1.7 Analyzing Puzzles and Games

GOAL

Determine, explain, and verify a reasoning strategy to solve a puzzle or to win a game.

EXPLORE...

- Three students are playing a game. Two of the students flip a coin, and the third student records their scores. Student 1 gets a point if the result is two heads, student 2 gets a point if the result is two tails, and student 3 gets a point if the result is a head and a tail. The first student to get 10 points wins. Explain whether you would prefer to be student 1, student 2, or student 3.

SAMPLE ANSWER

APPLY the Math

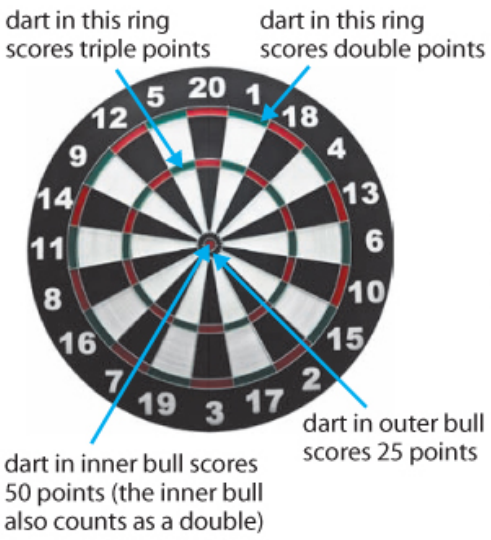
EXAMPLE 1 Using reasoning to determine possible winning plays

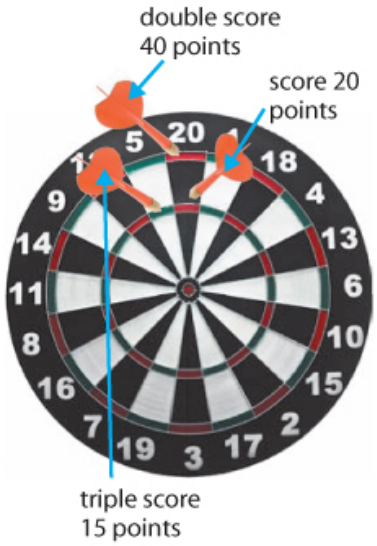
Frank and Tara are playing darts, using the given rules. Their scores are shown in the table below. To win, Frank must reduce his score to exactly zero and have his last counting dart be a double.

Rules

- Each player's score starts at 501.
- The goal is to reduce your score to zero.
- Players alternate turns.
- Each player throws three darts per turn.

Frank		Tara	
Turn Score	Total Score	Turn Score	Total Score
	501		501
100	401	85	416
95	306	85	331
140	166	140	191
130	36	91	100





Total score for turn: 75 points

What strategies for plays would give Frank a winning turn?

Frank's Solution

$2(18) = 36$ — I could win with a single dart in double 18.

$18 + 2(9) = 36$ — If I hit 18 instead of double 18, then I could use my second dart to try for double 9.

$18 + 9 = 27$
9 would be left. — If I hit 9 instead of double 9 with my second dart, then I couldn't win this turn. That's because I can't score 9 with a double.

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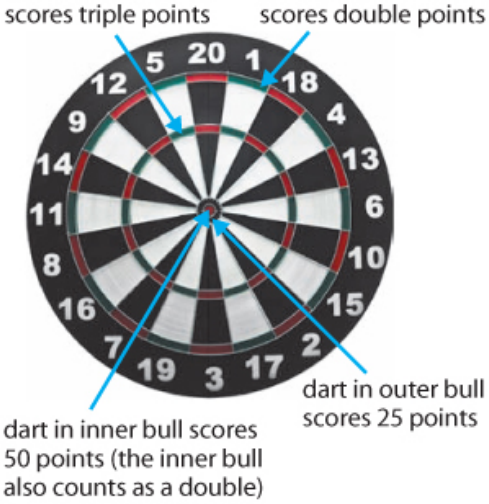
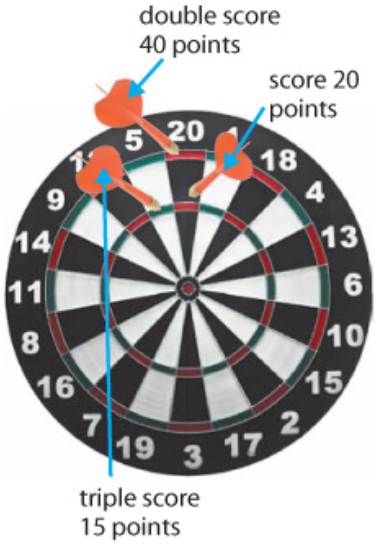




Total score for turn: 75 points

Your Turn

a) Describe two other ways that Frank could win the game on his turn.

b) If Frank does not win on his turn, describe a strategy that Tara could use to win on her next turn.

Answers

a)

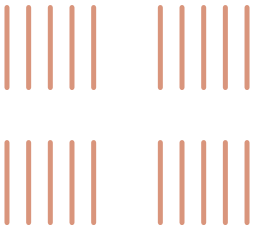
b)

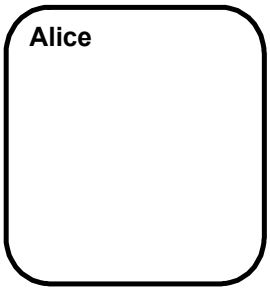
EXAMPLE 2 Using deductive and inductive reasoning to determine a winning strategy

Nadine and Alice are playing a toothpick game. They place a pile of 20 toothpicks on a desk and alternate turns. On each turn, the player can take one or two toothpicks from the pile. The player to remove the last toothpick is the winner. Nadine and Alice flip a coin to determine the starting player.

Is there a strategy Alice can use to ensure that she wins the game?

Try the game by dragging toothpicks into Nadine's and Alice's play areas.





Alice's Solution

I need to make sure that there are one or two toothpicks left after Nadine's last turn.

I will win the game if I can take the last toothpick. If I work backward, I might see a pattern I can use to win.

To make sure this happens, I have to leave three toothpicks on the desk for Nadine.

If I leave three toothpicks, Nadine has to take either one or two toothpicks. If she takes only one, I can take the two that are left and win. If she takes two, I can take the last one and win.

To make sure this happens, I have to leave six toothpicks on the desk for Nadine.

If I leave six toothpicks, Nadine has to take either one or two toothpicks. If she takes only one, I can take two, which would leave three. If she takes two, I can take one and leave her with three.

To make sure this happens, I have to leave nine toothpicks on the desk for Nadine.

If I leave nine toothpicks, Nadine has to take either one or two toothpicks. If she takes only one, then I can take two, which would leave six. If she takes two, I can take one and leave her with six.

I can see that I need to leave 12, 15, and 18 toothpicks for Nadine.

There is a pattern to the number of toothpicks I must leave for Nadine: 3, 6, 9, 12, 15, 18.

I will win if I go first and take two toothpicks. Each turn after that, I need to pick one or two so that I leave Nadine with a number of toothpicks that is a multiple of 3.

If Nadine goes first and knows this strategy, I can't win. If she goes first and doesn't know this strategy, however, I can win by arranging to leave her a number of toothpicks that is a multiple of 3.

EXAMPLE 2 Using deductive and inductive reasoning to determine a winning strategy

Nadine and Alice are playing a toothpick game. They place a pile of 20 toothpicks on a desk and alternate turns. On each turn, the player can take one or two toothpicks from the pile. The player to remove the last toothpick is the winner. Nadine and Alice flip a coin to determine the starting player.

Is there a strategy Alice can use to ensure that she wins the game?

Your Turn

a) Which part of Alice's strategy involved deductive reasoning? Explain.

b) Which part of Alice's strategy involved inductive reasoning? Explain.

Answers

a)

b)

In Summary

Key Idea

- Both inductive reasoning and deductive reasoning are useful for determining a strategy to solve a puzzle or win a game.

Need to Know

- Inductive reasoning is useful when analyzing games and puzzles that require recognizing patterns or creating a particular order.
- Deductive reasoning is useful when analyzing games and puzzles that require inquiry and discovery to complete.

http://akidsmath.com/mathgames/leapfrog.htm
http://www.combinationlock.com/
http://samgine.com/free/number-puzzles/

Assignment: pages 55-57
Questions: 2, 4, 5, 7ab, 9,10a,11
+
BONUS Sudoku Worksheet

SOLUTIONS ⇒ 1.7 Analyzing Puzzles and Games

2. Frank and Tara are playing another game of darts. Tara's game score is 66. List three different strategies she could use to win her turn.

SOLUTION

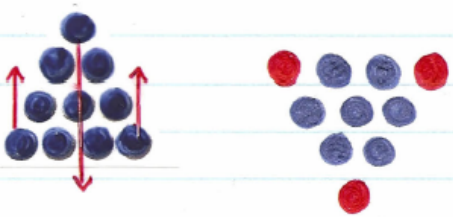
Tara could obtain a score of 66 using one of the following strategies:

- **10, 16, Double 20**
- **15, 15, Double 18**
- **Triple 20, Double 3**

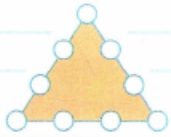
* These are just a sample of the ways to obtain 66, there are many more.

4. Rearrange three golf balls so that the arrowhead points down instead of up.

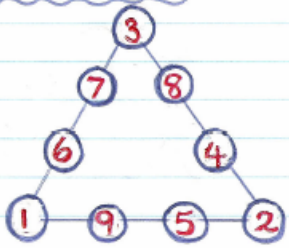
SOLUTION



5a) Draw a diagram like the one on the right. Place the numbers 1 through 9 in the circles so that the sum of the numbers on each side of the triangle is 17.



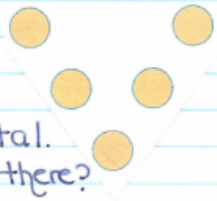
SOLUTION (For Example)



b) Explain the strategy you used.

I started with the smallest numbers in the three corners. I then placed one of the largest numbers along each side. I then used the three remaining digits to make each side of the triangle add to 17.

7. Place the numbers 1 to 5 in a V shape, as shown, so that the two arms of the V have the same total.



a) How many different solutions are there?

If the "1" stays in the center: 8 possibilities.

1 2 3 4

5 4 2 4 5 3 2 3

2 3 5 3 2 4 5 4

1 1 1 1

5 6 7 8

4 5 4 2 3 5 3 2

3 2 3 5 4 2 4 5

1 1 1 1

There are also 8 possibilities if the "3" stays in the middle. ex. 2 1

4 5

3

Finally there are 8 possibilities if the "5" stays in the middle. ex. 1 2

4 3

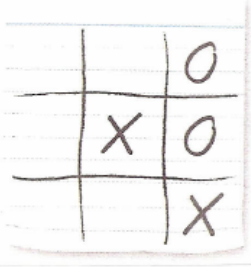
5

TOTAL: 8+8+8 = 24 different solutions.

b) What do you notice about all the solutions you found?

I notice that the number in the middle is always odd.

9. Who started this game of tic-tac-toe: player X or player O? Explain. Assume that both players are experienced at playing tic-tac-toe.



Player O started this game.
* How do we know?

10. Sudoku requires both inductive and deductive reasoning skills. The numbers that are used to complete a Sudoku puzzle relate to the size of the grid. The grid must be filled so that each column, row, or block contains all the numbers. No number can be repeated within any column, row, or block. Solve each of the Sudoku puzzles below.

a)

5	1	3	2	6	4
2	6	4	5	1	3
1	5	2	3	4	6
3	4	6	1	5	2
6	3	5	4	2	1
4	2	1	6	3	5

11. Fill in the missing numbers, from 1 to 9, so that the sum of the numbers in each row, column, and diagonal is 15.

a)

2	7	6
9	5	1
4	3	8

b)

2	9	4
7	5	3
6	1	8

SUDOKU

2	6	5	8	4	7	1	3	9
8	3	1	9	6	5	7	2	4
9	4	7	3	1	2	6	8	5
6	7	4	5	2	8	3	9	1
5	9	8	7	3	1	2	4	6
1	2	3	4	9	6	5	7	8
7	1	2	6	8	9	4	5	3
3	5	9	1	7	4	8	6	2
4	8	6	2	5	3	9	1	7

Attachments

1s7e1finalt2.mp4

1s7e2 finalt.mp4