Warm Up

Differentiate: $e^{xy^2} = 2x - 3xy + e^{\tan x}$

$$e^{xy^2}(x\,2yy' + (1)y^2) = 2 - (3xy' + 3y) + e^{\tan x} \cdot \sec^2 x(1)$$

$$2xyy'e^{xy^2} + y^2e^{xy^2} = 2 - 3xy' - 3y + e^{\tan x}\sec^2 x$$

$$2xyy'e^{xy^2} + 3xy' = 2 - 3y + e^{\tan x}\sec^2 x - y^2e^{xy^2}$$

$$y'(2xye^{xy^2} + 3x) = 2 - 3y + e^{\tan x}\sec^2 x - y^2e^{xy^2}$$

$$\boxed{y' = \frac{2 - 3y + e^{\tan x}\sec^2 x - y^2e^{xy^2}}{2xye^{xy^2} + 3x}}$$

Find the equation of the tangent line to the curve $y = 1 + xe^{2x}$ at the point where $x = 0$.

① Find y.

$y = 1 + xe^{2x}$

$y = 1 + (0)e^{2(0)}$

$y = 1 + 0$

$y = 1$

Point $(\underline{\underline{0}}, \underline{\underline{1}})$

② Find y':

$y = 1 + xe^{2x}$

$y' = 0 + x(e^{2x})(2) + (1)e^{2x}$

$y' = 2xe^{2x} + e^{2x}$

$y'(0) = 2(0)e^{2(0)} + e^{2(0)}$

$= 0 + 1$

$= \underline{\underline{1}}$

$\uparrow$

$m = 1$

slope of the tangent

③ Find equation:

$y - y_1 = m(x - x_1)$

$y - 1 = 1(x - 0)$

$y - 1 = x$

$\boxed{0 = x - y + 1}$

Questions from Homework

⑥ $e^{xy} = 2x+y$

$e^{xy}(xy' + 1y) = 2 + y'$

$xy'e^{xy} + ye^{xy} = 2 + y'$

$xy'e^{xy} - y' = 2 - ye^{xy}$

$y'(xe^{xy} - 1) = 2 - ye^{xy}$

$$\boxed{y' = \frac{2 - ye^{xy}}{xe^{xy} - 1}}$$

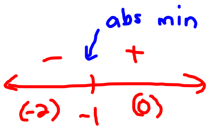
⑩ $F(x) = xe^{x}$

a) $F'(x) = x(e^{x})(1) + (1)e^{x}$

$= xe^{x} + e^{x}$

$= e^{x}(x+1)$

CV: $e^{x} = 0$ No CV. | $x+1=0$, $x=-1$



$F(-1) = (-1)e^{(-1)}$

$= -1 \cdot \frac{1}{e}$

$= \frac{-1}{e}$

$(-1, \frac{-1}{e})$ abs min

b) $F'(x) = e^{x}(x+1)$

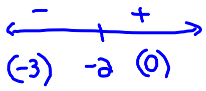
$F''(x) = e^{x}(1) + e^{x}(1)(x+1)$

$= e^{x} + e^{x}(x+1)$

$= e^{x}[1 + (x+1)]$

$= e^{x}(x+2)$

CV: $x=-2$



CU on $(-2,\infty)$

CD on $(-\infty,-2)$

c) Inflection Point when $x=-2$

$F(-2) = (-2)e^{(-2)}$

$= -2 \cdot \frac{1}{e^2}$

$= \frac{-2}{e^2}$

$(-2, -\frac{2}{e^2})$ Inflection Point

Derivatives of Logarithmic Functions

Let's work from the known...

- At this point you should know how to differentiate y = e^x.

$\log_e y = x$

$\ln y = x$

What other function could this model?

$$\ln y = x$$

Try to differentiate → y = ln x.

$\underline{e^y = x}$

Implicit Diff → $e^y \cdot y' = 1$

$y' = \frac{1}{e^y} = \frac{1}{\underline{x}}$

Differentiate: $y = \ln x^3$

$e^y = \underline{x^3}$

$e^y \cdot y' = 3x^2$

$y' = \frac{3x^2}{e^y} = \frac{3\cancel{x^2}}{\underline{\cancel{x^3}}} = \frac{3}{x}$

Rule: $d(\ln u) = \frac{1}{u}du$

$u = x^7$ $du = 7x^6$

Ex: $y = \ln x^7$

$y' = \frac{1}{x^{\cancel{7}}} \cdot 7\cancel{x^6} = \frac{7}{x}$

Practice Problem:

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$1 \div \frac{x+1}{x-1}$

$1 \times \frac{x-1}{x+1}$

j) $y = \ln\left(\frac{x+1}{x-1}\right)$

$$y' = \left(\frac{x-1}{x+1}\right)\left[\frac{\overset{x-1}{1(x-1)} - \overset{-x-1}{1(x+1)}}{(x-1)^2}\right]$$

$$y' = \left[\frac{\cancel{x-1}}{x+1}\right]\left[\frac{-2}{(x-1)^{\cancel{2}}}\right] = \frac{-2}{(x+1)(x-1)} = \frac{-2}{x^2-1}$$

Laws of Logarithms

$$\log_b M + \log_b N = \log_b (MN)$$

$$\log_b M - \log_b N = \log_b \left(\frac{M}{N} \right)$$

$$\log_b (N^p) = p \log_b (N)$$

We have now covered base "e"...both as an exponential and logarithmic function...

What about other bases??

Will need to know the change of base formula for logarithms:

$$\log_N M = \frac{\log_b M}{\log_b N}$$

Whatever new base you choose

Differentiate:

$y = \log_6 x^3$

$y = \log\left(5x^4\right)$

Rule: $d\left(\log_b u\right) = \frac{1}{u \ln b} du$

This leaves one form of exponential function remaining...

- What about a function such as $y = 3^{9x}$

Try this one... $y = \pi^{x^5}$

Rule:

$$d(b^{u}) = b^{u}(\ln b)du \text{ ,} where \;\; b \in R$$

Practice Problems:

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#5 #6 #7 #8