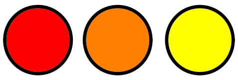
1.1 Making Conjectures: Inductive Reasoning

GOAL

Use reasoning to make predictions.

EXPLORE...

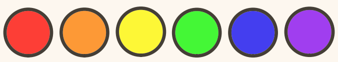
- If the first three colours in a sequence are red, orange, and yellow, what colours might be found in the rest of the sequence? Explain.



SAMPLE ANSWER

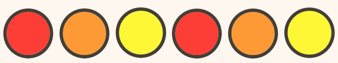
Here are three possible answers:

- If the colour sequence is red, orange, and yellow, the rest of the sequence may be green, blue, and purple. These colours are the primary and secondary colours seen on a colour wheel.



- If the colour sequence is red, orange, and yellow, the rest of the sequence may be green, blue, indigo, and violet. These colours are those of a rainbow.

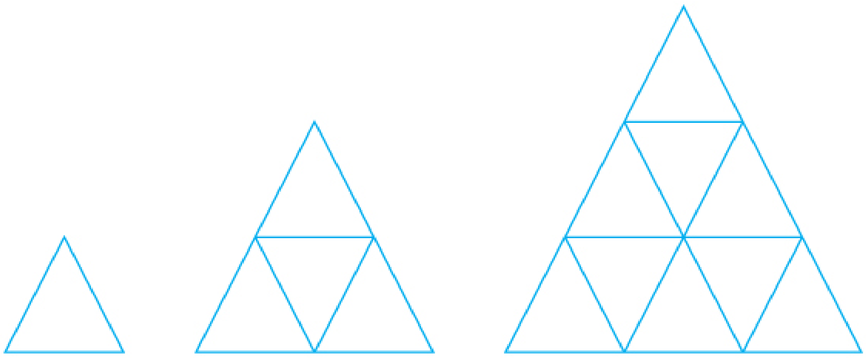
- If the colour sequence is red, orange, and yellow, the rest of the sequence may repeat these three colours.



INVESTIGATE the Math

Georgia, a fabric artist, has been patterning with equilateral triangles. Consider Georgia's **conjecture** about the following pattern.

conjecture
A testable expression that is based on available evidence but is not yet proved.



I think Figure 10 in this pattern will have 100 triangles, and all these triangles will be congruent to the triangle in Figure 1.

How did Georgia arrive at this conjecture?

A. Organize the information about the pattern in a table.

Figure	1	2	3	4	5	6	7	8	9	10
Number of Triangles	1	4	9	16	25	36	49	64	81	100

B. With a partner, discuss what you notice about the data in the table.

C. Extend the pattern for two more figures.

D. What numeric pattern do you see in the table?

Answers

A.

B.

C.

D.

Reflecting

E. Is Georgia's conjecture reasonable? Explain.

F. How did Georgia use **inductive reasoning** to develop her conjecture?

G. Is there a different conjecture you could make based upon the pattern you see? Explain.

inductive reasoning
Drawing a general conclusion by observing patterns and identifying properties in specific examples.

Answers

E.

F.

G.

APPLY the Math

EXAMPLE 1 Using inductive reasoning to make a conjecture about annual precipitation

Lila studied the following five-year chart for total precipitation in Vancouver.

Precipitation in Vancouver (mm)												
	Jan.	**Feb.**	**Mar.**	**Apr.**	**May**	**Jun.**	**Jul.**	**Aug.**	**Sep.**	**Oct.**	**Nov.**	**Dec.**
2003	150.5	27.1	133.7	139.8	49.3	12.8	19.8	4.1	40.2	248.2	167.4	113.2
2004	249.6	45.8	132.8	90.2	68.6	49.6	43.6	28.6	53.6	155.4	136.6	160.8
2005	283.6	57.0	92.4	70.0	42.8	54.4	25.2	4.8	39.4	57.8	350.8	146.0
2006	181.4	116.0	214.8	76.2	37.0	80.0	53.0	8.4	73.6	155.2	116.2	210.6
2007	137.6	68.6	75.2	62.2	43.2	43.0	15.8	75.8	30.6	99.6	177.0	197.2

Environment Canada, National Climate Data and Information Archive

What conjecture could Lila make based on the data?

Lila's Solution

	Jul.	**Aug.**	**Sep.**
	19.8	4.1	40.2
	43.6	28.6	53.6
	25.2	4.8	39.4
	53.0	8.4	73.6
	15.8	75.8	30.6
Totals:	157.4	121.7	237.4

I looked for patterns in the data. I noticed that the summer months seemed to have less precipitation than the other months. I checked the sum of the precipitation in July, August, and September over the five-year period.

	Jan.	**Feb.**	**Mar.**
	150.5	27.1	133.7
	249.6	45.8	132.8
	283.6	57.0	92.4
	181.4	116.0	214.8
	137.6	68.6	75.2
Totals:	1002.7	314.5	648.9

Then I looked for the months with the greatest precipitation, anticipating that the winter months might have greater precipitation. I checked the sums for January, February, and March.

	Nov.
	167.4
	136.6
	350.8
	116.2
	177.0
Total:	948.0

When I examined the information further, I saw that November had the highest value for precipitation: 350.8 mm. I checked the sum for November.

My conjecture is that fall and winter have more precipitation than spring and summer.

Since November is in the fall and January, February, and most of March are in the winter, I can make a conjecture about which seasons have the most precipitation.

Apr.	**May**	**Jun.**	**Jul.**	**Aug.**	**Sep.**
438.4	240.9	239.8	157.4	121.7	237.4

Total: 1435.6 mm

Oct.	**Nov.**	**Dec.**	**Jan.**	**Feb.**	**Mar.**
716.2	948.0	827.8	1002.7	314.5	648.9

Total: 4458.1 mm

I checked the totals for the five-year period. I found that spring and summer had a total of 1435.6 mm of precipitation, and fall and winter had a total of 4458.1 mm of precipitation.

The data support my conjecture.

APPLY the Math

EXAMPLE 1 Using inductive reasoning to make a conjecture about annual precipitation

Lila studied the following five-year chart for total precipitation in Vancouver.

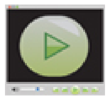
Precipitation in Vancouver (mm)												
	Jan.	Feb.	Mar.	Apr.	May	Jun.	Jul.	Aug.	Sep.	Oct.	Nov.	Dec.
2003	150.5	27.1	133.7	139.8	49.3	12.8	19.8	4.1	40.2	248.2	167.4	113.2
2004	249.6	45.8	132.8	90.2	68.6	49.6	43.6	28.6	53.6	155.4	136.6	160.8
2005	283.6	57.0	92.4	70.0	42.8	54.4	25.2	4.8	39.4	57.8	350.8	146.0
2006	181.4	116.0	214.8	76.2	37.0	80.0	53.0	8.4	73.6	155.2	116.2	210.6
2007	137.6	68.6	75.2	62.2	43.2	43.0	15.8	75.8	30.6	99.6	177.0	197.2

Environment Canada, National Climate Data and Information Archive

What conjecture could Lila make based on the data?

Your Turn

Make a different conjecture based on patterns in the precipitation chart.



Answer

EXAMPLE 2 Using inductive reasoning to develop a conjecture about integers

multiply

Make a conjecture about the product of two odd integers.

Jay's Solution

$(+3)(+7) = (+21)$

Odd integers can be negative or positive. I tried two positive odd integers first. The product was positive and odd.

$(-5)(-3) = (+15)$

Next, I tried two negative odd integers. The product was again positive and odd.

$(+3)(-3) = (-9)$

Then I tried the other possible combination: one positive odd integer and one negative odd integer. This product was negative and odd.

My conjecture is that the product of two odd integers is an odd integer.

I noticed that each pair of integers I tried resulted in an odd product.

$(-211)(-17) = (+3587)$

I tried other integers to test my conjecture. The product was again odd.

EXAMPLE 2 Using inductive reasoning to develop a conjecture about integers

Make a conjecture about the product of two odd integers.

Your Turn

Do you find Jay's conjecture convincing? Why or why not?

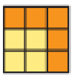
Answer

EXAMPLE 3 Using inductive reasoning to develop a conjecture about perfect squares

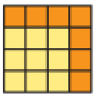
subtract

Make a conjecture about the difference between consecutive perfect squares. → 1, 4, 9, 16 . . .

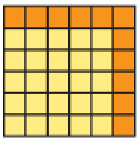
Steffan's Solution: Comparing the squares geometrically



I represented the difference using unit tiles for each perfect square. First, I made a 3 × 3 square in orange and placed a yellow 2 × 2 square on top. When I subtracted the 2 × 2 square, I had 5 orange unit tiles left.

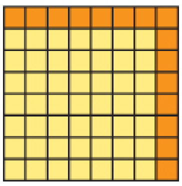


Next, I made 3 × 3 and 4 × 4 squares. When I subtracted the 3 × 3 square, I was left with 7 orange unit tiles. I decided to try greater squares.



My conjecture is that the difference between consecutive squares is always an odd number.

I saw the same pattern in all my examples: an even number of orange unit tiles bordering the yellow square, with one orange unit tile in the top right corner. So, there would always be an odd number of orange unit tiles left, since an even number plus one is always an odd number.



I tested my conjecture with the perfect squares 7 × 7 and 8 × 8. The difference was an odd number.

The example supports my conjecture.

EXAMPLE 3 Using inductive reasoning to develop a conjecture about perfect squares

Make a conjecture about the difference between consecutive perfect squares.

Francesca's Solution: Describing the difference numerically

$2^2 - 1^2 = 4 - 1$
$2^2 - 1^2 = 3$

I started with the smallest possible perfect square and the next greater perfect square: 1^2 and 2^2. The difference was 3.

$4^2 - 3^2 = 7$
$9^2 - 8^2 = 17$

Then I used the perfect squares 3^2 and 4^2. The difference was 7. So, I decided to try even greater squares.

My conjecture is that the difference between consecutive perfect squares is always a prime number.

I thought about what all three differences—3, 7, and 17—had in common. They were all prime numbers.

$12^2 - 11^2 = 23$

To test my conjecture, I tried the perfect squares 11^2 and 12^2. The difference was a prime number.

The example supports my conjecture.

EXAMPLE 3 Using inductive reasoning to develop a conjecture about perfect squares

Make a conjecture about the difference between consecutive perfect squares.

Your Turn

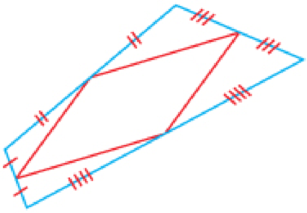
How is it possible to have two different conjectures about the same situation? Explain.

Answer

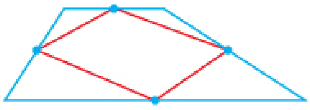
EXAMPLE 4 Using inductive reasoning to develop a conjecture about quadrilaterals

Make a conjecture about the shape that is created by joining the midpoints of adjacent sides in any quadrilateral.

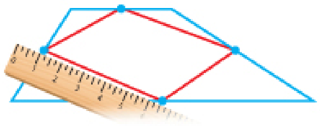
Marc's Solution: Using a protractor and ruler



I drew an irregular quadrilateral on tracing paper. I used my ruler to determine the midpoints of each side. I joined the midpoints of adjacent sides to form a new quadrilateral. This quadrilateral looked like a parallelogram.



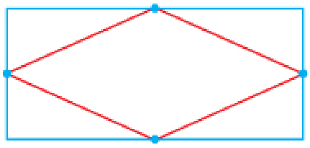
Next, I drew a trapezoid with sides that were four different lengths. I determined the midpoints of the sides. When the midpoints were joined, the new quadrilateral looked like a parallelogram.



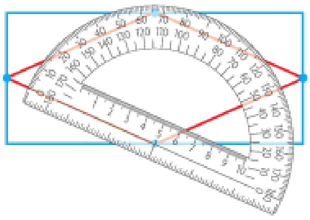
I used my ruler to confirm that the opposite sides were equal.

My conjecture is that joining the adjacent midpoints of any quadrilateral will create a parallelogram.

Each time I joined the midpoints, a parallelogram was formed.



To check my conjecture one more time, I drew a rectangle. I determined its midpoints and joined them. This quadrilateral also looked like a parallelogram.



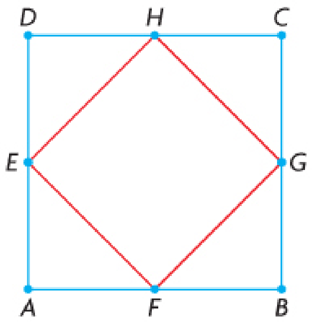
I checked the measures of the angles in the new quadrilateral. The opposite angles were equal. The new quadrilateral was a parallelogram, just like the others were.

The rectangle example supports my conjecture.

EXAMPLE 4 Using inductive reasoning to develop a conjecture about quadrilaterals

Make a conjecture about the shape that is created by joining the midpoints of adjacent sides in any quadrilateral.

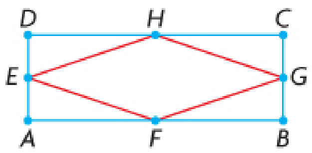
Tracey's Solution: Using dynamic geometry software



$\overline{HE} = 2.4$ cm
$\overline{EF} = 2.4$ cm
$\overline{FG} = 2.4$ cm
$\overline{GH} = 2.4$ cm

$\angle EFG = 90°$
$\angle FGH = 90°$
$\angle GHE = 90°$
$\angle HEF = 90°$

I constructed a square and the midpoints of the sides. Then I joined the adjacent midpoints. *EFGH* looked like a square. I checked its side lengths and angle measures to confirm that it was a square.



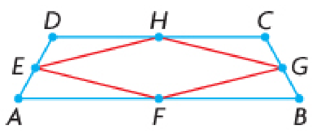
$\overline{HE} = 1.6$ cm
$\overline{EF} = 1.6$ cm
$\overline{FG} = 1.6$ cm
$\overline{GH} = 1.6$ cm

$\angle EFG = 143°$
$\angle FGH = 37°$
$\angle GHE = 143°$
$\angle HEF = 37°$

Next, I constructed a rectangle and joined the adjacent midpoints to create a new quadrilateral, *EFGH*. The side lengths and angle measures of *EFGH* showed that *EFGH* was a rhombus but not a square.

My conjecture is that the quadrilateral formed by joining the adjacent midpoints of any quadrilateral is a rhombus.

Since a square is a rhombus with right angles, both of my examples resulted in a rhombus.



$\overline{HE} = 1.6$ cm
$\overline{EF} = 1.6$ cm
$\overline{FG} = 1.6$ cm
$\overline{GH} = 1.6$ cm

$\angle EFG = 152°$
$\angle FGH = 28°$
$\angle GHE = 152°$
$\angle HEF = 28°$

To check my conjecture, I tried an isosceles trapezoid. The new quadrilateral, *EFGH*, was a rhombus.

The isosceles trapezoid example supports my conjecture.

EXAMPLE 4 Using inductive reasoning to develop a conjecture about quadrilaterals

Make a conjecture about the shape that is created by joining the midpoints of adjacent sides in any quadrilateral.

Your Turn

a) Why did the students draw different conjectures?

b) Do you think that both conjectures are valid? Explain.

Answers

a)

b)

In Summary

Key Idea

- Inductive reasoning involves looking at specific examples. By observing patterns and identifying properties in these examples, you may be able to make a general conclusion, which you can state as a conjecture.

Need to Know

- A conjecture is based on evidence you have gathered.
- More support for a conjecture strengthens the conjecture, but does not prove it.

Assignment: pgs. 12 - 13
1, 2, 3, 6, 7, 8a, 9, 11, 13

SOLUTIONS => 1.1 Making Conjectures: Inductive Reasoning

1. Three types of downhill skis are available:
   - ↳ parabolic
   - ↳ twin tip
   - ↳ powder.

The manager of the store ordered 100 pairs of each type.
What conjecture did the manager make?

The manager made the conjecture that each type of ski would sell equally as well as the others.

2. Tomas gathered the following evidence and noticed a pattern.

$17(11) = 187$ $\quad$ $23(11) = 253$

$41(11) = 451$ $\quad$ $62(11) = 682$

Tomas made this conjecture: When you multiply a two-digit number by 11, the first and last digits of the product are the digits of the original number.

Is Tomas's conjecture reasonable?

Develop evidence to test his conjecture and determine whether it is reasonable.

↳ $85(11) = 935$

↳ $98(11) = 1078$

Since evidence can be found that does not support this conjecture, Tomas's conjecture is not reasonable.

3. Make a conjecture about the sum of two even integers. Develop evidence to test your conjecture.

Evidence

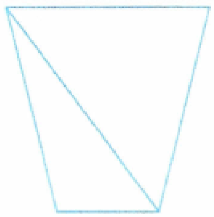
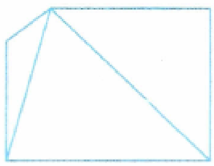
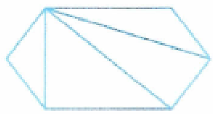
$2+4=6$
$-12+16=4$
$-20-36=-56$

Conjecture

The sum of two even integers is always even.

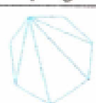
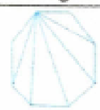
For example: $72+104=176$

6. Use the evidence given in the chart below to make a conjecture. Provide more evidence to support your conjecture.

Polygon	quadrilateral	pentagon	hexagon
Fewest Number of Triangles	 2	 3	 4

Conjecture: The fewest number of triangles in a polygon is the number of sides subtracted by 2.

More Evidence:

Polygon	heptagon	octagon	nonagon
Fewest Number of Triangles	 5	 6	 7

7. Sonia noticed a pattern when dividing the square of an odd number by 4. Determine the pattern and make a conjecture.

Evidence

$$\frac{(3)^2}{4} = \frac{9}{4} = 2.25 \qquad \frac{(11)^2}{4} = \frac{121}{4} = 30.25 \qquad \frac{(33)^2}{4} = \frac{1089}{4} = 272.25$$

Conjecture

When you divide the square of an odd number by 4, the result is always an even number ending with a decimal of .25.

8. Dan noticed a pattern in the digits of the multiples of 3. He created the following table to show the pattern.

Multiples of 3	12	15	18	21	24	27	30
Sum of the Digits	3	6	9	3	6	9	3

a) Make a conjecture based on the pattern in the table.

Conjecture

The sums of the digits of multiples of 3 are always 3, 6, or 9.

9. Make a conjecture about the sum of one odd integer and one even integer. Test your conjecture with at least three examples.

Evidence	Conjecture
$2+3=5$	The sum of one odd integer and one even integer is always odd.
$-10+21=11$	
$-150-75=-225$	

11. Paula claims that whenever you square an odd integer, the result is an odd number. Is her conjecture reasonable? Justify your decision.

Evidence

3^2	13^2	25^2
$= (3)(3)$	$= (13)(13)$	$= (25)(25)$
$= 9$	$= 169$	$= 625.$

Paula's conjecture is reasonable. When you multiply an odd digit with an odd digit, the result is odd.

13. Text messages often include cryptic abbreviations, such as L2G (love to go), 2MI (too much information, LOL (laugh out loud), and MTF (more to follow). Make a conjecture about the cryptic abbreviations used in text messages, and provide evidence to support your conjecture.

Conjecture

Abbreviations in text messages reduce the difficult typing that needs to be done using small keypads or keyboards. For example: LOL is 3 characters, while "laugh out loud" is 14 characters.

PM11-1s1.gsp

1s1e1 finalt highquality.mp4

1s1e2 finalt.mp4

1s1e4 finalt.mp4