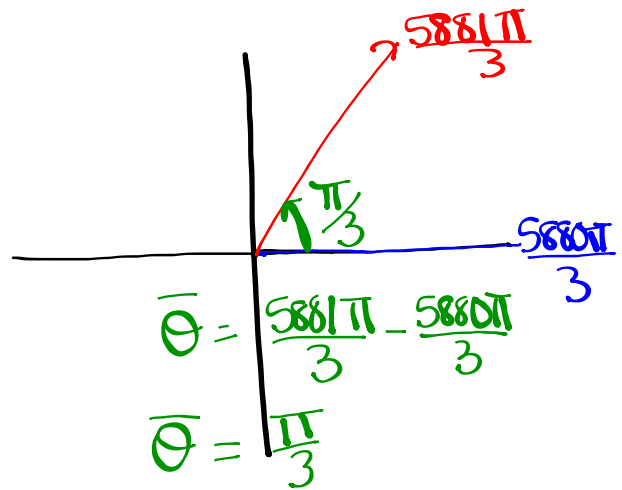Sketch the following and determine a negative angle co-terminal with:

$(i) \quad \dfrac{5881\pi}{3}$

$\dfrac{5880\pi}{3}, \dfrac{5881\pi}{3}, \dfrac{5882\pi}{3}$

1960π

$\overline{\theta} = \dfrac{5881\pi}{3} - \dfrac{5880\pi}{3}$

$\overline{\theta} = \dfrac{\pi}{3}$

Negative coterminal angle:
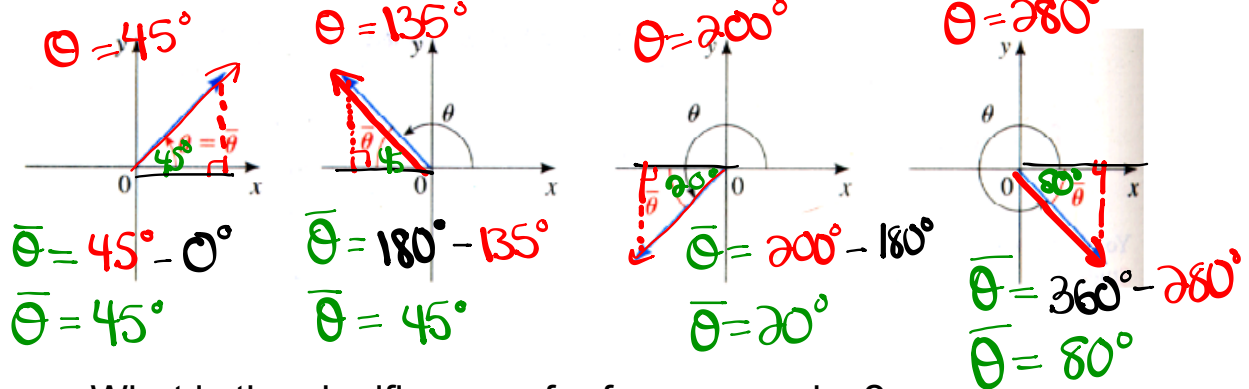
$\dfrac{5881\pi}{3} - \dfrac{1962\pi}{1}$

$\dfrac{5881\pi}{3} - \dfrac{5886\pi}{3}$

$\dfrac{-5\pi}{3}$

Reference Triangles:

Definition 17 *The reference angle $\bar{\theta}$ of an angle θ in standard position is the acute angle (between 0 and 90°) the terminal side makes with the x-axis.*
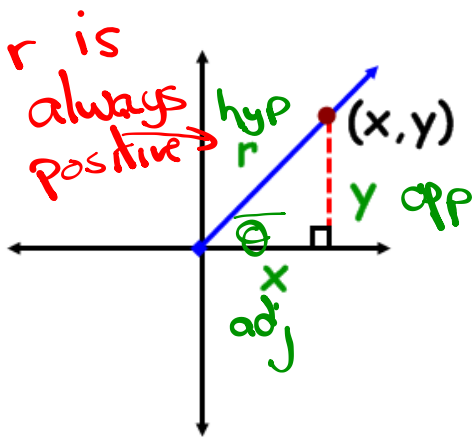
arm

The picture below illustrates this concept.

$\theta = 45°$

$\bar{\theta} = 45° - 0°$

$\bar{\theta} = 45°$

$\theta = 135°$

$\bar{\theta} = 180° - 135°$

$\bar{\theta} = 45°$

$\theta = 200°$

$\bar{\theta} = 200° - 180°$

$\bar{\theta} = 20°$

$\theta = 280°$

$\bar{\theta} = 360° - 280°$

$\bar{\theta} = 80°$

What is the significance of reference angles?

Angles on the Cartesian Plane

- **Reference Angle** - an acute angle formed between the terminal arm and the x-axis.

- **Reference Triangle** - a triangle formed by drawing a perpendicular line from a point on the terminal to the x-axis.

r is always positive → r hyp

(x,y)

y opp

θ

x adj

Notice what will happen if the rotation moves into other quadrants?

TRIG RATIOS on the CARTESIAN PLANE

$$\sin \theta = \frac{y}{r} = \frac{o}{h} \qquad \csc \theta = \frac{r}{y} = \frac{h}{o}$$
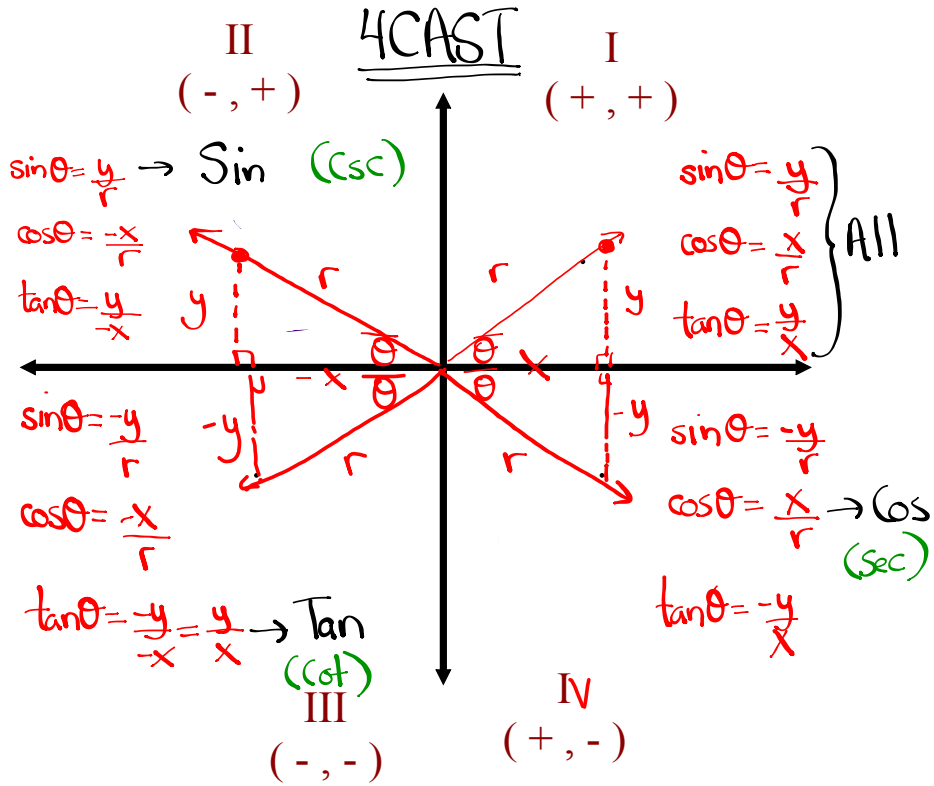
$$\cos \theta = \frac{x}{r} = \frac{a}{h} \qquad \sec \theta = \frac{r}{x} = \frac{h}{a}$$

$$\tan \theta = \frac{y}{x} = \frac{o}{a} \qquad \cot \theta = \frac{x}{y} = \frac{a}{o}$$

"Primary"   "Reciprocal"

3

TRIG RATIOS IN ALL 4 QUADRANTS

What primary trig ratios are POSITIVE in...

II
$(-,+)$

4CAST

I
$(+,+)$

$\sin\theta = \frac{y}{r} \rightarrow$ Sin (Csc)

$\cos\theta = \frac{-x}{r}$

$\tan\theta = \frac{y}{-x}$

$\sin\theta = \frac{y}{r}$

$\cos\theta = \frac{x}{r}$ $\Big\}$ All

$\tan\theta = \frac{y}{x}$

$\sin\theta = \frac{-y}{r}$

$\cos\theta = \frac{x}{r}$

$\tan\theta = \frac{y}{-x} = \frac{y}{x} \rightarrow$ Tan (Cot)

$\sin\theta = \frac{-y}{r}$

$\cos\theta = \frac{x}{r} \rightarrow$ Cos (Sec)

$\tan\theta = \frac{-y}{x}$

III
$(-,-)$

IV
$(+,-)$

Where is θ if...

cscθ<0

$\sin\theta < 0$

S	A
T	C

Quad 3 + 4

sinθ<0 & tanθ<0

S	A
T	Ⓒ

Quad 4

cscθ>0 & cotθ<0

$\sin\theta > 0 + \tan\theta < 0$

Ⓢ	A
T	C

Quad 2

Homework

$\sin\theta > 0$

$\sec\theta < 0$ ⟶ $\cos\theta < 0$

Quad 2

S ✓	A
✓ T	C

If $\sec\theta = -\sqrt{10}$ and $\sin\theta > 0$, determine the value of $\csc\theta = \dfrac{h}{o} = \dfrac{r}{y}$

$\sec\theta = \dfrac{-\sqrt{10}}{1} = \dfrac{hyp}{adj} = \dfrac{r}{x}$

$r = \sqrt{10}$

$r = \sqrt{10}$ (always +)

$x = -1$

$y = 3$ $x = -1$

① Find y:

$x^2 + y^2 = r^2$

$(-1)^2 + y^2 = (\sqrt{10})^2$

$1 + y^2 = 10$

$y^2 = 9$

$y = \pm 3$

$y = 3$ (Q2)

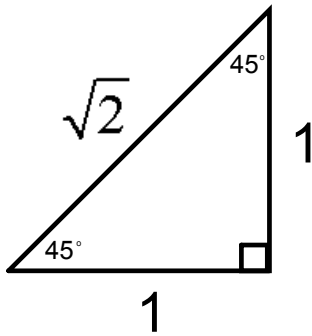② $\csc\theta = \dfrac{hyp}{opp} = \dfrac{r}{y} = \dfrac{\sqrt{10}}{3}$

Determine the measure (in radians) of an angle whose terminal arm passes through the ordered pair $(-2\sqrt{3}, -4)$
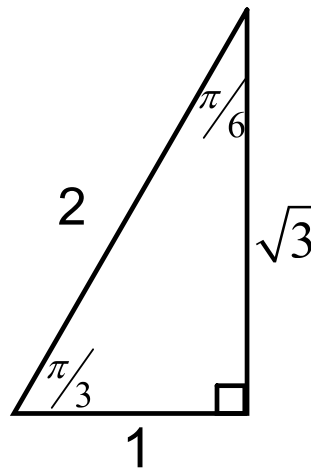
$x = -2\sqrt{3}$

$y = -4$

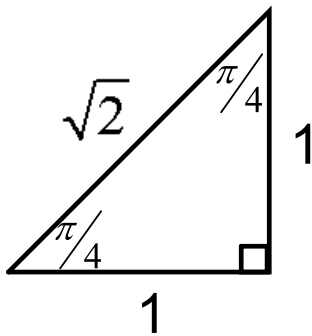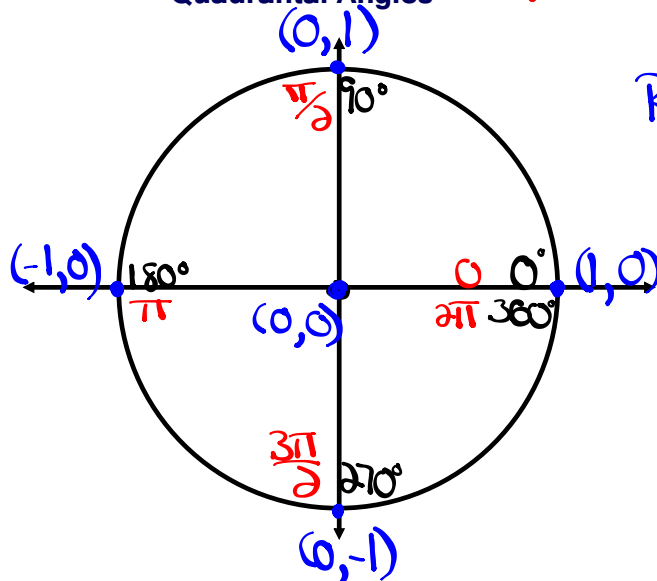$x = -2\sqrt{3}$

$y = -4$

① Find $\bar{\theta}$

$\tan\bar{\theta} = \dfrac{y}{x} = \dfrac{-4}{-2\sqrt{3}} = \dfrac{2}{\sqrt{3}}$

$\tan\bar{\theta} = 1.1547$

Convert calculator to rads ⟶ $\bar{\theta} = \tan^{-1}(1.1547)$

$\bar{\theta} = 0.86 \text{ rads}$

② Find θ

$\theta = \pi + \bar{\theta}$

$\theta = 3.14 + 0.86$

$\theta = 4 \text{ rads}$

Special Angles

In Degrees:

$\sqrt{2}$ 45° 1 1 45°

2 30° $\sqrt{3}$ 60° 1

In Radians:

$\sqrt{2}$ $\pi/4$ 1 1 $\pi/4$

2 $\pi/6$ $\sqrt{3}$ $\pi/3$ 1

Quadrantal Angles   (Multiples of 90° or $\frac{\pi}{2}$ rads)

$(0,1)$ $\frac{\pi}{2}$ 90°

$(-1,0)$ 180° π O 0° $(1,0)$

$(0,0)$ 2π 360°

$\frac{3\pi}{2}$ 270°

$(0,-1)$

Radius = 1

$$690° - 360° = 330°$$

Solving Trig Expressions by Sketching Angles
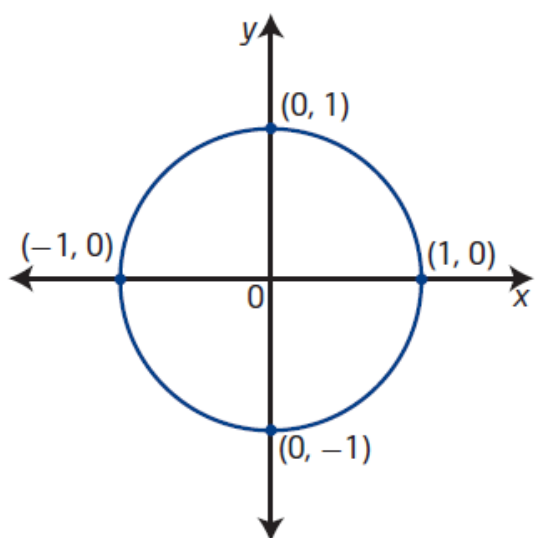
Ex. Evaluate the $\sin 690° = \sin 330° = \dfrac{-1}{2}$

S A

$x = \sqrt{3}$

$30°$ 4

$60°$ $y = -1$

T $r = 2$ C

$\overline{\theta} = 360° - 330°$

$\overline{\theta} = 30°$

Ex. $\cos \dfrac{13\pi}{3} = \dfrac{1}{2}$

$\dfrac{12\pi}{3}, \dfrac{13\pi}{3}, \dfrac{14\pi}{3}$

4π

A

$r = 2$

S $\pi/6$ $y = \sqrt{3}$

ODD $\pi/3$ EVEN

$x = 1$

$\overline{\theta} = \dfrac{13\pi}{3} - \dfrac{12\pi}{3}$

$\overline{\theta} = \dfrac{\pi}{3}$ C

1

Unit Circle

y

(0, 1)

(−1, 0)

0

(1, 0)

x

(0, −1)

unit circle

- a circle with radius 1 unit $(r = 1)$

- a circle of radius 1 unit with centre at the origin on the Cartesian plane is known as *the* unit circle

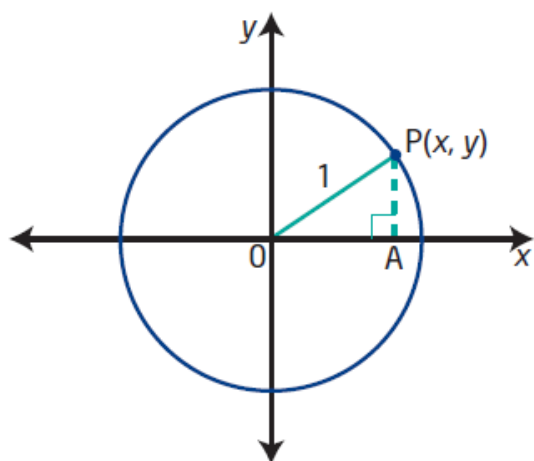$$\sin\theta = \frac{o}{h} = \frac{y}{r} = \frac{y}{1} = y$$

$$\csc\theta = \frac{1}{y}$$

$$\cos\theta = \frac{a}{h} = \frac{x}{r} = \frac{x}{1} = x$$

$$\sec\theta = \frac{1}{x}$$

$$\tan\theta = \frac{o}{a} = \frac{y}{x}$$

$$\cot\theta = \frac{x}{y}$$

$r = 1$

$$x^2 + y^2 = r^2$$

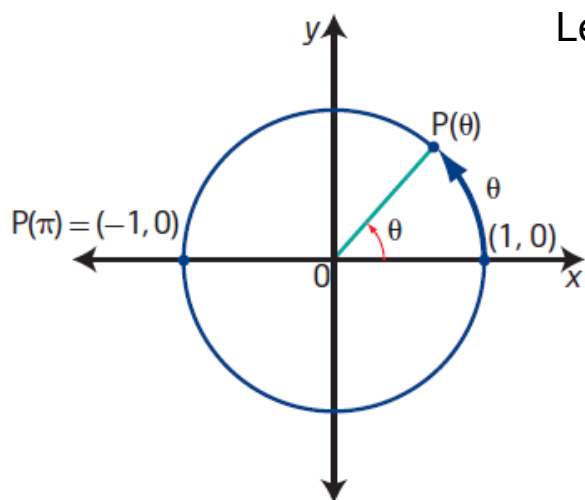The equation of the unit circle is $x^2 + y^2 = 1$.

Determine the equation of a circle with centre at the origin and radius 6.

$$x^2 + y^2 = r^2$$
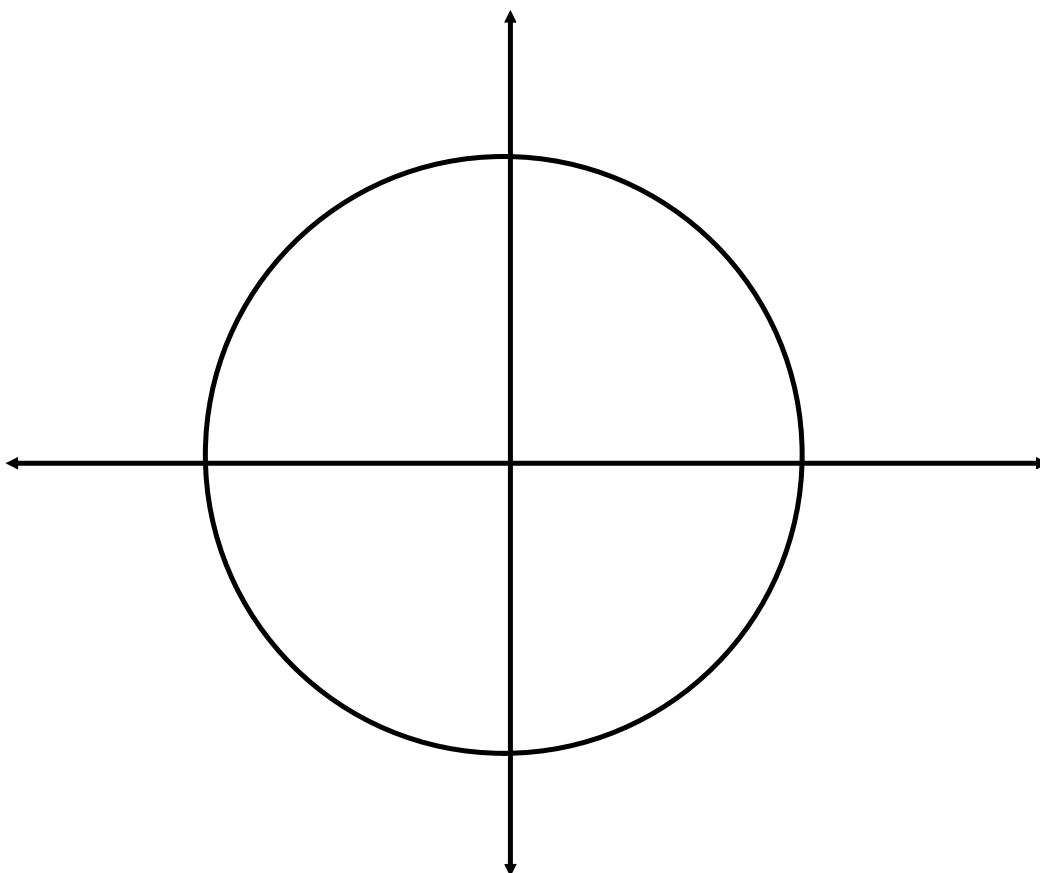
$$x^2 + y^2 = (6)^2$$

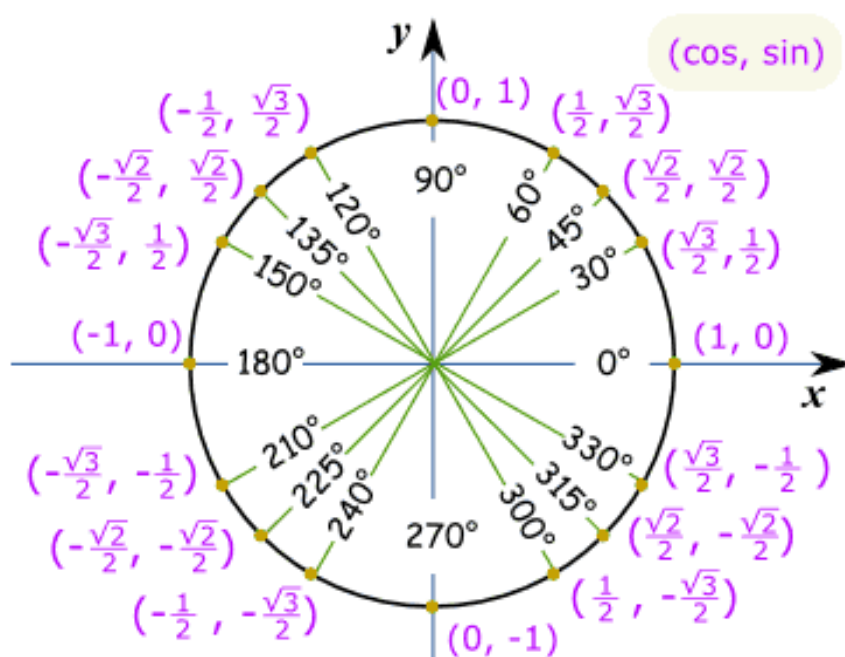$$\boxed{x^2 + y^2 = 36}$$

Special Angles on the Unit Circle:

Let's use $\dfrac{\pi}{4}$ as our reference angle

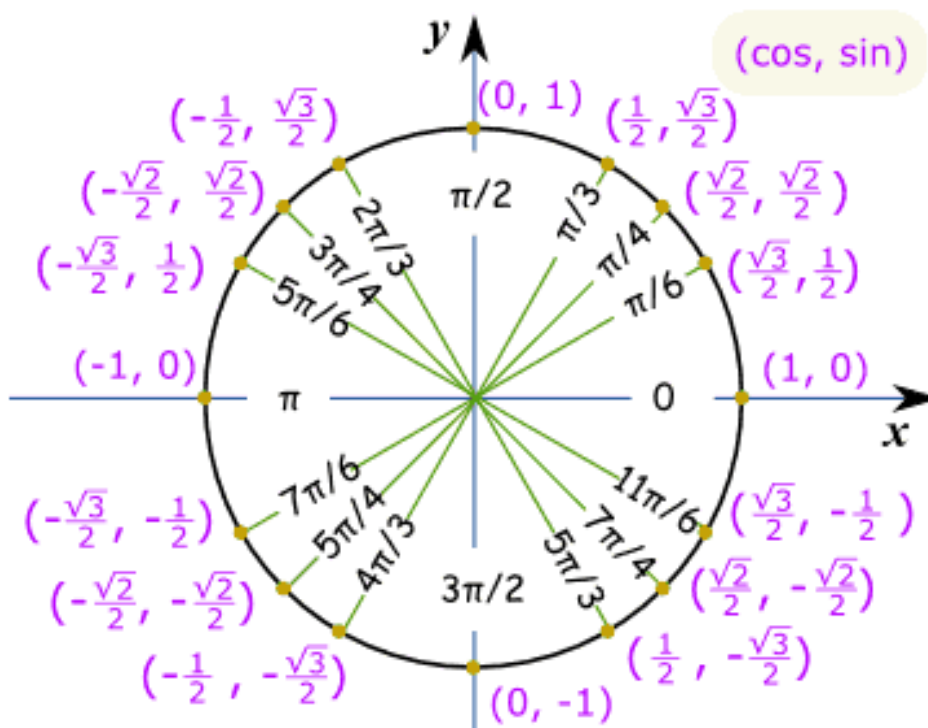Construct reference triangles for all multiples of π/4

between 0 and 2π

Unit Circle of Special Angles in Degrees

This is lovely...so what is it used for????
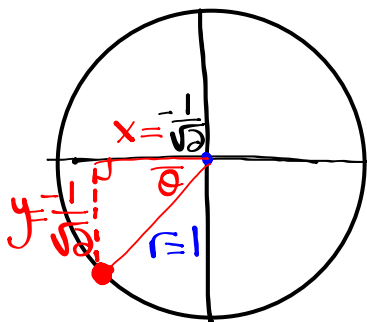
Unit Circle of Special Angles in Radians

Problems Involving the Unit Circle:

Determine Coordinates for Points of the Unit Circle

Determine the coordinates (x, y) for all points on the unit circle $r = 1$ that satisfy the conditions given. Draw a diagram in each case.

- the y-coordinate is $-\dfrac{1}{\sqrt{2}}$ and the point is in quadrant III

$$x^2 + y^2 = r^2$$

$$x^2 + \left(\frac{-1}{\sqrt{2}}\right)^2 = (1)^2$$

$$x^2 + \frac{1}{2} = 1$$

$$x^2 = 1 - \frac{1}{2}$$

$$x^2 = \frac{2}{2} - \frac{1}{2}$$

$$x^2 = \frac{1}{2}$$

$$x = \pm \frac{\sqrt{1}}{\sqrt{2}}$$

$$x = \pm \frac{1}{\sqrt{2}}$$

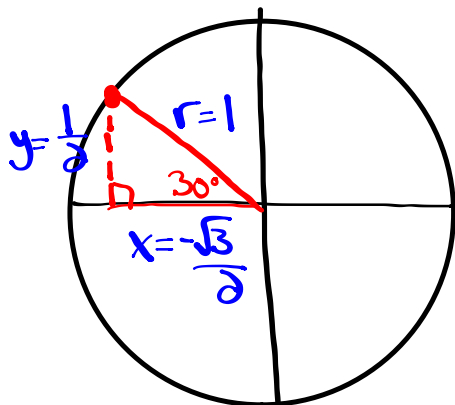$$x = -\frac{1}{\sqrt{2}} \text{ or } -\frac{\sqrt{2}}{2} \qquad \text{(Quad 3)}$$

Coordinates are:

$$\left(-\frac{1}{\sqrt{2}}, -\frac{1}{\sqrt{2}}\right) \text{ or } \left(\frac{-\sqrt{2}}{2}, -\frac{\sqrt{2}}{2}\right)$$

Problems Involving the Unit Circle:

If $P(150°)$ is the point at which the terminal arm of an angle θ in standard position intersects the unit circle, determine the exact coordinates of...
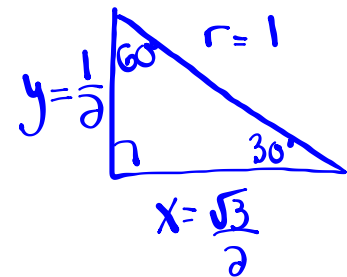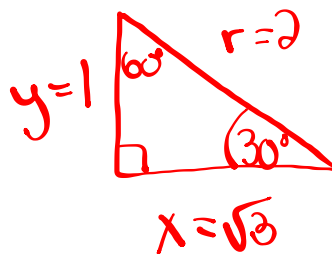
$r = 1$

(x, y)

We will have to scale the special triangle so that $r = 1$

$\overline{\theta} = 180° - 150°$

$\theta = 30°$

$y = \frac{1}{2}$　$r = 1$

$x = -\frac{\sqrt{3}}{2}$

$y = 1$　$r = 2$　$60°$

$30°$

$x = \sqrt{3}$

$y = \frac{1}{2}$　$60°$　$r = 1$

$30°$

$x = \frac{\sqrt{3}}{2}$

Coordinates are:

$\left(-\frac{\sqrt{3}}{2}, \frac{1}{2} \right)$

Questions from Homework

Scale to fit unit circle

① c) $\dfrac{10\pi}{6}$, $\dfrac{11\pi}{6}$, $\boxed{\dfrac{12\pi}{6}}$

2π

$x = \sqrt{3}$
$\dfrac{\pi}{6}$
$r = 2$ $\dfrac{\pi}{3}$ $y = 1$

$x = \dfrac{\sqrt{3}}{2}$
$r = 1$ $y = \dfrac{1}{2}$

$x = \dfrac{\sqrt{3}}{2}$
$\dfrac{\pi}{6}$
$r = 1$ $\dfrac{\pi}{3}$ $y = -\dfrac{1}{2}$

Coordinates are:
$\left(\dfrac{\sqrt{3}}{2} , -\dfrac{1}{2} \right)$

$\bar{\theta} = \dfrac{12\pi}{6} - \dfrac{11\pi}{6}$

$\bar{\theta} = \dfrac{\pi}{6}$
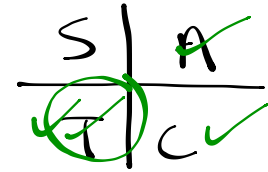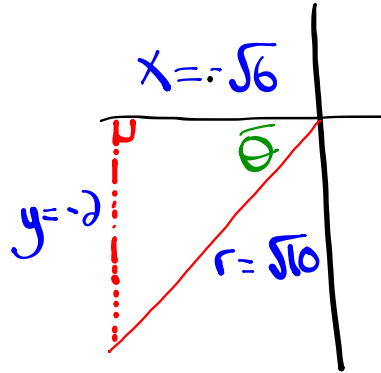
Questions from Homework

③ $\csc\theta = -\dfrac{\sqrt{10}}{2}$ and $\tan\theta > 0$

$\sin\theta < 0$

Given:

$\text{hyp} = r = \sqrt{10}$

$\text{opp} = y = -2$

$x = -\sqrt{6}$

$y = -2$

$r = \sqrt{10}$

θ

① Find x

$x^2 + y^2 = r^2$

$x^2 + (-2)^2 = (\sqrt{10})^2$

$x^2 + 4 = 10$

$x^2 = 6$

$x = \pm\sqrt{6}$

$x = -\sqrt{6}$ (Q3)

$\sin\theta = \dfrac{-2}{\sqrt{10}} = \dfrac{-2\sqrt{10}}{10} = \dfrac{-\sqrt{10}}{5}$

$\cos\theta = \dfrac{-\sqrt{6}}{\sqrt{10}} = \dfrac{-\sqrt{60}}{10} = \dfrac{-2\sqrt{15}}{10} = \dfrac{-\sqrt{15}}{5}$

$\tan\theta = \dfrac{-2}{-\sqrt{6}} = \dfrac{2\sqrt{6}}{6} = \dfrac{\sqrt{6}}{3}$
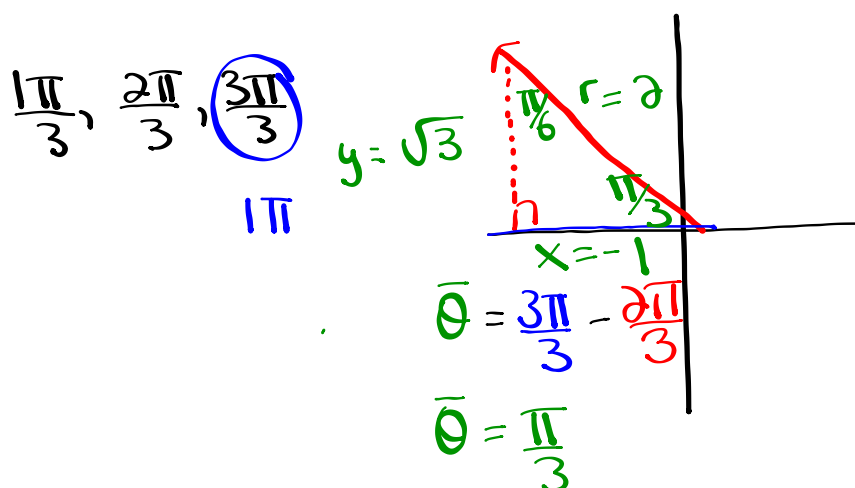
$\csc\theta = -\dfrac{\sqrt{10}}{2}$ (Given)

$\sec\theta = -\dfrac{\sqrt{10}}{\sqrt{6}} = -\dfrac{\sqrt{60}}{6} = \dfrac{-2\sqrt{15}}{6} = -\dfrac{\sqrt{15}}{3}$

$\cot\theta = \dfrac{-\sqrt{6}}{-2} = \dfrac{\sqrt{6}}{2}$

Questions from Homework

④ b) $\sec \dfrac{2\pi}{3} = \dfrac{2}{1} = -2$

$\dfrac{1\pi}{3}, \dfrac{2\pi}{3}, \boxed{\dfrac{3\pi}{3}}$

1π

$y = \sqrt{3}$

$\dfrac{\pi}{6}$ $r = 2$

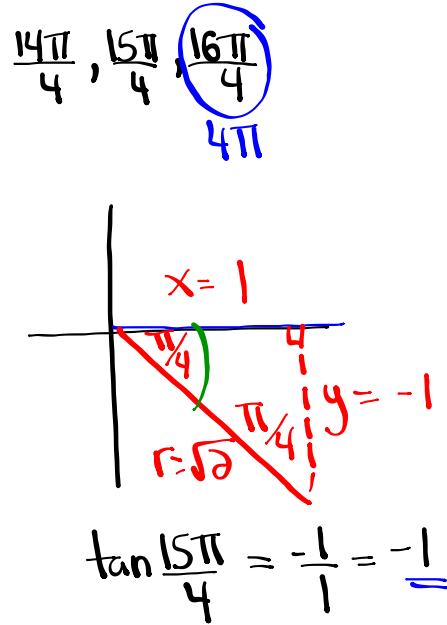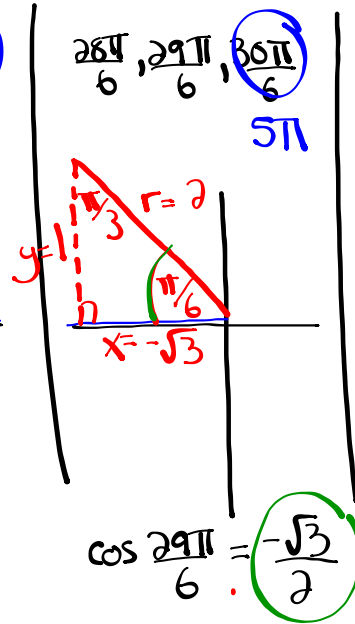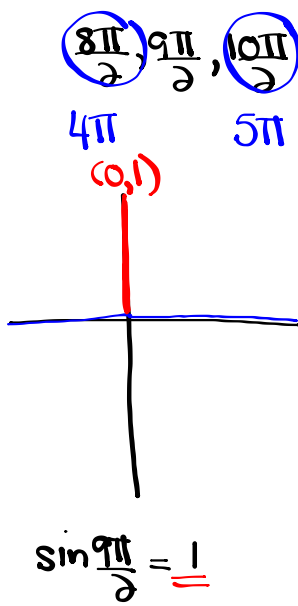h $\dfrac{\pi}{3}$

$x = -1$

$\bar{\theta} = \dfrac{3\pi}{3} - \dfrac{2\pi}{3}$

$\bar{\theta} = \dfrac{\pi}{3}$

Evaluate without the use of a calculator:

① Sketch angles
② Find θ
③ Fill in ref. triangle
④ Find trig ratio

$$\sin\frac{9\pi}{2} - \cos^2\left(\frac{29\pi}{6}\right)\tan\left(\frac{15\pi}{4}\right)$$

$\dfrac{8\pi}{2}, \dfrac{9\pi}{2}, \dfrac{10\pi}{2}$

$4\pi \qquad\qquad 5\pi$

(0,1)

$$\sin\frac{9\pi}{2} = 1$$

$\dfrac{26\pi}{6}, \dfrac{29\pi}{6}, \dfrac{30\pi}{6}$

5π

$r = 2$
$y = 1$
$\frac{\pi}{6}$
$x = -\sqrt{3}$

$$\cos\frac{29\pi}{6} = \boxed{\dfrac{-\sqrt{3}}{2}}$$

$\dfrac{14\pi}{4}, \dfrac{15\pi}{4}, \dfrac{16\pi}{4}$

4π

$x = 1$
$\frac{\pi}{4}$
$\frac{\pi}{4}$
$y = -1$
$r = \sqrt{2}$

$$\tan\frac{15\pi}{4} = \frac{-1}{1} = \underline{-1}$$

$$\sin\frac{9\pi}{2} - \cos^2\left(\frac{29\pi}{6}\right)\tan\left(\frac{15\pi}{4}\right)$$

$$(1) - \left(\frac{\sqrt{3}}{2}\right)^2(-1)$$

$$1 - \left(\frac{3}{4}\right)\frac{(-1)}{1} \quad\longrightarrow\quad \text{or} \quad 1 - \left(\frac{-3}{4}\right)$$

$$1 + \frac{3}{4}$$

$$\frac{4}{4} + \frac{3}{4}$$

$$\boxed{\frac{7}{4}}$$

Evaluate without the use of a calculator:

$$\cos\left(\frac{16\pi}{3}\right)\tan^2\left(\frac{23\pi}{6}\right) + \csc\left(\frac{11\pi}{2}\right) + \sin^2\left(\frac{27\pi}{4}\right)$$

(Q3)   (Q4)   Border of 3 and 4   (Q2)

$$\left(-\frac{1}{2}\right)\left(-\frac{1}{\sqrt{3}}\right)^2 + \left(\frac{1}{-1}\right) + \left(+\frac{1}{\sqrt{2}}\right)^2$$

$$-\frac{1}{2}\left(\frac{1}{3}\right) + (-1) + \frac{1}{2}$$

$$-\frac{1}{6} - 1 + \frac{1}{2}$$

$$\frac{-1}{6} - \frac{6}{6} + \frac{3}{6}$$

$$\frac{-4}{6}$$

$$\boxed{-\frac{2}{3}}$$

Homework: Worksheet - Sketching Angles in Radians.doc

Solutions...

1. $-\dfrac{5}{3}$

2. $\dfrac{-\sqrt{6}}{3}$

3. $-2-\sqrt{3}$

4. $\dfrac{-5}{3}$

5. $\dfrac{4+3\sqrt{3}}{6}$

6. $\dfrac{-10}{3}$

7. 0

8. $\dfrac{3+3\sqrt{3}}{-2}$

Attachments

Worksheet - Sketching Angles in Radians.doc