Warm Up

2. Factor each of the following:

$x^{27} - 1$ $\qquad$ $(x^2+1)^{\frac{1}{2}} + 3(x^2+1)^{-\frac{1}{2}}$

$(x^9-1)(x^{18}+x^9+1)$

$(x^3-1)(x^6+x^3+1)(x^{18}+x^9+1)$

$(x-1)(x^2+x+1)(x^6+x^3+1)(x^{18}+x^9+1)$

$(x^2+1)^{\frac{1}{2}} + 3(x^2+1)^{-\frac{1}{2}}$

$(x^2+1)^{-1/2}(x^2+1+3)$

$(x^2+1)^{-1/2}(x^2+4)$

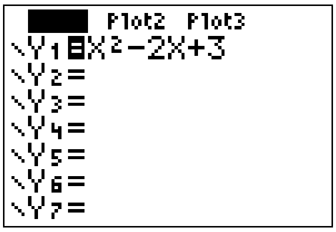
$\frac{(x^2+1)^{1/2}}{(x^2+1)^{-1/2}} = (x^2+1)^1$

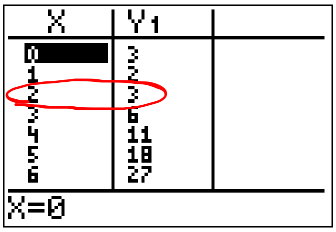
$\frac{3(x^2+1)^{-1/2}}{(x^2+1)^{-1/2}} = 3(x^2+1)^0$

$= 3$

Limit of a Function

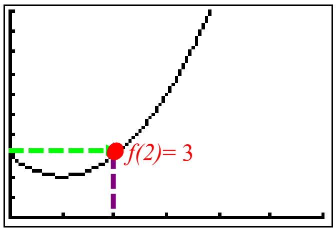
Let's examine the function $f(x)=x^2-2x+3$ (parabola)



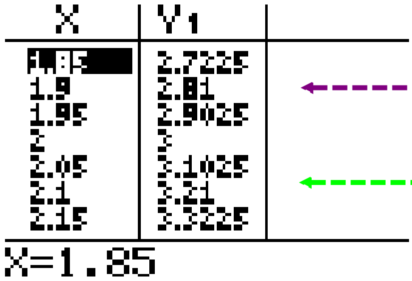


X	Y1
0	3
1	2
2	3
3	6
4	11
5	18
6	27

X=0



We can see that *f(2)*= 3...let's check the behaviour of *f* as we get closer and closer to $x = 2$.



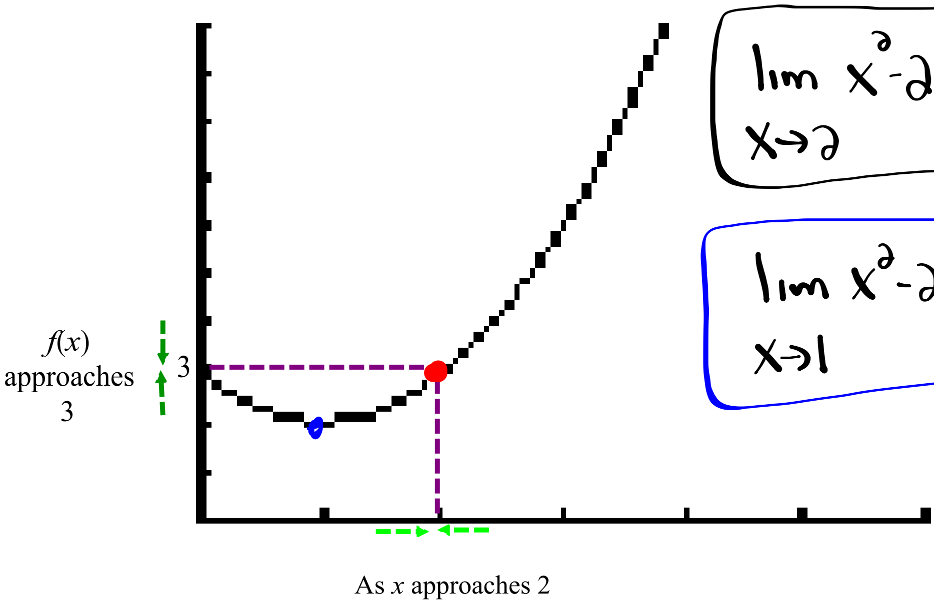
X	Y1
1.85	2.7225
1.9	2.81
1.95	2.9025
2	3
2.05	3.1025
2.1	3.21
2.15	3.3225

X=1.85

As x gets closer to 2 from the left y is getting closer to 3.

As x gets closer to 2 from the right y is getting closer to 3.

From the above, the notion of the limit of a function arises...



height

$$\boxed{\lim_{x\to 2} x^2-2x+3 = 3}$$

height

$$\boxed{\lim_{x\to 1} x^2-2x+3 = 2}$$

Notation: $\lim_{x\to 2} f(x)=3$

"The limit of the function $f(x)$ as x approaches 2 is equal to 3."

The common sense definition of a limit...

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What *is* a limit?

The functions intended height

A formal definition of a limit...

We write $\lim_{x \to a} f(x) = L$ if we can make the values of $f(x)$ arbitrarily close to L

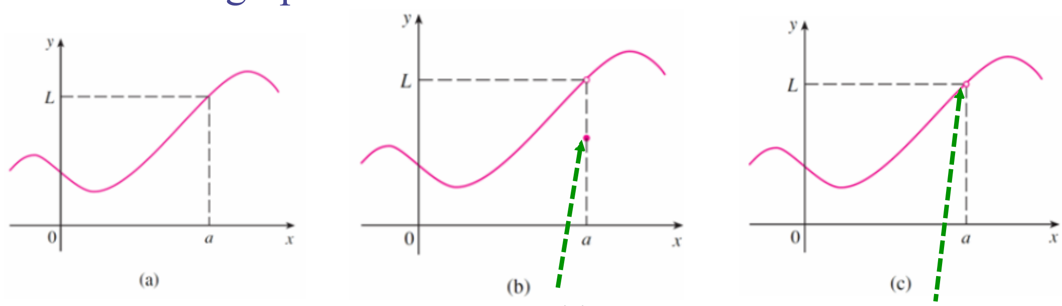
- (as close to L as we like)

by taking x to be sufficiently close to a

- (on either side of a)

but not equal to a.

Look at the graphs of these three functions...



Notice $f(a) \neq L$

Notice $f(a)$ is undefined

But in each case, regardless of what happens at a, it is true that

$$\lim_{x \to a} f(x) = L$$

Evaluating Limits

I. Using a Graph:

- We looked at this in the previous two examples

II. Algebraically:

- Direct Substitution...

Examples:

$$\lim_{x\to -2} \frac{x^2-2x+1}{x+3}$$

(y-value) height

$$\lim_{x\to -2} \frac{(-2)^2-2(-2)+1}{(-2)+3} = \frac{9}{1} = 9$$

$$\lim_{x\to 3}(16-x^2)$$

(y-value) height

$$\lim_{x\to 3}(16-(3)^2) = 7$$

- Indeterminate limits... $\Rightarrow$ Direct substitution leads to $\frac{0}{0}$

$\Rightarrow$ Factor
$\Rightarrow$ Rationalize (Radicals, square roots)
$\Rightarrow$ Expand
$\Rightarrow$ Find Common Denominators (Complex Fraction)

Examples:

$$\lim_{x\to 4} \frac{x^2-16}{x-4}$$

intended height

$$\lim_{x\to 4} \frac{(x+4)\cancel{(x-4)}}{\cancel{x-4}} = 8$$

$$\lim_{h\to 0} \frac{\left(\sqrt{4+h}-2\right)}{h} \cdot \frac{\left(\sqrt{4+h}+2\right)}{\left(\sqrt{4+h}+2\right)}$$

$$\lim_{h\to 0} \frac{4+h-4}{h(\sqrt{4+h}+2)}$$

$$\lim_{h\to 0} \frac{\cancel{h}}{\cancel{h}(\sqrt{4+h}+2)}$$

$$\lim_{h\to 0} \frac{1}{(\sqrt{4+0}+2)} = \frac{1}{4}$$

Try these...remember to use your algebra skills to try and eliminate the **indeterminate form**.

$$\lim_{x\to 0}\frac{x^2+3x}{(x+2)^2-(x-2)^2}$$

$$\lim_{x\to 0}\frac{x(x+3)}{[x+2+(x-2)][x+2-(x-2)]}$$

$$\lim_{x\to 0}\frac{x(x+3)}{[x+2+x-2][x+2-x+2]}$$

$$\lim_{x\to 0}\frac{x(x+3)}{(2x)(4)}$$

$$\lim_{x\to 0}\frac{\cancel{x}(x+3)}{8\cancel{x}}=\frac{3}{8}$$

$$\lim_{x\to 2}\frac{2x\cdot\frac{1}{x}-\frac{1}{2}\cdot 2x}{(x-2)\,2x}$$

$$\lim_{x\to 2}\frac{\cancel{(2-x)}^{-1}}{2x\cancel{(x-2)}}=\frac{-1}{4}$$

Homework