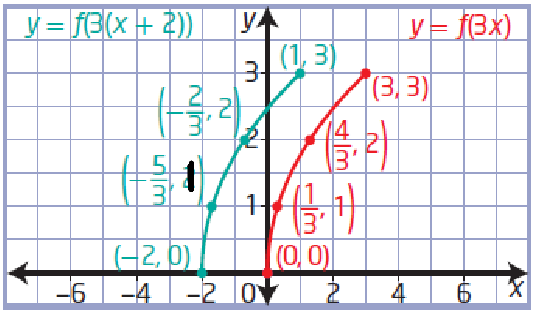
$y=1f[\underline{3}(x+\underline{2})]+\underline{0}$

The graph of $y = f(x)$ is horizontally stretched about the y-axis by a factor of $\frac{1}{3}$ and then horizontally translated 2 units to the left.

$(x,y) \rightarrow \left(\frac{1}{3}x - 2, y\right)$



(0,0)	(-2, 0)
(1,1)	$(-\frac{5}{3}, 1)$
(4,2)	$(-\frac{2}{3}, 2)$
(9,3)	(1 ,3)



$\frac{1}{3}\left(\frac{1}{1}\right) - \frac{2}{1}$ $\frac{1}{3}\left(\frac{4}{1}\right) - 2$ $\frac{1}{3}\left(\frac{9}{1}\right) - 2$

$\frac{1}{3} - \frac{6}{3}$ $\frac{4}{3} - \frac{6}{3}$ $\frac{9}{3} - 2$

$\frac{-5}{3}$ $\frac{-2}{3}$ $3 - 2$

1

Questions From Homework

3. Copy and complete the table by describing the transformations of the given functions, compared to the function $y = f(x)$.

	Function	Reflections	Vertical Stretch Factor	Horizontal Stretch Factor	Vertical Translation	Horizontal Translation
(i)	$y - 4 = f(x - 5)$	–	–	–	4	5
(ii)	$y + 5 = 2f(3x)$	–	2	$\frac{1}{3}$	-5	–
(iii)	$y = \frac{1}{2}f\left(\frac{1}{2}(x - 4)\right)$	–	$\frac{1}{2}$	2	–	4
(iv)	$y + 2 = -3f(2(x + 2))$	↑	3	$\frac{1}{2}$	-2	-2

vertical reflection in x-axis

(i) $y = f(x-5) + 4$
$a=1$ $b=1$ $h=5$ $k=4$

(ii) $y = 2f(3x) - 5$
$a=2$ $b=3$ $h=0$ $k=-5$

(iii) $y = \frac{1}{2}f\left(\frac{1}{2}(x-4)\right)$
$a=\frac{1}{2}$ $b=\frac{1}{2}$ $h=4$ $k=0$

(iv) $y = -3f(2(x+2)) - 2$
$a=-3$ $b=2$ $h=-2$ $k=-2$

6. The key point $(-12, 18)$ is on the graph of $y = f(x)$. What is its image point under each transformation of the graph of $f(x)$?

e) $y + 3 = -\frac{1}{3}f[2(x+6)]$

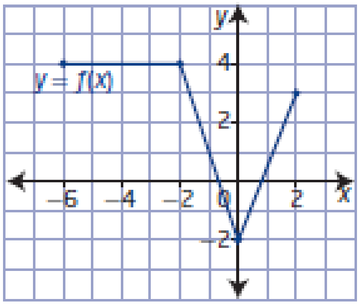
$y = -\frac{1}{3}f[2(x+6)] - 3$

$a = -\frac{1}{3}$ $b = 2$ $h = -6$ $k = -3$

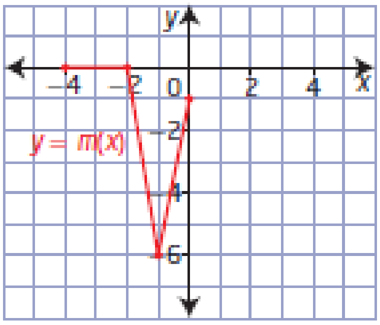
$(x, y) \rightarrow \left(\frac{1}{2}x - 6, -\frac{1}{3}y - 3\right)$

$(-12, 18) \rightarrow \boxed{(-12, -9)}$

4. Using the graph of $y = f(x)$, write the equation of each transformed graph in the form $y = af(b(x - h)) + k$.



b)



$f(x)$	$m(x)$
$(-6,4)$	$(-4,0)$
$(-2,4)$	$(-2,0)$
$(0,-2)$	$(-1,-6)$
$(2,3)$	$(0,-1)$

$(x,y) \longrightarrow \left(\frac{1}{2}x-1,\ y-4\right)$

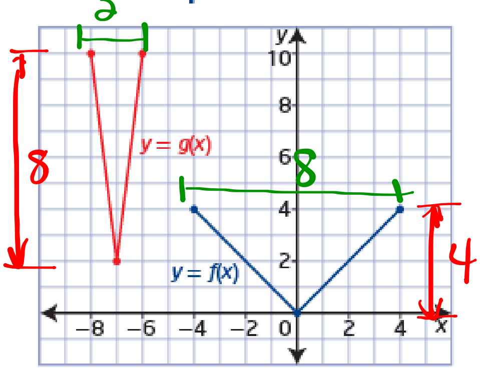
$a=1 \quad b=2 \quad h=-1 \quad k=-4$

$m(x) = 1f(2(x+1)) - 4$

Example 3

Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.



Solution

Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.

$(-4, 4) \rightarrow (-8, 10)$
$(0, 0) \rightarrow (-7, 2)$
$(4, 4) \rightarrow (-6, 10)$

The equation of the transformed function is $g(x) = 2f(4(x + 7)) + 2$.

① Reflections: None

② Vertical stretch factor $= \frac{8}{4} = 2$ $a = 2$
(Compare Range $\frac{\text{New}}{\text{Old}}$)

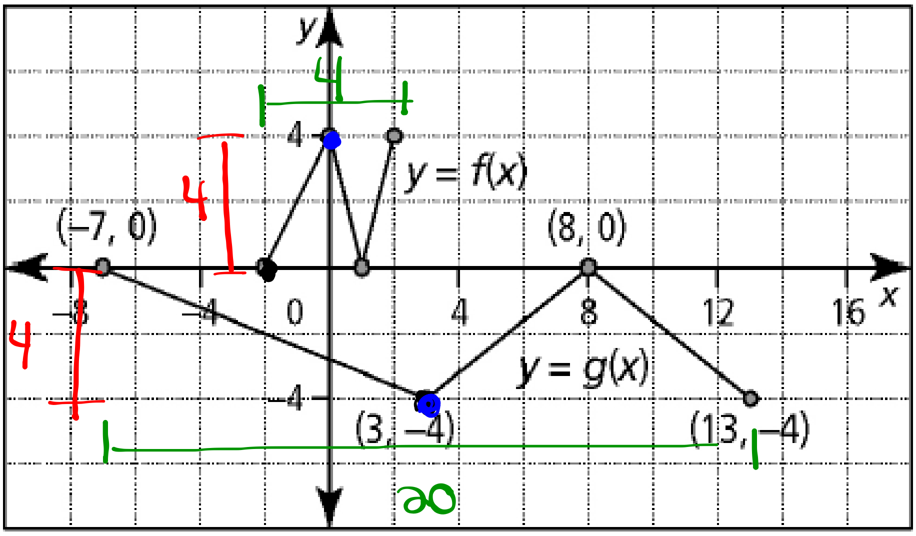
③ horizontal stretch factor $= \frac{2}{8} = \frac{1}{4}$ $b = 4$
(Compare domain $\frac{\text{New}}{\text{Old}}$)

④ horizontal translation: $(0,0) \rightarrow (-7,2)$ left 7 units
(Pick a point on $f(x)$ with an x-value of 0) $h = -7$

⑤ vertical translation: $(0,0) \rightarrow (-7,2)$ up 2 units
(Pick a point on $f(x)$ with an y-value of 0) $k = 2$

⑥ Equation: $g(x) = 2f(4(x+7)) + 2$

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$.



① Reflection: vertical in x-axis $(a<0)$

② Vertical Stretch factor $= \frac{4}{4} = 1$ $\boxed{a=-1}$

③ horizontal stretch factor $= \frac{20}{4} = 5$ $\boxed{b=\frac{1}{5}}$

④ horizontal translation: $(\underline{0},4) \rightarrow (\underline{3},-4)$ right 3 $\boxed{h=3}$

⑤ vertical translation: $(-2,\underline{0}) \rightarrow (-7,\underline{0})$ $\boxed{k=0}$

⑥ $g(x) = -1f\left(\frac{1}{5}(x-3)\right)+0$

$$\boxed{g(x) = -f\left(\frac{1}{5}(x-3)\right)}$$

$y = -f\left(\frac{1}{5}(x-3)\right)$

Homework

Page 38 # 3-6
Plus 7, 8, 9 (a, c, e) and 10

① f) $3y-6 = f(-2x+12)$

$3y = f(-2x+12)+6$ Factor

$\frac{3y}{3} = \frac{1}{3}f[-2(x-6)] + \frac{6}{3}$ (Only divide/multiply the a & k)

$y = \frac{1}{3}f[-2(x-6)]+2$

$a=\frac{1}{3}$ $b=-2$ $h=6$ $k=2$

$a=\frac{1}{3} \rightarrow$ A vertical compression about the x-axis by a factor of $\frac{1}{3}$

$b=-2 \rightarrow$ A horizontal compression about the y-axis by a factor of $\frac{1}{2}$ and a horizontal reflection in the y-axis.

$h=6 \rightarrow$ translated 6 units right

$k=2 \rightarrow$ translated 2 units up.