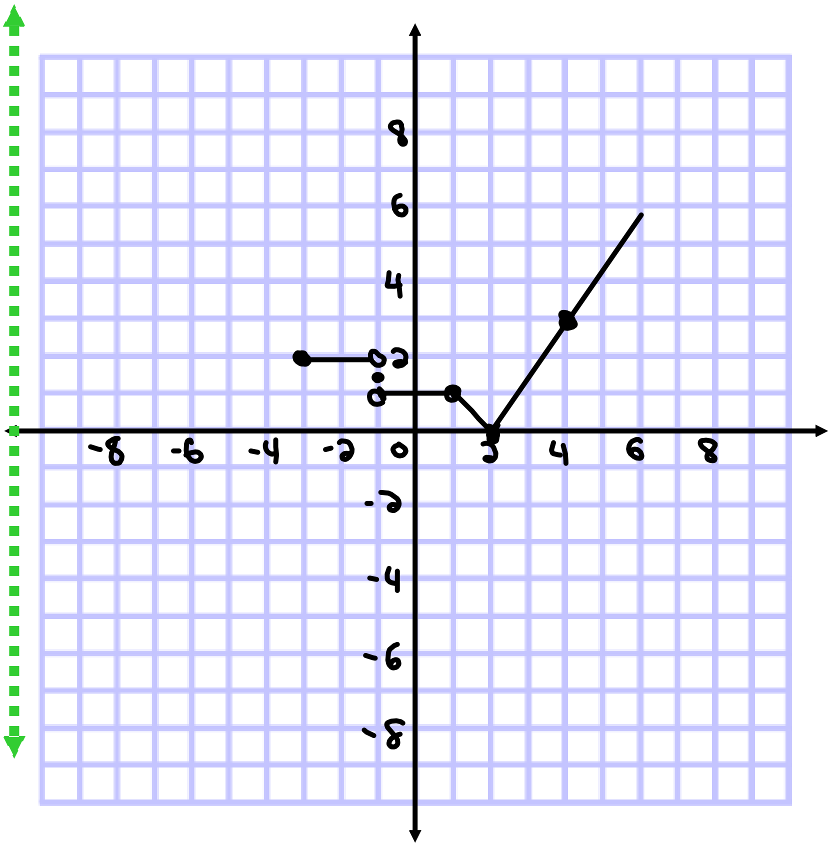
Questions From Homework



a) $\lim_{x \to -3^+} g(x) = 2$

b) $\lim_{x \to -1^-} g(x) = 2$

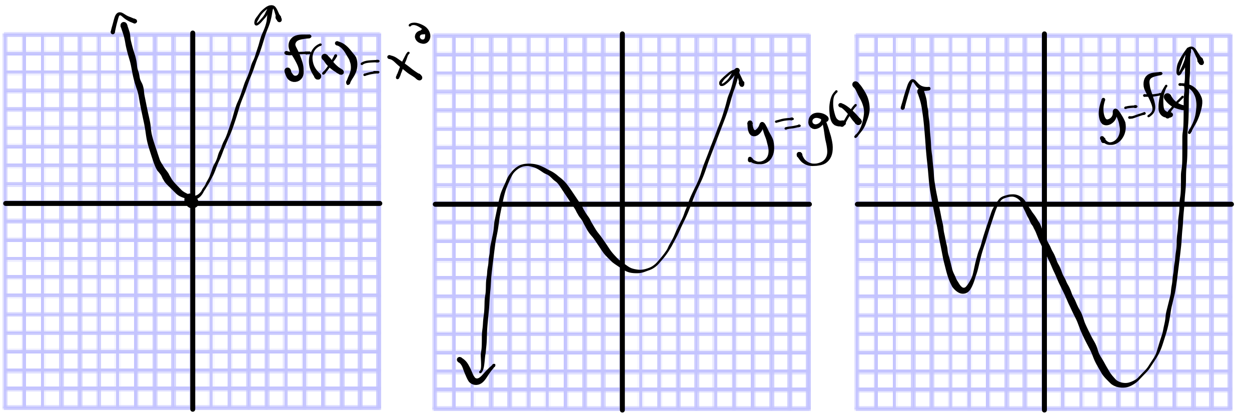
c) $\lim_{x \to -1^+} g(x) = 1$

d) $\lim_{x \to -1} g(x) = DNE$

* $g(-1) = 1.5$

Recall from our previous discussions that ...

1 Theorem $\lim_{x \to a} f(x) = L$ if and only if $\lim_{x \to a^-} f(x) = L = \lim_{x \to a^+} f(x)$



These graphs have limits that exist at every x value and are what we call ***continuous***

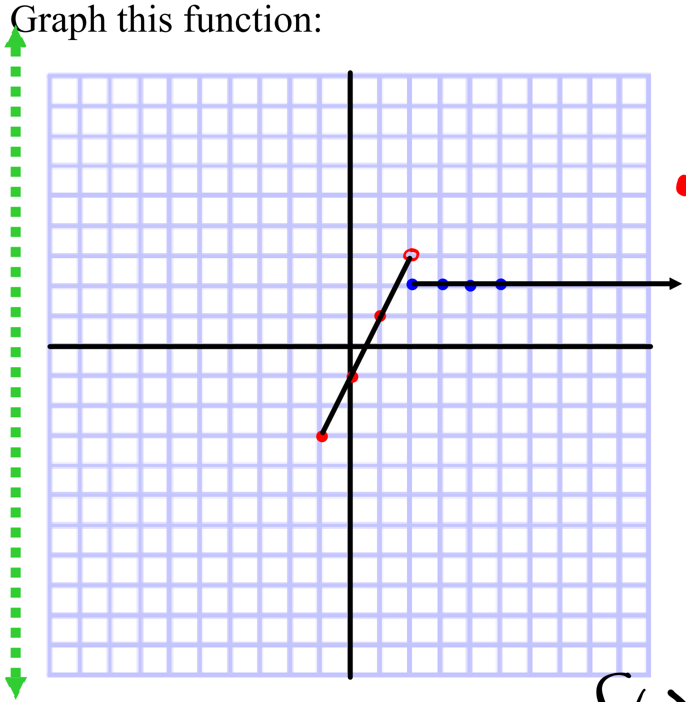
We also want to be able to check limits of piecewise defined functions...

$\geq, \leq, = \rightarrow$ closed dot
$>, < \rightarrow$ open dot

Example:

$$f(x) = \begin{cases} 2x - 1 & \text{if } -1 \leq x < 2 \\ 2 & \text{if } x \geq 2 \end{cases}$$

Graph this function:



$2x - 1$

	x	$f(x)$
•	-1	-3
•	0	-1
•	1	1
○	2	3

2

	x	$f(x)$
•	2	2
•	3	2
•	4	2
•	5	2

$f(2) = 2$ (closed dot)

Evaluate the following limits:

$\lim_{x \to 2^-} f(x) = 3 \qquad \lim_{x \to 2^+} f(x) = 2 \qquad \lim_{x \to 2} f(x) = \text{DNE}$

Continuity

Definition

- We noticed in the preceding section that…
  - the limit of a function as x approaches a

  can often be found simply by…
  - calculating the value of the function at a.
- Functions with this property are called *continuous* at a:

1 Definition A function f is **continuous at a number** a if

$$\lim_{x \to a} f(x) = f(a)$$

- This definition implicitly requires three things if f is continuous at a:
  1. $f(a)$ is defined
     - That is, a is in the domain of f
  2. $f(x)$ has a limit as x approaches a
  3. This limit is actually equal to $f(a)$.

no holes or breaks in the curve

In English!

- **The graph must be defined at that x-value**
- **A limit must exist at that x-value**
- **Limit must be the same as the defined height**

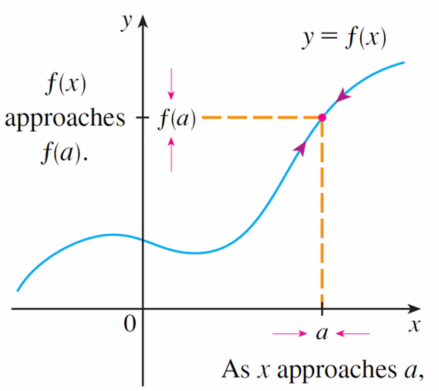


FIGURE 1

Examine the graph shown below
for points of discontinuity...

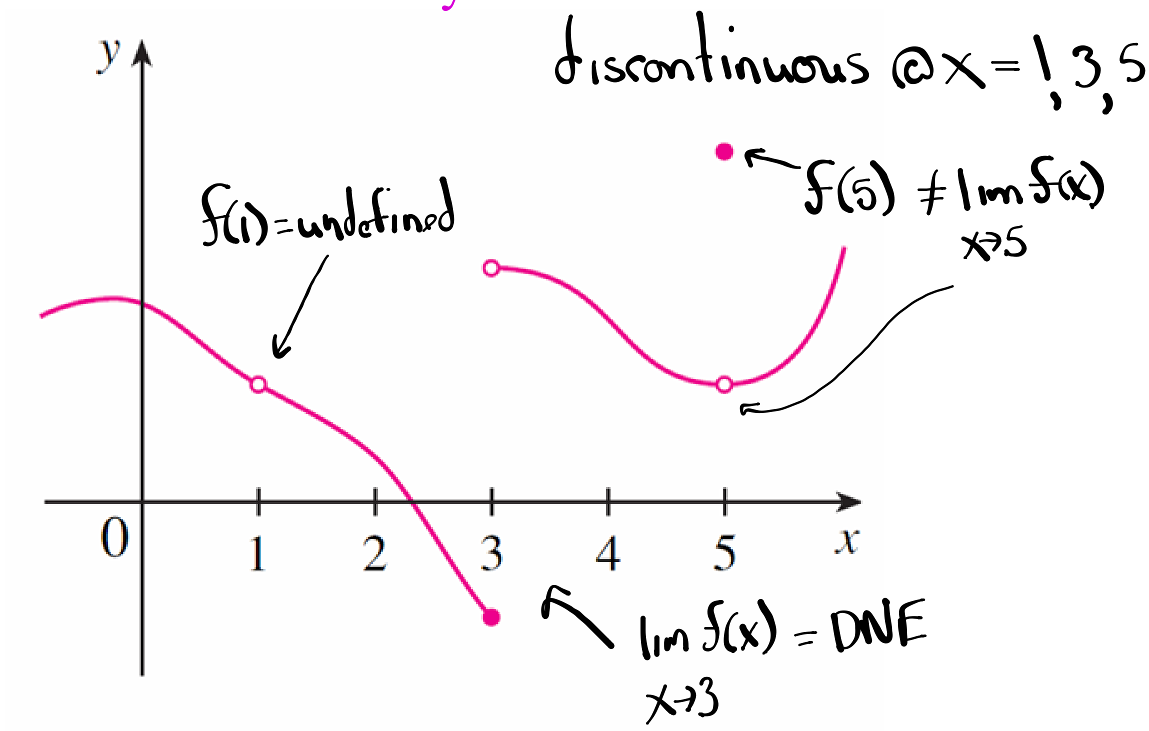


FIGURE 2

- f is discontinuous at 1 because $f(1)$ is not defined…
  - …despite the fact that f has a limit at $a = 1$
- f is also discontinuous at 3 , but for a different reason:
  - $f(3)$ is defined, but f has no limit at $a = 3$.
- f has both a value and a limit at 5 , but they are different; thus f is discontinuous at 5 .

Let's simplify things...

A function whose graph has holes or breaks is considered discontinuous at these particular points.

If you have to lift your pencil from the page to sketch the graph, it is discontinuous anywhere you lift your pencil

Examples:

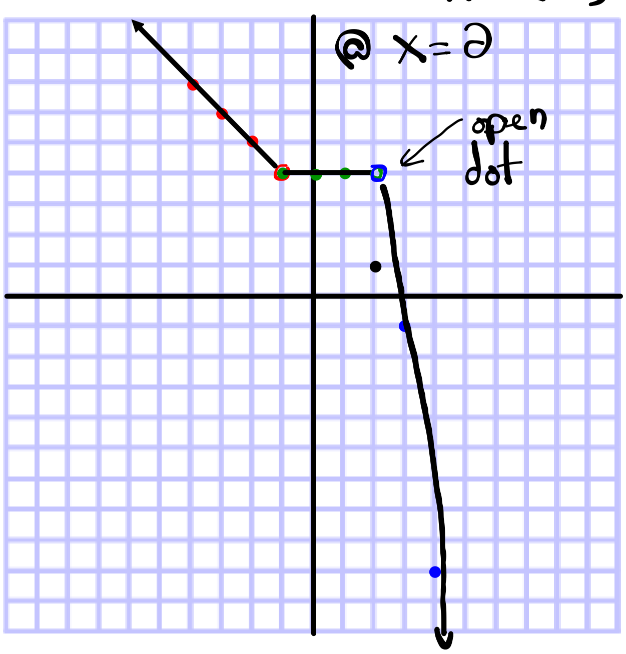
Given the function

$$f(x) = \begin{cases} 3 - x, & \text{if } x < -1 \\ 4, & \text{if } -1 \le x < 2 \\ 1, & \text{if } x = 2 \\ 8 - x^2, & \text{if } x > 2 \end{cases}$$

(a) Check $f(x)$ for any points of discontinuity. Provide a mathematical reason to validate any point(s) where the function is discontinuous.

(b) Sketch $f(x)$.

discontinuous @ $x = 2$



3-x

	x	$f(x)$
o	-1	4
•	-2	5
•	-3	6
•	-4	7

4

	x	$f(x)$
•	-1	4
•	0	4
•	1	4
o	2	4

1

	x	$f(x)$
•	2	1

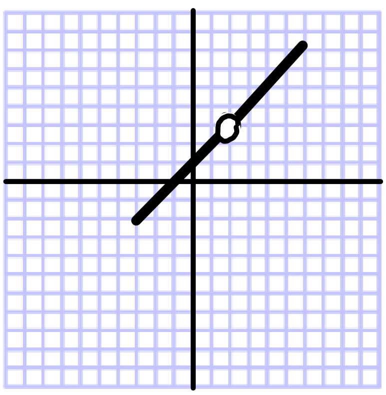
$8-x^2$

	x	$f(x)$
o	2	4
•	3	-1
•	4	-8

In English!
- The graph must be defined at that x-value
- A limit must exist at that x-value
- Limit must be the same as the defined height

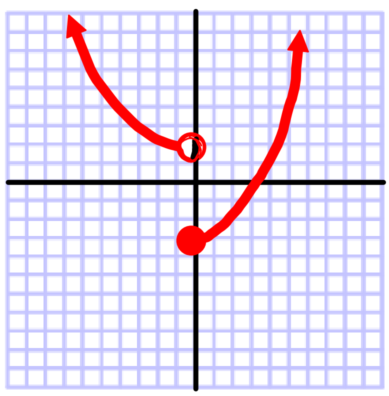
Summary of Continuity:

Hole Function

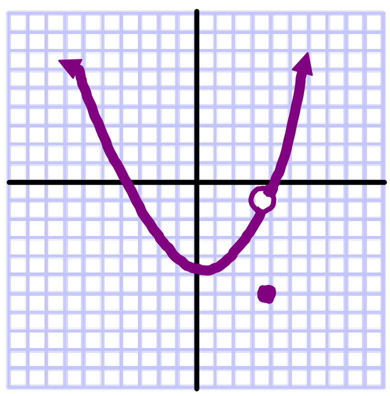


Limit exists
but it is not
defined
Not Continuous

Step Graph



Limit does not
exist
Not Continuous



Limit is not the
same as the y
value
Not Continuous

Homework

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Try this one...

$$f(x) = \begin{cases} 2 - x^2 & if\ x < 1 \\ 3 & if\ x = 1 \\ 2x - 1 & if\ 1 < x \le 3 \\ (x-4)^2 & if\ x > 3 \end{cases}$$

Evaluate:

$\lim_{x \to 1} f(x)$

$\lim_{x \to 3} f(x)$

Given the function $f(x) = \begin{cases} 2 - x^2 & \text{if } x < 1 \\ 3 & \text{if } x = 1 \\ 2x - 1 & \text{if } 1 < x \le 3 \\ (x-4)^2 & \text{if } x > 3 \end{cases}$

(a) Check $f(x)$ for any points of discontinuity. Provide a mathematical reason to validate any point(s) where the function is discontinuous.

(b) Sketch $f(x)$.