## Questions from Homework

$F'(x) = f(x)g'(x) + f'(x)g(x)$ $\qquad$ $F'(x) = \dfrac{g(x)f'(x) - f(x)g'(x)}{[g(x)]^2}$

① e) $f(x) = \dfrac{\sqrt{x}}{x^2+1} = \dfrac{x^{1/2}}{x^2+1}$ $\quad$ $f(x)$ $\quad$ $g(x)$

$$f'(x) = \frac{\frac{1}{2}x^{-1/2}(x^2+1) - 2x(x^{1/2})}{(x^2+1)^2}$$

$$f'(x) = \frac{\frac{1}{2\sqrt{x}}(x^2+1) - 2x^{3/2}}{(x^2+1)^2}$$

$$f'(x) = \frac{\cancel{2\sqrt{x}}\,\frac{x^2+1}{\cancel{2\sqrt{x}}} - 2x^{3/2}\cdot 2\sqrt{x}}{2\sqrt{x}\,(x^2+1)^2}$$

$$f'(x) = \frac{x^2+1-4x^2}{2\sqrt{x}\,(x^2+1)^2} = \frac{1-3x^2}{2\sqrt{x}\,(x^2+1)^2}$$

g) $g(x) = \dfrac{x^3-1}{x^2+x+1}$

$$g'(x) = \frac{3x^2(x^2+x+1) - (2x+1)(x^3-1)}{(x^2+x+1)^2}$$

$$g'(x) = \frac{3x^4+3x^3+3x^2-(2x^4-2x+x^3-1)}{(x^2+x+1)^2}$$

$$g'(x) = \frac{x^4+2x^3+3x^2+2x+1}{(x^2+x+1)^2} \longleftarrow \text{expand}$$

$$g'(x) = \frac{\cancel{x^4+2x^3+3x^2+2x+1}}{\cancel{x^4+2x^3+3x^2+2x+1}} = 1$$

h) $f(x) = \dfrac{1-\frac{1}{x}}{x+1} = \dfrac{1-x^{-1}}{x+1}$

$$f'(x) = \frac{x^{-2}(x+1) - 1(1-x^{-1})}{(x+1)^2}$$

$$f'(x) = \frac{x^{-1}+x^{-2}-1+x^{-1}}{(x+1)^2}$$

$$f'(x) = \frac{x^{-2}+2x^{-1}-1}{(x+1)^2}$$

$$f'(x) = \frac{\frac{1}{x^2}\cdot x^2 + \frac{2}{x}\cdot x^2 - 1\cdot x^2}{x^2(x+1)^2} = \frac{1+2x-x^2}{x^2(x+1)^2}$$

h) $f(x) = \dfrac{x\left(1-\frac{1}{x}\right)x}{x(x+1)} = \dfrac{x-1}{x^2+x}$

③ a) $y = \dfrac{x}{x-2} \rightarrow (4,2)$ $\quad$ $x_1 = 4$, $y_1 = 2$

① Find $y'$

$y = \dfrac{x}{x-2}$

$$y' = \frac{1(x-2) - 1(x)}{(x-2)^2}$$

$$y' = \frac{x-2-x}{(x-2)^2} = \frac{-2}{(x-2)^2}$$

② Find $y'(4)$ "m"

$$y'(4) = \frac{-2}{(4-2)^2}$$

$$y'(4) = \frac{-2}{4} = \frac{-1}{2}$$

m

③ $y - y_1 = m($

$y - 2 = -\frac{1}{2}(x$

$y - 2 = -\frac{1}{2}x +$

$\boxed{y = -\frac{1}{2}x +}$

$y = -\frac{1}{x}$

# Questions from Homework

2 c) $f(x) = \frac{1}{(x+1)(2x-3)} = \frac{1}{2x^2-3x+2x-3} = \frac{1}{2x^2-x-3}$ $\frac{f(x)}{g(x)}$

$f'(x) = \frac{(0)(2x^2-x-3) - 1(4x-1)}{(2x^2-x-3)^2}$

$f'(x) = \frac{0-4x+1}{(2x^2-x-3)^2} = \frac{-4x+1}{(2x^2-x-3)^2}$

3 c) $y = \frac{1}{x^2+1}$ @ $(-2, \frac{1}{5})$

$x_1 = -2$ $m = \frac{4}{25}$
$y_1 = \frac{1}{5}$

i) Find $y'$

$y' = \frac{(0)(x^2+1) - 1(2x)}{(x^2+1)^2}$

$y' = \frac{-2x}{(x^2+1)^2}$

ii) sub in $x=-2$

$y' = \frac{-2(-2)}{((-2)^2+1)^2}$

$y' = \frac{4}{25}$

$m = \frac{4}{25}$

iii) $y - y_1 = m(x - x_1)$

$y - \frac{1}{5} = \frac{4}{25}(x+2)$

$y - \frac{1}{5} = \frac{4x}{25} + \frac{8}{25}$

$y = \frac{4x}{25} + \frac{8}{25} + \frac{1}{5}$

$y = \frac{4x}{25} + \frac{8}{25} + \frac{5}{25}$

$\boxed{y = \frac{4x}{25} + \frac{13}{25}}$ ✓

or $25y = 4x + 13$

$\boxed{0 = 4x - 25y + 13}$

# Questions from Homework

⑥ $y = \dfrac{x^2}{2x+5}$

horizontal slope $= \dfrac{0}{1}$

① Find $y'$

$$y' = \frac{2x(2x+5) - 2(x^2)}{(2x+5)^2}$$

$$y' = \frac{4x^2 + 10x - 2x^2}{(2x+5)^2}$$

$$y' = \frac{2x^2 + 10x}{(2x+5)^2}$$

$$\frac{0}{1} = \frac{2x^2 + 10x}{(2x+5)^2}$$

$$2x^2 + 10x = 0$$

$$2x(x+5) = 0$$

| | |
|---|---|
| $2x = 0$ | $x+5=0$ |
| $x = 0$ | $x = -5$ |

Solve for y

| | |
|---|---|
| $x = 0$ | $x = 5$ |
| $y = \dfrac{(0)^2}{2(0)+5} = 0$ | $y = \dfrac{(5)^2}{2(5)+5} = \dfrac{25}{15} = \dfrac{5}{3}$ |
| $(0,0)$ | $(5, 5/3)$ |

The points are $(0,0)$ and $(5, 5/3)$

# Questions from Homework

④ $$\left(\frac{f}{g}\right)'(x) = \frac{f'(x)g(x) - f(x)g'(x)}{[g(x)]^2}$$

$$\left(\frac{f}{g}\right)'(2) = \frac{f'(2)g(2) - f(2)g'(2)}{[g(2)]^2}$$

Given:

$f(2) = 3$

$f'(2) = 5$

$g(2) = -1$

$g'(2) = -4$

$$\left(\frac{f}{g}\right)'(2) = \frac{(5)(-1) - (3)(-4)}{[-1]^2} = \frac{-5+12}{1} = 7$$

# Chain Rule:

**The Chain Rule** If $f$ and $g$ are both differentiable and $F = f \circ g$ is the composite function defined by $F(x) = f(g(x))$, then $F$ is differentiable and $F'$ is given by the product

$$F'(x) = f'(g(x))g'(x)$$

**Work from the outside to the inside**

Examples:

$$f(x) = (5x^3 + 1)^{10}$$

$$f'(x) = 10(5x^3+1)^9(15x^2)$$

$$f'(x) = 150x^2(5x^3+1)^9$$

$$F(x) = \sqrt{2x^2+3} = (2x^2+3)^{1/2}$$

$$F'(x) = \frac{1}{\cancel{2}}(2x^2+3)^{-1/2}(\overset{2}{\cancel{4}}x) = 2x(2x^2+3)^{-1/2} = \frac{2x}{(2x^2+3)^{1/2}} = \frac{2x}{\sqrt{2x^2+3}}$$

$$h(x) = \sqrt[3]{5-3x^4} = (5-3x^4)^{1/3}$$

$$h'(x) = \frac{1}{\cancel{3}}(5-3x^4)^{-2/3}(\overset{-4}{\cancel{-12}}x^3) = -4x^3(5-3x^4)^{-2/3} = \frac{-4x^3}{(5-3x^4)^{2/3}} = \frac{-4x^3}{\sqrt[3]{(5-3x^4)^2}}$$

$$F'(x) = f'(g(x))\,g'(x)$$

suppose $f(1)=5$, $f'(1)=3$, $g(1)=2$, $g'(1)=-1$ and $f'(2)=7$ find $F'(1)$

$$F'(1) = f'(g(1)) \cdot g'(1)$$

$$= f'(2) \cdot g'(1)$$

$$= 7 \cdot -1$$

$$= -7$$

# Homework

③ $g(x) = (x^3+x^2-2)^{3/4}$

$g'(x) = \frac{3}{4}(x^3+x^2-2)^{-1/4}(3x^2+2x)$

$g'(x) = \frac{3(3x^2+2x)}{4(x^3+x^2-2)^{1/4}} = \frac{9x^2+6x}{4\sqrt[4]{x^3+x^2-2}}$

⑦ $y = \frac{1}{(x^3+2x^2+1)^2} = (x^3+2x^2+1)^{-2}$

$y' = -2(x^3+2x^2+1)^{-3}(3x^2+4x)$

$y' = \frac{-2(3x^2+4x)}{(x^3+2x^2+1)^3} = \boxed{\frac{-6x^2-8x}{(x^3+2x^2+1)^3}}$

⑦ $y = \frac{1}{(x^3+2x^2+1)^2}$ (Using Quotient Rule)

$y' = \frac{\cancel{0(x^3+2x^2+1)^2} - 2(x^3+2x^2+1)(3x^2+4x)}{(x^3+2x^2+1)^4}$

$y' = \frac{-2\cancel{(x^3+2x^2+1)}(3x^2+4x)}{(x^3+2x^2+1)^{\cancel{4}\,3}} = \boxed{\frac{-6x^2-8x}{(x^3+2x^2+1)^3}}$

⑯ $y = \sqrt{x+\sqrt{x}} = (x+x^{1/2})^{1/2}$

$y' = \frac{1}{2}(x+x^{1/2})^{-1/2}\left(1+\frac{1}{2}x^{-1/2}\right)$

$y' = \frac{1}{2}(x+\sqrt{x})^{-1/2}\left(1+\frac{1}{2\sqrt{x}}\right)$

$y' = \left(\frac{1}{2\sqrt{x+\sqrt{x}}}\right)\left(\frac{2\sqrt{x}}{2\sqrt{x}}+\frac{1}{2\sqrt{x}}\right)$

$y' = \left(\frac{1}{2\sqrt{x+\sqrt{x}}}\right)\left(\frac{2\sqrt{x}+1}{2\sqrt{x}}\right) = \frac{2\sqrt{x}+1}{4\sqrt{x}\sqrt{x+\sqrt{x}}}$

$$g(x) = 9x^{-3}\left(5x^3 - 1\right)^6$$

$$g(x) = \frac{\left(x^2 - 5x + 1\right)^8}{\left(1 - x^{-7}\right)^{20}}$$