Laws of Logarithms

$$\log_b M + \log_b N = \log_b(MN)$$

$$\log_b M - \log_b N = \log_b\left(\frac{M}{N}\right)$$

$$\log_b(N^p) = p\log_b(N)$$

Warm Up

Review of laws of logarithms...

Given that $\log_x M = -3$, $\log_x N = 5$ and $\log_x P = 4$, evaluate the following logarithmic expression:

$$\log_x\left[\frac{(M^3N)^2\sqrt{P}}{MP}\right]$$

$$\log_x\left[\frac{M^6N^2P^{1/2}}{MP}\right]$$

$$\log_x\left[M^5N^2P^{-1/2}\right]$$

$$\log_x\left[\frac{M^5N^2}{P^{1/2}}\right]$$

$$\log_x M^5 + \log_x N^2 - \log_x P^{1/2}$$

$$5\log_x M + 2\log_x N - \frac{1}{2}\log_x P$$

$$5(-3) + 2(5) - \frac{1}{2}(4)$$

$$-15 + 10 - 2$$

$$-7$$

Solve the following equation: $\dfrac{3^{x-1}}{5*2^{3x}} = 6^{1-2x}$

take log of both sides

$$\log\left(\frac{3^{x-1}}{5\cdot 2^{3x}}\right) = \log 6^{1-2x}$$

$$\log 3^{x-1} - \log 5 - \log 2^{3x} = \log 6^{1-2x}$$

$$(x-1)\log 3 - \log 5 - 3x\log 2 = (1-2x)\log 6$$

$$x\log 3 - \log 3 - \log 5 - 3x\log 2 = \log 6 - 2x\log 6$$

$$x\log 3 - 3x\log 2 + 2x\log 6 = \log 6 + \log 3 + \log 5$$

$$x(\log 3 - 3\log 2 + 2\log 6) = \log 6 + \log 3 + \log 5$$

$$x = \frac{\log 6 + \log 3 + \log 5}{\log 3 - 3\log 2 + 2\log 6}$$

$$x = \frac{\log 6 + \log 3 + \log 5}{\log 3 - \log 2^3 + \log 6^2}$$

$$x = \frac{\log(6\cdot 3\cdot 5)}{\log\left(\frac{3\cdot 36}{8}\right)}$$

$$x = \frac{\log 90}{\log 13.5} \approx 1.73$$

Questions from Homework

Rule: $d(\ln u) = \dfrac{1}{u}du = \dfrac{du}{u}$

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$u = \left(\dfrac{x}{2x+3}\right)^{1/2} = \dfrac{(x)^{1/2}}{(2x+3)^{1/2}}$

① k) $y = \ln\sqrt{\dfrac{x}{2x+3}} = \ln\left(\dfrac{x}{2x+3}\right)^{1/2}$

$y' = \dfrac{(2x+3)^{1/2}}{x^{1/2}}\left[\dfrac{1}{2}\left(\dfrac{x}{2x+3}\right)^{-1/2} \cdot \dfrac{1(2x+3)-2x}{(2x+3)^2}\right]$

$y' = \dfrac{(2x+3)^{1/2}}{x^{1/2}}\left[\dfrac{1}{2}\cdot\dfrac{(2x+3)^{1/2}}{x^{1/2}} \cdot \dfrac{3}{(2x+3)^2}\right]$

$y' = \dfrac{3\cancel{(2x+3)}}{2x\,(2x+3)^{\cancel{2}}} = \dfrac{3}{2x(2x+3)} = \dfrac{3}{4x^2+6x}$

m) $y = \ln(\sec x + \tan x)$

$u = \sec x + \tan x$

$du = \sec x \tan x \cdot 1 + \sec^2 x \cdot$

$= \sec x \tan x + \sec^2 x$

$y' = \dfrac{\sec x \tan x + \sec^2 x}{\sec x + \tan x}$

$y' = \dfrac{\sec x\,\cancel{(\tan x + \sec x)}}{\cancel{(\sec x + \tan x)}} = \sec x$

We have now covered base "e"...both as an exponential and logarithmic function...

What about other bases??

Will need to know the change of base formula for logarithms:

$$\log_N M = \frac{\log_b M}{\log_b N}$$

Whatever new base you choose

Rule: $d(\log_b u) = \frac{1}{u \ln b} du = \frac{du}{u \ln b}$

Differentiate:

$y = \log_6 x^3$

$b=6$
$u=x^3$
$du=3x^2$

$y' = \frac{1}{x^3 \ln 6} \cdot 3x^2$

$y' = \frac{3\cancel{x^2}}{\cancel{x^3} \ln 6} = \frac{3}{x \ln 6}$

$y = \log(5x^4)$

$b=10$
$u=5x^4$
$du=20x^3$

$y' = \frac{20x^3}{5x^4 \ln 10} = \frac{4}{x \ln 10}$

This leaves one form of exponential function remaining...

- What about a function such as $y = 3^{9x}$

$b = 3$
$u = 9x$
$du = 9$

Rule:

$$d(b^u) = b^u(\ln b)du \text{ , where } b \in R$$

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$y = 3^{9x}$

$y' = 3^{9x}(\ln 3)9$

Try this one... $y = \pi^{x^5}$

$b = \pi$
$u = x^5$
$du = 5x^4$

$y' = \pi^{x^5}(\ln \pi)5x^4$

Practice Problems:

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#1 #2 a #3 #4

#5 #6 #7 #8