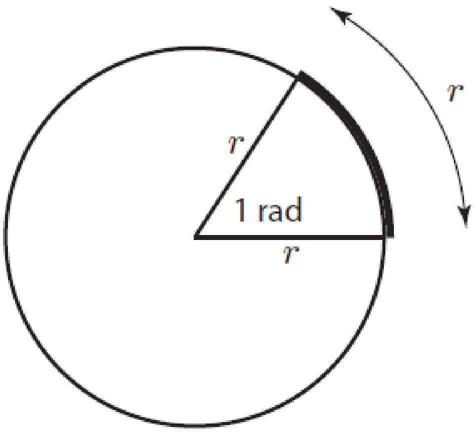
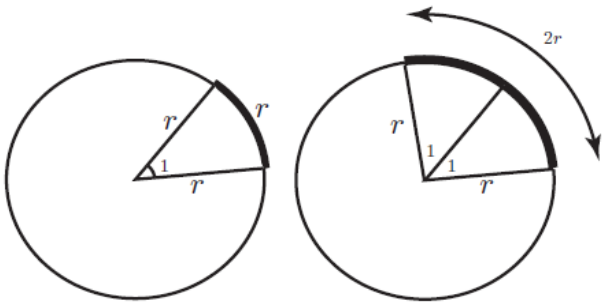
Radian Measure

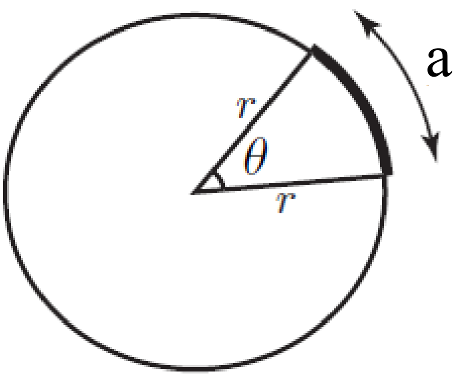
A radian is the angle subtended by an arc of length r (radius)





An angle of 2 rad subtends an arc of 2r...
7 rads subtends an arc of??? 7r

Use the above information to develop a formula to connect arc length, radius and the measure of an angle in radian measure...



has to be in radians

$$a = \theta r$$

Check-Up...

Arrange the following angles in descending order:

$340°$ $4.28\,rad$ $\frac{9\pi}{5}$ $(10\pi)°$

$340°$ $4.28\left(\frac{180}{\pi}\right)$ $\frac{9\pi}{5}\left(\frac{180}{\pi}\right)$ $(10(3.14))°$

$\frac{770.4}{\pi}$ $\frac{1620\pi}{5\pi}$ $31.4°$

$245.2°$ $324°$

$340°, \frac{9\pi}{5}, 4.28\text{rads}, (10\pi)°$

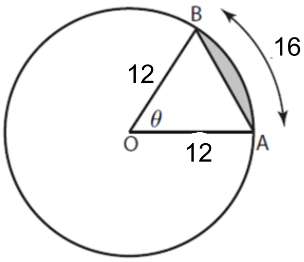
Find all angles coterminal to 150⁰

$150° \pm (360°)n, n \in N$

Homework

Page 176: #14, 15, 16

Find the area of the shaded region



Given:

$r = 12$

$a = 16$

Find θ

$$\theta = \frac{a}{r} = \frac{16}{12} = \frac{4}{3} \approx 1.\overline{3}$$

rads

$$A_{circle} = \pi r^2$$

$$A_{circle} = \pi (12)^2$$

$$A_{circle} = 144\pi \approx 452.4 \text{ units}^2$$

① $$\frac{A_{sector}}{A_{circle}} = \frac{\theta}{2\pi}$$

$$\cancel{452.4} \times \frac{A_{sector}}{\cancel{452.4}} = \frac{1.\overline{3}}{6.28} \times 452.4$$

$$A_{sector} = \frac{603.2}{6.28} = 96 \text{ units}^2$$

Switch calculator to rads

② $$A_{\Delta} = \frac{1}{2} r^2 \sin\theta$$

$$A_{\Delta} = \frac{1}{2}(12)^2 \sin(1.\overline{3})$$

$$A_{\Delta} = 69.97 \text{ units}^2$$

$$A_{\Delta} = 70 \text{ units}^2$$

③ $$A_{segment} = A_{sector} - A_{\Delta}$$

$$A_{segment} = 96 - 70 = 26 \text{ units}^2$$

Questions from Homework

14. A rotating water sprinkler makes one revolution every 15 s. The water reaches a distance of 5 m from the sprinkler.

a) What is the arc length of the sector watered when the sprinkler rotates through $\frac{5\pi}{3}$? Give your answer as both an exact value and an approximate measure, to the nearest hundredth.

b) Show how you could find the area of the sector watered in part a).

c) What angle does the sprinkler rotate through in 2 min? Express your answer in radians and degrees.

Given:

$r = 5m$

$\theta = \frac{5\pi}{3}$

a) $a = \theta r$

$a = \left(\frac{5\pi}{3}\right)(5)$

$a = \frac{25\pi}{3}$ m

$a \doteq 26.2m$

b) (i) $A_{circle} = \pi r^2 = \pi(5)^2$

$= 25\pi \ m^2$

(ii) $\frac{A_{sector}}{A_{circle}} = \frac{\theta}{2\pi}$

$(\cancel{25\pi})\frac{A_{sector}}{\cancel{25\pi}} = \frac{\frac{5\pi}{3}}{2\pi}(25\pi)$

$A_{sector} = \left(\frac{5\pi}{3}\right)(25\pi)\left(\frac{1}{2\pi}\right)$

$A_{sector} = \frac{125\pi^{\cancel{2}}}{6\cancel{\pi}} = \frac{125\pi}{6} m^2$

$\doteq 65.4 m^2$

15. Angular velocity describes the rate of change in a central angle over time. For example, the change could be expressed in revolutions per minute (rpm), radians per second, degrees per hour, and so on. All that is required is an angle measurement expressed over a unit of time.

a) Earth makes one revolution every 24 h. Express the angular velocity of Earth in three other ways.

b) An electric motor rotates at 1000 rpm. What is this angular velocity expressed in radians per second?

c) A bicycle wheel completes 10 revolutions every 4 s. Express this angular velocity in degrees per minute.

$V_a = \frac{\theta}{t}$

a) $V_a = \frac{1 \text{ rev}}{24 \text{ hrs}}$

(i) $V_a = \frac{360°}{24 \text{ hrs}}$ (iii) $V_a = \frac{360°}{1 \text{ day}}$

(ii) $V_a = \frac{2\pi}{24 \text{ hrs}} = \frac{\pi}{12 \text{ hrs}}$

b) (i) $\theta = 1000 \cancel{\text{revs}} \times 2\pi \text{ rads} = 2000\pi \text{ rads}$

(ii) $V_a = \frac{\theta}{t} = \frac{2000\pi \text{ rads}}{\text{min}} = \frac{2000\pi \text{ rads}}{60 \text{ sec.}} \doteq 104.7 \text{ rads/sec.}$

c) Given:

10 revs in 4 sec.

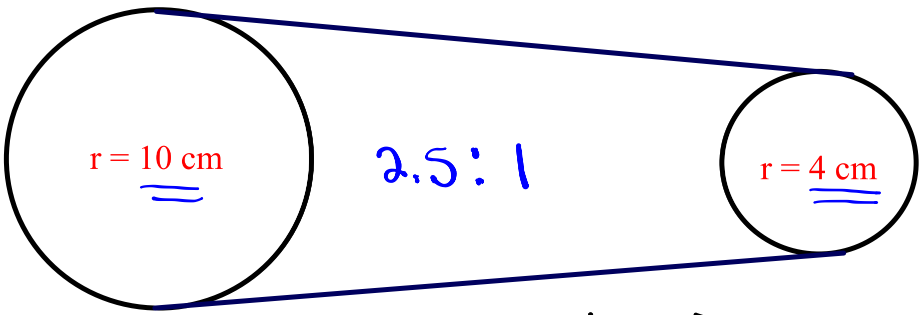
(i) $\theta = 10 \text{ revs} \times 360° = 3600°$

(ii) $V_a = \frac{\theta}{t} = \frac{3600°}{4 \text{ sec}}\,{}^{\times 15}_{\times 15} = \frac{54000°}{60 \text{ sec}} = \boxed{54000°/\text{min}}$

Applying our knowledge of rotations and radians...

Ex. (a) If the large wheel rotates $2\pi/3$ radians, how many radians does the smaller wheel rotate?
(b) If the large wheel completes three revolutions, how much does the small wheel rotate in radians?
(c) If the small wheel rotates $-15\pi/4$ radians, how many radians does the larger wheel rotate?

Ratio
10 : 4
5 : 2
2.5 : 1



(Big → Small multiply)

a) $\frac{2\pi}{3} \cdot 2.5 = \frac{5\pi}{3}$ The small wheel rotates $\frac{5\pi}{3}$

b) 3 revs = $3 \times 2\pi = 6\pi$

$6\pi \cdot 2.5 = 15\pi$ The small wheel rotates 15π

c) $-\frac{15\pi}{4} \div 2.5$ (Small → Big divide)

$-\frac{15\pi}{4} \cdot \frac{1}{2.5} = -\frac{15\pi}{10} = -\frac{3\pi}{2}$ The large wheel rotates $-\frac{3\pi}{2}$

Angular Velocity

Angular velocity - amount of rotation around a central point per unit of time

$$v_a = \frac{\theta}{t} \qquad \qquad \theta = \frac{a}{r}$$

θ = angle (radians)

v_a = angular velocity

t = time

a = arc length

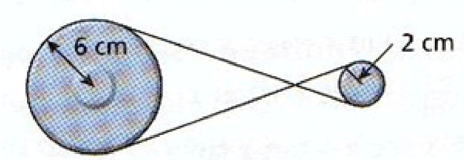
r = radius

Ex. The roller on a computer printer makes 2200 rpm (revolution per minute).
Find the roller's angular velocity.

$\theta = 2200 \times 2\pi = 4400\pi$ rads

$v_a = \frac{\theta}{t} = \frac{4400\pi \text{ rads}}{\text{min}} = \frac{4400\pi \text{ rads}}{60\text{s}} = 230.38$ rads/sec.

Two flywheels are connected by a belt, as shown in the diagram below. The larger one has a radius of 6 cm and the smaller one has a radius of 2 cm.



Ratio:
6:2
3:1

(a) If the small wheel rotates –300°, then through how many radians does the large wheel rotate?

arc length

(b) If the large wheel rotates $\frac{7\pi}{6}$ radians, what distance would a point on the circumference of the small wheel rotate?

Small

a) (i) $-300^\circ\left(\frac{\pi}{180}\right)$

$\frac{-300^\circ\pi}{180^\circ}$

$\theta = \frac{-5\pi}{3}$

Large

(ii) $\frac{-5\pi}{3} \div 3$

$\frac{-5\pi}{3} \cdot \frac{1}{3}$

$\theta = \frac{-5\pi}{9} \rightarrow \frac{5\pi}{9}$

switch sign if chain is crossed

Large Wheel

b) $\theta = \frac{7\pi}{6}$

Small Wheel

$\theta = \frac{7\pi}{6} \cdot 3 = \frac{21\pi}{6} = -\frac{7\pi}{2}$

$r = 2\text{cm}$ (Given)

$a = \theta r$

$a = \left(\frac{-7\pi}{\cancel{2}}\right)(\cancel{2}) = -\frac{14\pi}{2} = \boxed{-7\pi\text{cm}}$

Ex. A small electrical motor turns at 2200 rpm.

(a) Express the angular velocity in rad/s.
(b) Find the distance a point 0.8cm from the center of rotation travels in 0.008 s.

arc length

a) (i) $\theta = 2200 \text{ revs} \times 2\pi = 4400\pi \text{ rads}$

(ii) $V_a = \frac{\theta}{t} = \frac{4400\pi \text{ rads}}{60 \text{sec}} \doteq \boxed{230.38 \text{ rads/sec}}$

b) Given:

$r = 0.8\text{cm}$

$t = 0.008\text{sec.}$

$a =$

(i) Find θ

$\theta = V_a \times t$

$\theta = 230.38 \frac{\text{rads}}{\cancel{\text{sec}}} \times 0.008 \cancel{\text{sec}}$

$\theta = 1.84 \text{ rads}$

(ii) Find a

$a = \theta r$

$\boxed{a = (1.84)(0.8) = 1.47\text{cm}}$

Homework

Ex. A Ferris Wheel rotates 3 times each minute. The passengers sit in seats that are 5 m from the center of the wheel. What is the angular velocity of the wheel in radians per second? What distance do the passengers travel in 6.5 seconds?

Ex. A bicycle wheel has a radius of 36 cm and is turning at 4.8m/s. Determine the angu velocity of this wheel?

Homework

Ex. A Ferris Wheel rotates 3 times each minute. The passengers sit in seats that are 5 m from the center of the wheel. What is the angular velocity of the wheel in radians per second? What distance do the passengers travel in 6.5 seconds?

solve for a

$\theta = 3 \times 2\pi = 6\pi$ rads

$V_a = \frac{\theta}{t} = \frac{6\pi \text{ rads}}{1 \text{ min}} = \frac{6\pi \text{ rads}}{60 \text{ sec}} = 0.314 \text{ rads/sec}$

Given:

$r = 5m$

$t = 6.5sec$

(i) $\theta = V_a \times t$

$\theta = 0.314 \frac{\cancel{\text{rads}}}{\cancel{\text{sec}}} \times 6.5 \cancel{\text{sec}}$

$\theta = 2.04$ rads

(ii) $a = \theta r$

$a = (2.04)(5m)$

$a = 10.2m$

Ex. A bicycle wheel has a radius of 36 cm and is turning at 4.8m/s. Determine the angu velocity of this wheel?

Given:

$r = 36cm = 0.36m$

$V_a = ?$

$t = 1$ sec

After 1 sec

$a = 4.8m$

(i) $\theta = \frac{a}{r} = \frac{4.8m}{0.36m} = 13.\bar{3}$ rads

(ii) $V_a = \frac{\theta}{t} = \frac{13.3 \text{rads}}{1 \text{sec}} = \boxed{13.\bar{3} \text{ rads/sec}}$