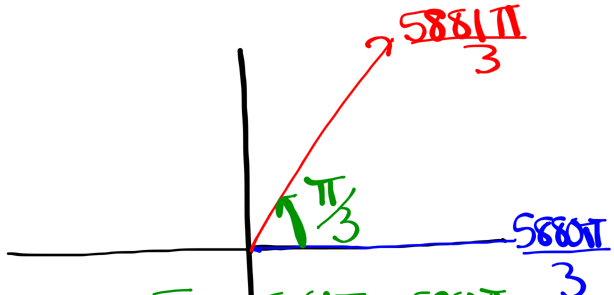
Sketch the following and determine a negative angle co-terminal with:

(*i*) $\frac{5881\pi}{3}$



$\frac{5880\pi}{3}, \frac{5881\pi}{3}, \frac{5882\pi}{3}$

1960π

$\bar{\theta} = \frac{5881\pi}{3} - \frac{5880\pi}{3}$

$\bar{\theta} = \frac{\pi}{3}$

Negative coterminal angle:

$\frac{5881\pi}{3} - \frac{1962\pi}{1}$

$\frac{5881\pi}{3} - \frac{5886\pi}{3}$

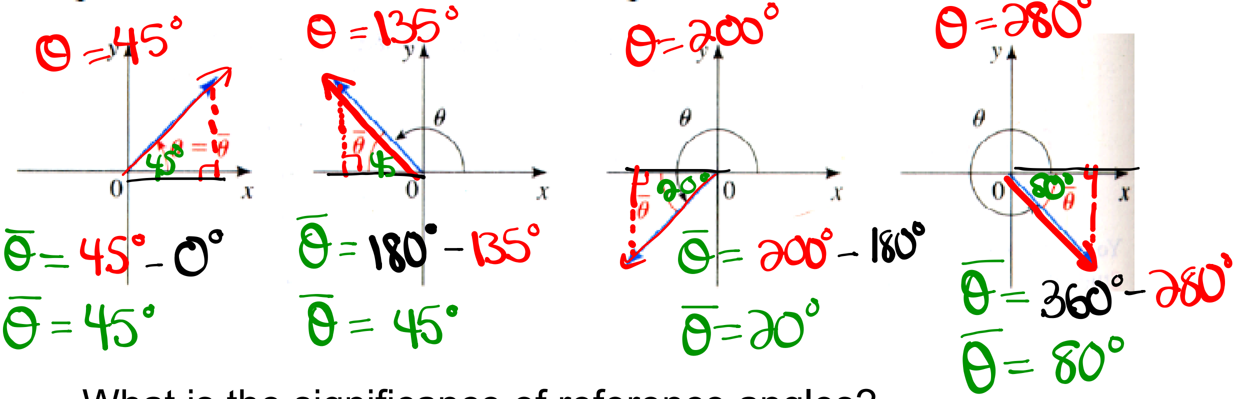
$\frac{-5\pi}{3}$

Reference Triangles:

Definition 17 *The reference angle* $\bar{\theta}$ *of an angle* θ *in standard position is the acute angle (between 0 and 90°) the terminal side makes with the x-axis.*

arm

The picture below illustrates this concept.



$\bar{\theta} = 45° - 0°$
$\bar{\theta} = 45°$

$\bar{\theta} = 180° - 135°$
$\bar{\theta} = 45°$

$\bar{\theta} = 200° - 180°$
$\bar{\theta} = 20°$

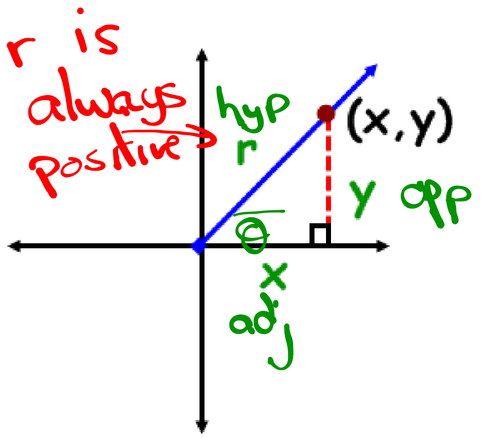
$\bar{\theta} = 360° - 280°$
$\bar{\theta} = 80°$

What is the significance of reference angles?

Angles on the Cartesian Plane

- **Reference Angle** - an acute angle formed between the terminal arm and the x-axis.

- **Reference Triangle** - a triangle formed by drawing a perpendicular line from a point on the terminal to the x-axis.



TRIG RATIOS on the CARTESIAN PLANE

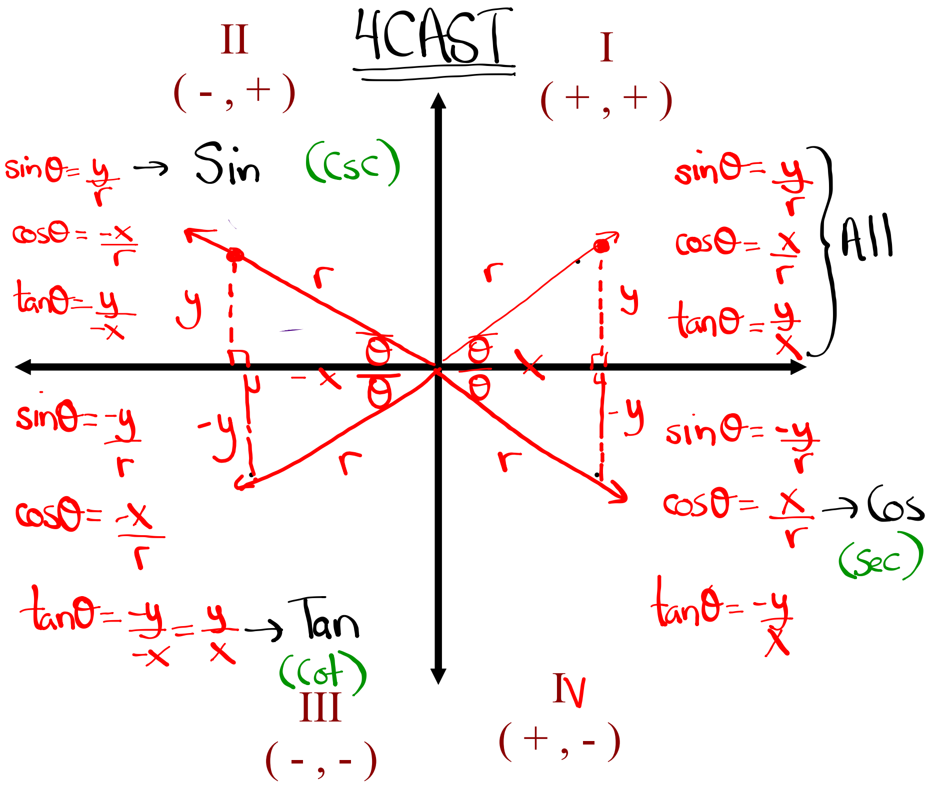
Primary	Reciprocal
$\sin\theta = \frac{y}{r} = \frac{o}{h}$	$\csc\theta = \frac{r}{y} = \frac{h}{o}$
$\cos\theta = \frac{x}{r} = \frac{a}{h}$	$\sec\theta = \frac{r}{x} = \frac{h}{a}$
$\tan\theta = \frac{y}{x} = \frac{o}{a}$	$\cot\theta = \frac{x}{y} = \frac{a}{o}$

"Primary" "Reciprocal"

Notice what will happen if the rotation moves into other quadrants?

TRIG RATIOS IN ALL 4 QUADRANTS

What primary trig ratios are POSITIVE in...



Where is θ if...

cscθ<0
sinθ<0

S	A
T	C

Quad 3+4

sinθ<0 & tanθ<0

S	A
T	C

Quad 4

cscθ>0 & cotθ<0
sinθ >0 + tanθ<0

S	A
T	C

Quad 2

Homework

$\sin\theta > 0$
$\cos\theta < 0$
Quad 2

S	A
T	C

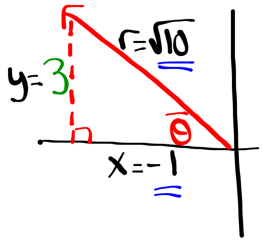
$\sec\theta < 0$

If $\sec\theta = -\sqrt{10}$ and $\sin\theta > 0$, determine the value of $\csc\theta = \frac{h}{o} = \frac{r}{y}$

$\sec\theta = \frac{-\sqrt{10}}{1} = \frac{hyp}{adj} = \frac{r}{x}$

$r = \sqrt{10}$ (always +)

$x = -1$



① Find y:

$x^2 + y^2 = r^2$

$(-1)^2 + y^2 = (\sqrt{10})^2$

$1 + y^2 = 10$

$y^2 = 9$

$y = \pm 3$

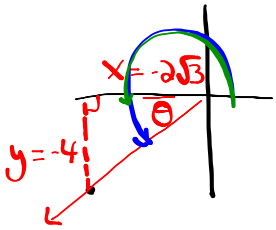
$y = 3$ (Q2)

② $\csc\theta = \frac{hyp}{opp} = \frac{r}{y} = \frac{\sqrt{10}}{3}$

Determine the measure (in radians) of an angle whose terminal arm passes through the ordered pair ~~$(-2\sqrt{3}, -4)$~~

$x = -2\sqrt{3}$

$y = -4$



① Find $\bar{\theta}$

$\tan\bar{\theta} = \frac{y}{x} = \frac{-4}{-2\sqrt{3}} = \frac{2}{\sqrt{3}}$

$\tan\bar{\theta} = 1.1547$

convert calculator to rads $\rightarrow$ $\bar{\theta} = \tan^{-1}(1.1547)$

$\bar{\theta} = 0.86$ rads

② Find θ

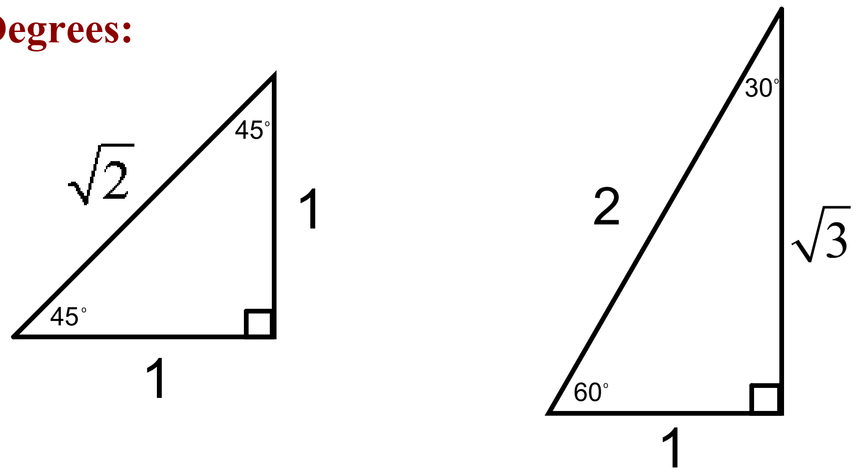
$\theta = \pi + \bar{\theta}$

$\theta = 3.14 + 0.86$

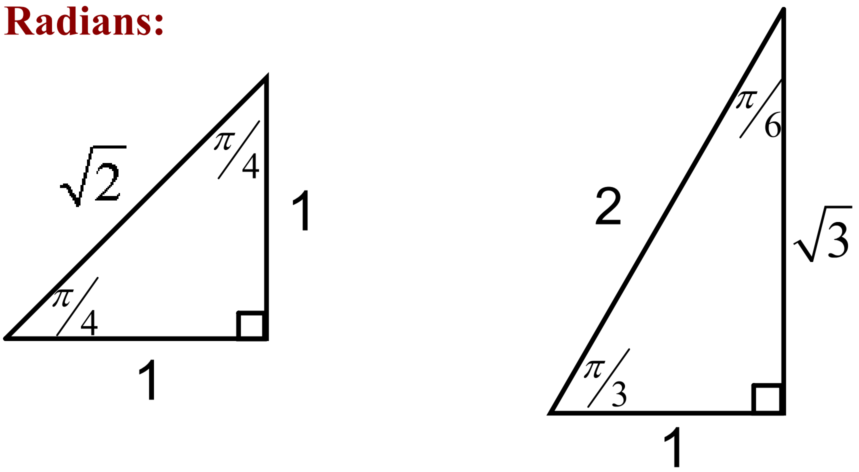
$\theta = 4$ rads

Special Angles

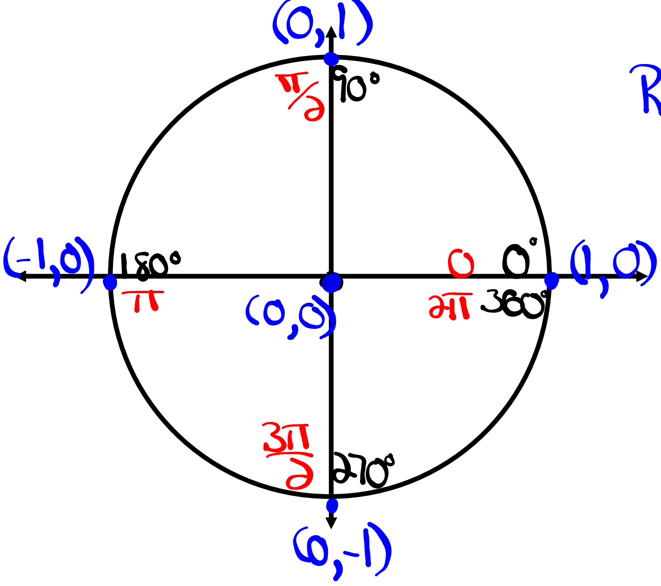
In Degrees:



In Radians:



Quadrantal Angles



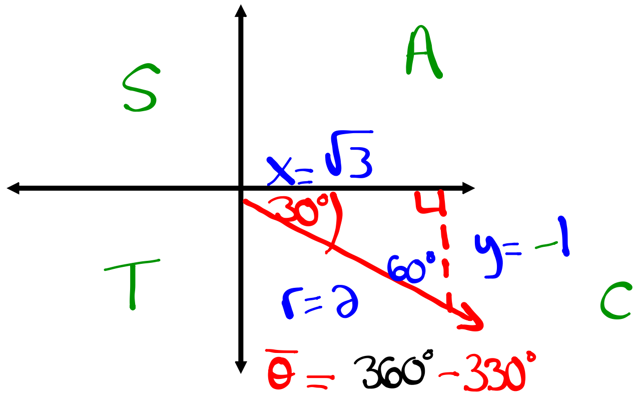
(Multiples of 90° or $\pi/2$ rads)

Radius = 1

$690^\circ - 360^\circ = 330^\circ$

Solving Trig Expressions by Sketching Angles

Ex. Evaluate the $\sin 690^\circ = \sin 330^\circ = \frac{-1}{2}$



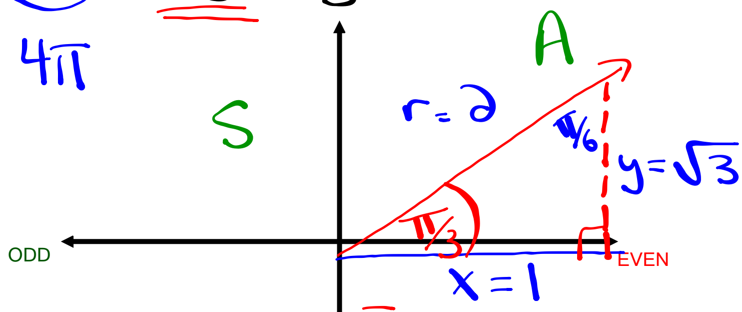
$\bar{\theta} = 360^\circ - 330^\circ$

$\bar{\theta} = 30^\circ$

Ex. $\cos\frac{13\pi}{3} = \frac{1}{2}$

$\frac{12\pi}{3}, \frac{13\pi}{3}, \frac{14\pi}{3}$

4π

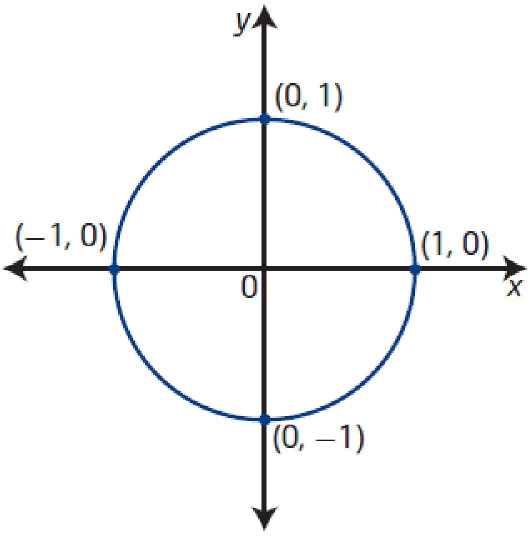


$\bar{\theta} = \frac{13\pi}{3} - \frac{12\pi}{3}$

$\bar{\theta} = \frac{\pi}{3}$

1

Unit Circle



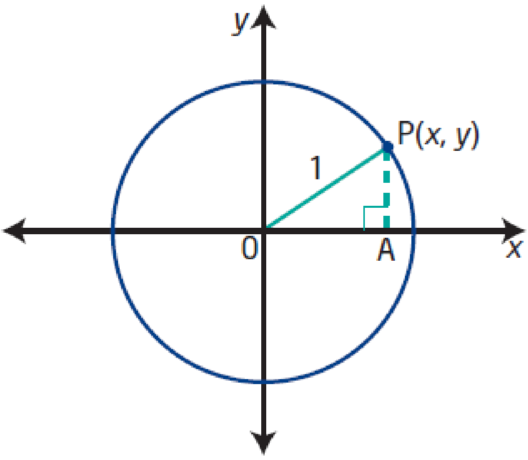
unit circle

- a circle with radius 1 unit $(r = 1)$
- a circle of radius 1 unit with centre at the origin on the Cartesian plane is known as *the* unit circle

$$\sin\theta = \frac{o}{h} = \frac{y}{r} = \frac{y}{1} = y \qquad \csc\theta = \frac{1}{y}$$

$$\cos\theta = \frac{a}{h} = \frac{x}{r} = \frac{x}{1} = x \qquad \sec\theta = \frac{1}{x}$$

$$\tan\theta = \frac{o}{a} = \frac{y}{x} \qquad \cot\theta = \frac{x}{y}$$



$r = 1$

$x^2 + y^2 = r^2$

The equation of the unit circle is $x^2 + y^2 = 1$.

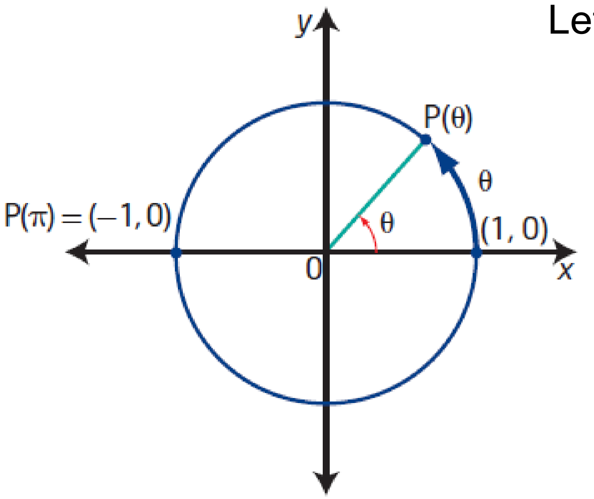
Determine the equation of a circle with centre at the origin and radius 6.

$x^2 + y^2 = r^2$

$x^2 + y^2 = (6)^2$

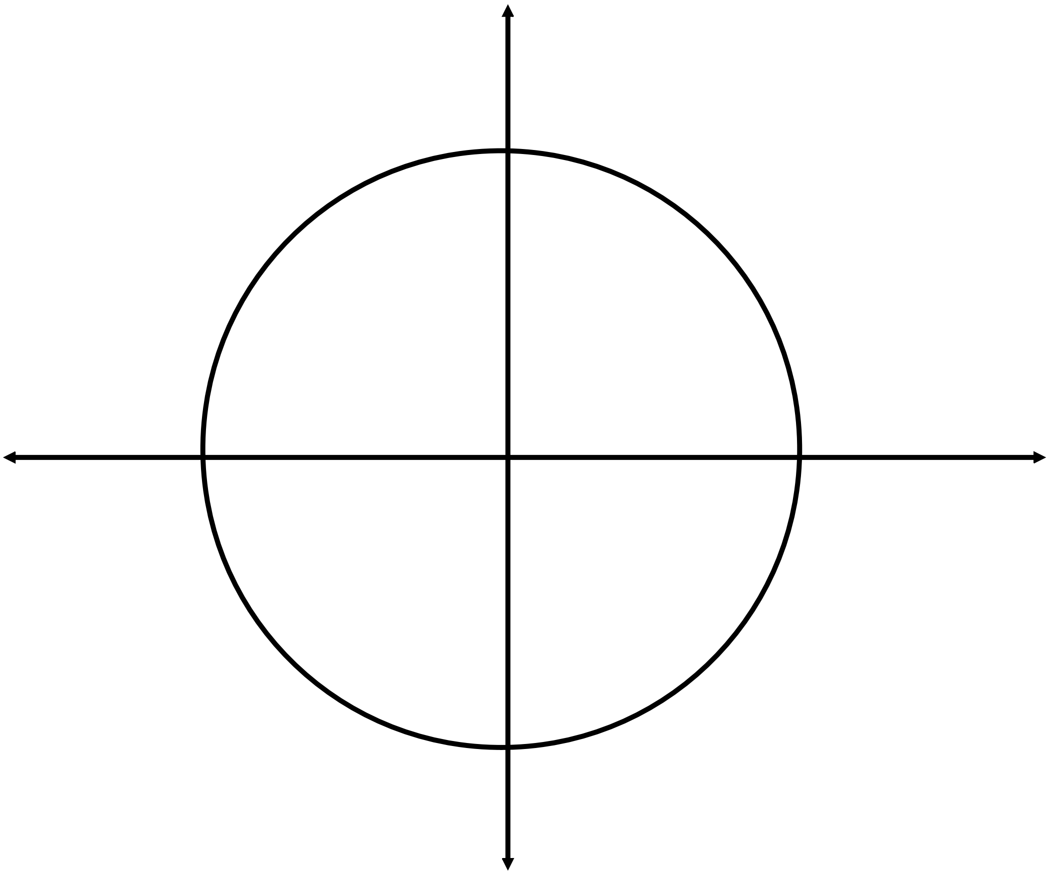
$\boxed{x^2 + y^2 = 36}$

Special Angles on the Unit Circle:

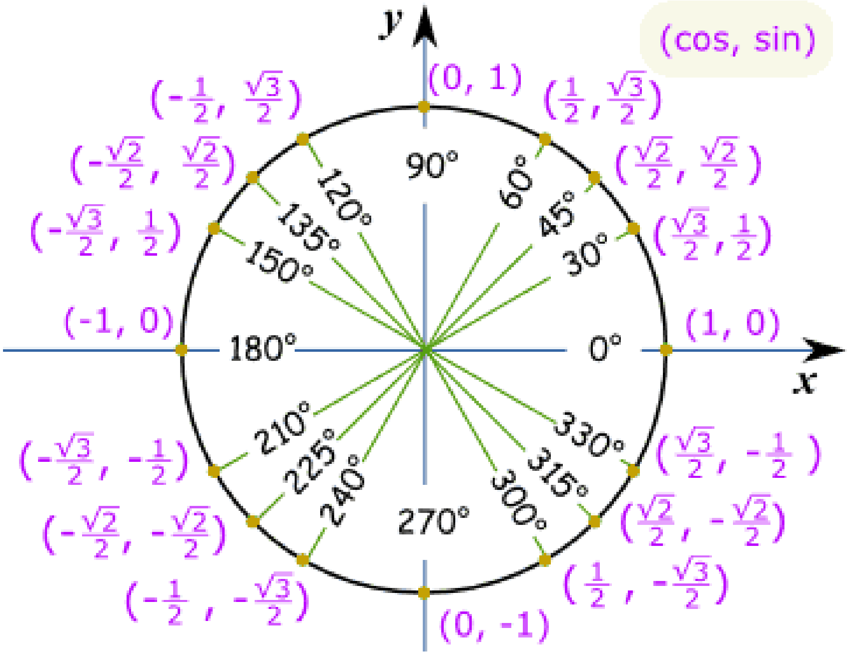


Let's use $\frac{\pi}{4}$ as our reference angle

Construct reference triangles
for all multiples of $\pi/4$
between 0 and 2π

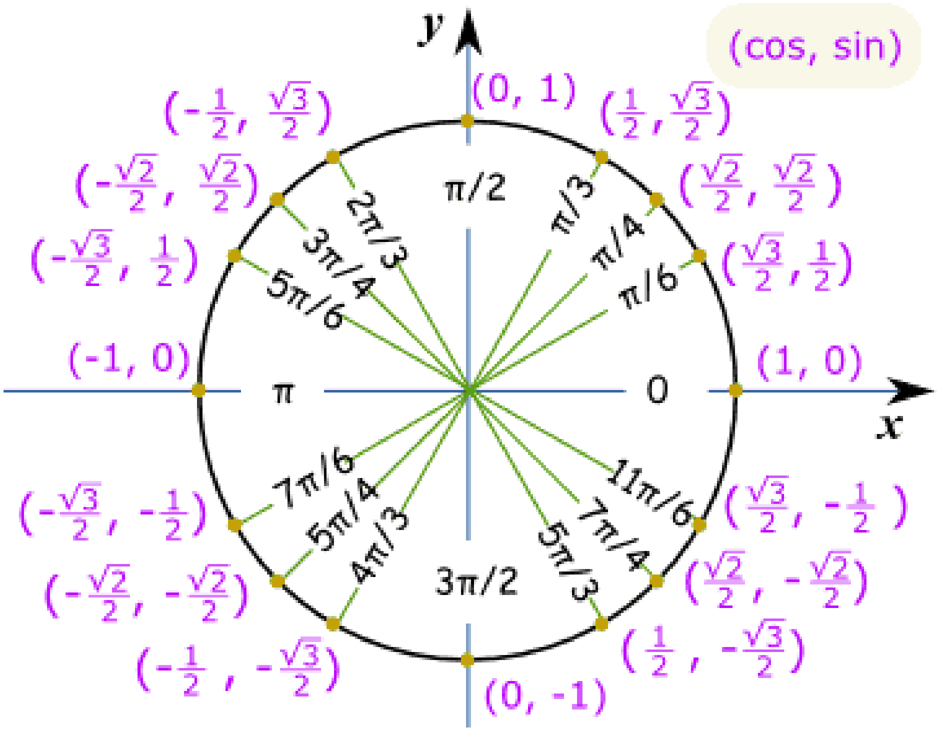


Unit Circle of Special Angles in Degrees



This is lovely...so what is it used for????

Unit Circle of Special Angles in Radians



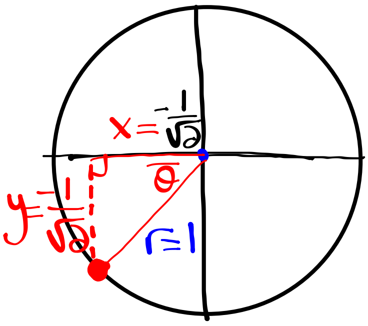
Problems Involving the Unit Circle:

Determine Coordinates for Points of the Unit Circle

(x,y) r=1

Determine the coordinates for all points on the unit circle that satisfy the conditions given. Draw a diagram in each case.

- the y-coordinate is $-\frac{1}{\sqrt{2}}$ and the point is in quadrant III



$x^2+y^2=r^2$

$x^2+\left(-\frac{1}{\sqrt{2}}\right)^2=(1)^2$

$x^2+\frac{1}{2}=1$

$x^2=1-\frac{1}{2}$

$x^2=\frac{2}{2}-\frac{1}{2}$

$x^2=\frac{1}{2}$

$x=\pm\frac{\sqrt{1}}{\sqrt{2}}$

$x=\pm\frac{1}{\sqrt{2}}$

$x=-\frac{1}{\sqrt{2}}$ or $-\frac{\sqrt{2}}{2}$ (Quad 3)

Coordinates are:

$\left(-\frac{1}{\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$ or $\left(-\frac{\sqrt{2}}{2},-\frac{\sqrt{2}}{2}\right)$

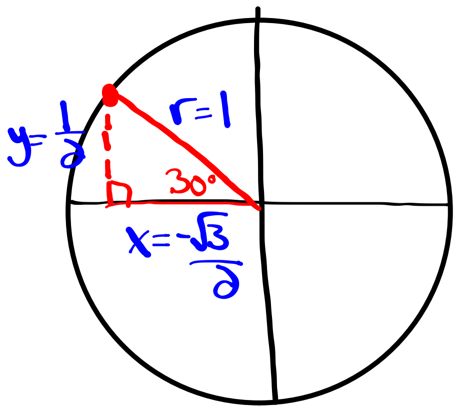
Problems Involving the Unit Circle:

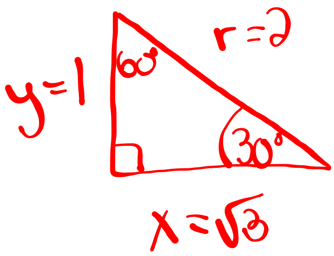
If $P(150°)$ is the point at which the terminal arm of an angle θ in standard position intersects the unit circle, determine the exact coordinates of...

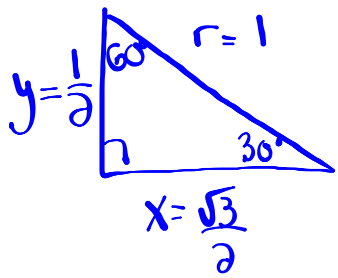
$r=1$

(x,y)

We will have to scale the special triangle so that $r = 1$







$\bar{\theta} = 180° - 150°$

$\bar{\theta} = 30°$

Coordinates are:

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2}\right)$

Evaluate without the use of a calculator:

$$\sin\frac{9\pi}{2} - \cos^2\left(\frac{29\pi}{6}\right)\tan\left(\frac{15\pi}{4}\right)$$

Evaluate without the use of a calculator:

$$\cos\left(\frac{16\pi}{3}\right)\tan^2\left(\frac{23\pi}{6}\right)+\csc\left(\frac{11\pi}{2}\right)+\sin^2\left(\frac{27\pi}{4}\right)$$

Homework:

Worksheet - Sketching Angles in Radians.doc

Solutions...

1. $-\frac{5}{3}$

2. $\frac{-\sqrt{6}}{3}$

3. $-2-\sqrt{3}$

4. $\frac{-5}{3}$

5. $\frac{4+3\sqrt{3}}{6}$

6. $\frac{-10}{3}$

7. 0

8. $\frac{3+3\sqrt{3}}{-2}$

Attachments

Worksheet - Sketching Angles in Radians.doc