② f) $f(t) = (at+b)(ct^2-d)$

$f'(t) = (a)(ct^2-d) + (at+b)(2ct)$

$f'(t) = act^2 - ad + 2act^2 + 2bct$

$f'(t) = 3act^2 + 2bct - ad$

③ a) $y = (1-2x)(3x-4)$, $x=2$

① Find y':

$y' = -2(3x-4) + (1-2x)(3)$

$y' = -6x+8+3-6x$

$y' = -12x+11$

② sub in $x=2$

$y' = -12x+11$

$y' = -12(2)+11$

$y' = -13$

$m=-13$

⑤ $y = (2-\sqrt{x})(1+\sqrt{x}+3x)$ @ $(1,5)$

$y = (2-x^{1/2})(1+x^{1/2}+3x)$

① find y':

$y' = \left(-\frac{1}{2}x^{-1/2}\right)(1+x^{1/2}+3x) + (2-x^{1/2})\left(\frac{1}{2}x^{-1/2}+3\right)$

$y' = \left(\frac{-1}{2\sqrt{x}}\right)(1+\sqrt{x}+3x) + (2-\sqrt{x})\left(\frac{1}{2\sqrt{x}}+3\right)$

② sub in $x=1$

$y' = \left(\frac{-1}{2\sqrt{1}}\right)(1+\sqrt{1}+3(1)) + (2-\sqrt{1})\left(\frac{1}{2\sqrt{1}}+3\right)$

$y' = \left(-\frac{1}{2}\right)(5) + (1)\left(\frac{7}{2}\right)$

$y' = \frac{-5}{2} + \frac{7}{2} = \frac{2}{2} = 1$

$m=1$

③ $y-y_1 = m(x-x_1)$

$y-5 = 1(x-1)$..

$y-5 = x-1$

$y = x+4$

or $x-y+4=0$

Differentiation Rules

Product Rule:

The Product Rule If f and g are both differentiable, then

$$\frac{d}{dx}[f(x)g(x)] = f(x)\frac{d}{dx}[g(x)] + g(x)\frac{d}{dx}[f(x)]$$

Express the product rule verbally if you are considering a function of the form...

f(x) = (First) X (Second)

In words, ***the Product Rule*** says that the ***derivative of a product of two functions is:*** ***the derivative of the first function times the second function, plus the first function times the derivative of the second function***

Get in the habit of verbalizing the rule as you differentiate...it will help when the functions get more complicated.

$$(fg)'x = f'(x)g(x) + f(x)g'(x)$$

Differentiate the following function and simplify your answer:

$$h(t) = \underbrace{(t^3 - 5t)}_{f(x)}\underbrace{(6\sqrt{t} - t^{-5})}_{g(x)}$$

$$h'(t) = (3t^2 - 5)(6t^{1/2} - t^{-5}) + (t^3 - 5t)(3t^{-1/2} + 5t^{-6})$$

$$h'(t) = 18t^{5/2} - 3t^{-3} - 30t^{1/2} + 5t^{-5} + 3t^{5/2} + 5t^{-3} - 15t^{1/2} - 25t^{-5}$$

$$h'(t) = 21t^{5/2} - 45t^{1/2} + 2t^{-3} - 20t^{-5}$$

$$\boxed{h'(t) = 21t^{5/2} - 45t^{1/2} + \frac{2}{t^3} - \frac{20}{t^5}}$$

Quotient Rule:

The Quotient Rule If f and g are differentiable, then

$$\frac{d}{dx}\left[\frac{f(x)}{g(x)}\right] = \frac{g(x)\frac{d}{dx}[f(x)] - f(x)\frac{d}{dx}[g(x)]}{[g(x)]^2}$$

Express the quotient rule verbally if you are considering a function of the form...

$$f(x) = \frac{\text{(First)}}{\text{(Second)}}$$

In words, ***the Quotient Rule*** says that the ***derivative of a quotient is:***
the denominator times the derivative of the numerator, minus the numerator times the derivative of the denominator, all divided by the square of the denominator.

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

Examples:

$$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{g(x)^2}$$

Differentiate the following functions and simplify your answers:

$$F(x) = \frac{x^2 + 2x - 3}{x^3 + 1} \quad \begin{matrix} f(x) \\ g(x) \end{matrix}$$

$$F'(x) = \frac{(2x+2)(x^3+1) - 3x^2(x^2+2x-3)}{(x^3+1)^2}$$

$$F'(x) = \frac{2x^4 + 2x + 2x^3 + 2 - 3x^4 - 6x^3 + 9x^2}{(x^3+1)^2}$$

$$F'(x) = \frac{-x^4 - 4x^3 + 9x^2 + 2x + 2}{(x^3+1)^2}$$

$$F(x) = \frac{\sqrt{x}}{1 + 2x} \quad \begin{matrix} f(x) \\ g(x) \end{matrix} \quad = \frac{x^{1/2}}{1+2x}$$

$$F'(x) = \frac{\frac{1}{2}x^{-1/2}(1+2x) - 2\sqrt{x}}{(1+2x)^2}$$

$$F'(x) = \frac{\frac{1}{2\sqrt{x}}(1+2x) - 2\sqrt{x}}{(1+2x)^2}$$

$$F'(x) = \frac{\cancel{2\sqrt{x}}\,\frac{1+2x}{\cancel{2\sqrt{x}}} - 2\sqrt{x} \cdot 2\sqrt{x}}{2\sqrt{x}(1+2x)^2}$$

CD: $2\sqrt{x}$

$$F'(x) = \frac{1 + 2x - 4x}{2\sqrt{x}(1+2x)^2} = \frac{1 - 2x}{2\sqrt{x}(1+2x)^2}$$

Differentiate the following functions, do not simplify your answers:

$$f(x) = \frac{8 - 9x^7}{3x - 7}$$

$$f'(x) = \frac{-63x^6(3x-7) - 3(8-9x^7)}{(3x-7)^2}$$

$$f(x) = \frac{x^3 - 7x^2 + 2}{x^8 - 4x^5}$$

$$f'(x) = \frac{(3x^2-14x)(x^8-4x^5) - (x^3-7x^2+2)(8x^7-20x^4)}{(x^8-4x^5)^2}$$

Homework

$$y = \frac{x}{x-2} \qquad (4,2)$$

① $$y' = \frac{1(x-2) - x}{(x-2)^2} = \frac{x-2-x}{(x-2)^2} = \frac{-2}{(x-2)^2}$$

② $$y' = \frac{-2}{(x-2)^2} = \frac{-2}{(4-2)^2} = \frac{-2}{(2)^2} = \frac{-2}{4} = -\frac{1}{2}$$

③ $$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{1}{2}(x-4)$$

$$y - 2 = -\frac{1}{2}x + 2$$

$$y = -\frac{1}{2}x + 4$$

$$f(x)=\frac{\left(10x^{-5}+x\right)\left(3x^{3}+5\right)}{\left(-2x^{6}+\sqrt[3]{x}\right)}$$

$$f(x)=\frac{\left(x-7\right)\left(2x^{6}-x^{4}+5\right)}{\left(6x-x^{5}\right)\left(4x^{3}+2\right)}$$