Questions From Homework

③ (ii) $y = x^4 - 8x^2$

$y' = 4x^3 - 16x$

$y' = 4x(x^2-4)$

$y' = 4x(x-2)(x+2)$

CV: $x = -2, 0, 2$

$y'' = 12x^2 - 16$

$y'' = 4(3x^2-4)$

$3x^2 - 4 = 0$

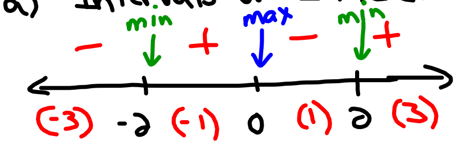
$3x^2 = 4$

$x^2 = \frac{4}{3}$

$x = \pm \frac{2}{\sqrt{3}}$

CV: $x = \pm \frac{2}{\sqrt{3}} \doteq \pm 1.15$

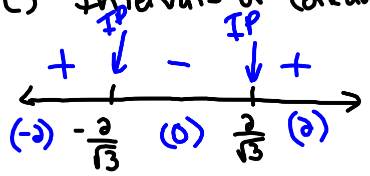
a) Intervals of Inc/Dec.



Increasing on $(-2,0)$ & $(2,\infty)$

Decreasing on $(-\infty,-2)$ & $(0,2)$

c) Intervals of Concavity



CU on $(-\infty, \frac{-2}{\sqrt{3}})$ & $(\frac{2}{\sqrt{3}}, \infty)$

CD on $(\frac{-2}{\sqrt{3}}, \frac{2}{\sqrt{3}})$

Max Min

b) $f(-2) = -16$ $(-2,-16)$ min

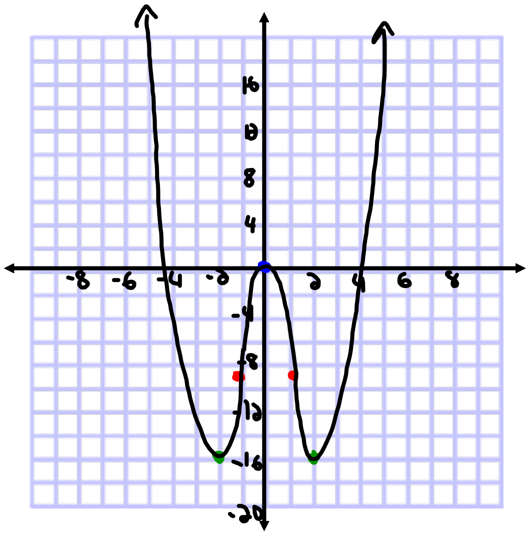
$f(0) = 0$ $(0,0)$ max

$f(2) = -16$ $(2,-16)$ min

d) Inflection Points:

$f(\frac{-2}{\sqrt{3}}) = -\frac{80}{9}$ $(\frac{-2}{\sqrt{3}}, \frac{-80}{9})$

$f(\frac{2}{\sqrt{3}}) = -\frac{80}{9}$ $(\frac{2}{\sqrt{3}}, \frac{-80}{9})$



Making a Complete Sketch

Example:

Examine the function $f(x) = x^4 - 4x^3$ with respect to…

- ✓ Intercepts
- ✓ Symmetry
- ✓ Asymptotes (Not a rational Function) ← fraction
- Intervals of Increase or Decrease · $f'(x) = 4x^3 - 12x^2$
- Local Maximum and Minimum values
- Concavity and Points of Inflection $f''(x) = 12x^2 - 24x$
- Sketch the Curve

① x-int $\underline{y=0}$

$f(x) = x^4 - 4x^3$

$0 = x^4 - 4x^3$ (factor)

$0 = x^3(x-4)$

$x^3 = 0$ | $x - 4 = 0$

$x = 0$ | $x = 4$

$(0,0)$ $(4,0)$

② y-int $\underline{x=0}$

$f(x) = x^4 - 4x^3$

$f(0) = (0)^4 - 4(0)^3 = 0$

$(0,0)$

③ Symmetry

$f(x) = x^4 - 4x^3$

$f(-x) = (-x)^4 - 4(-x)^3$

$f(-x) = x^4 - 4(-x^3)$

$f(-x) = x^4 + 4x^3$

No symmetry

④ Assymptotes:

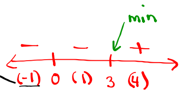
No assymptotes

⑤ Intervals of Inc/Dec.

$f(x) = x^4 - 4x^3$

$f'(x) = 4x^3 - 12x^2$

$f'(x) = 4x^2(x-3)$ ←



cv: $4x^2 = 0$ | $x - 3 = 0$

$x^2 = 0$ | $x = 3$

$x = 0$

Increasing on $(3, \infty)$

$x > 3$

Decreasing on $(-\infty, 3)$

$x < 3$

⑥ Local max/min

when $x = 3$

$f(x) = x^4 - 4x^3$

$f(3) = (3)^4 - 4(3)^3$

$f(3) = 81 - 108$

$f(3) = -27$

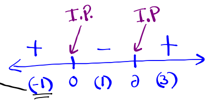
local min @ $(3, -27)$

⑦ Intervals of concavity

$f'(x) = 4x^3 - 12x^2$

$f''(x) = 12x^2 - 24x$

$f''(x) = 12x(x-2)$ ←



cv: $12x = 0$ | $x - 2 = 0$

$x = 0$ | $x = 2$

∪ Concave up on $(-\infty, 0) + (2, \infty)$

$x < 0$ + $x > 2$

∩ Concave down on $(0, 2)$

$0 < x < 2$

⑧ Inflection Points:

when $x = 0$

$f(x) = x^4 - 4x^3$

$f(0) = (0)^4 - 4(0)^3$

$f(0) = 0$

I.P. @ $(0,0)$

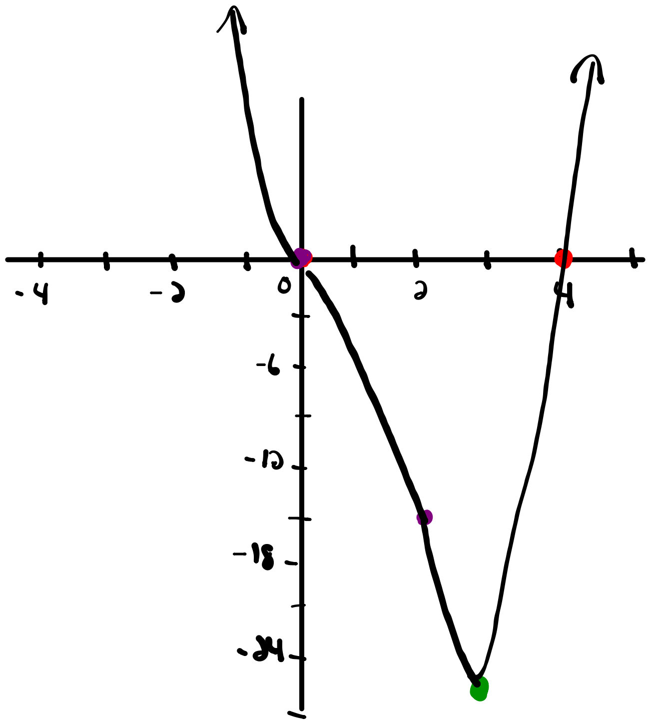
when $x = 2$

$f(x) = x^4 - 4x^3$

$f(2) = (2)^4 - 4(2)^3$

$f(2) = 16 - 32 = -16$

I.P @ $(2, -16)$



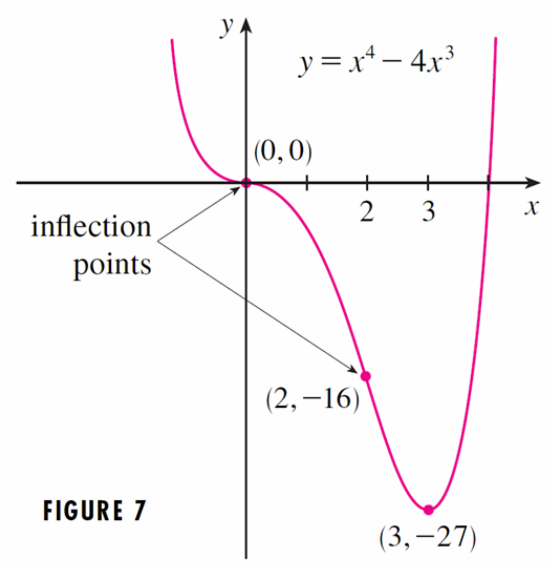


FIGURE 7

Examine the function $f(x) = 3x^5 - 5x^3$ with respect to…

- Intercepts
- Symmetry
- Asymptotes
- Intervals of Increase or Decrease
- Local Maximum and Minimum values
- Concavity and Points of Inflection
- Sketch the Curve

$f(x) = 3x^5 - 5x^3$

$f(x) = x^3(3x^2-5)$

$f'(x) = 15x^4 - 15x^2$

$f'(x) = 15x^2(x^2-1)$

$f''(x) = 60x^3 - 30x$

$f''(x) = 30x(2x^2-1)$

① Intercepts:

x int $(y=0)$

$x = 0, \pm\sqrt{5/3}$

(0,0) (1.29,0) (-1.29,0)

y int $(x=0)$

$y = 0$

(0,0)

② Symmetry:

$f(-x) = 3(-x)^5 - 5(-x)^3$

$= -3x^5 + 5x^3$

$\therefore f(-x) = -f(x)$ Odd

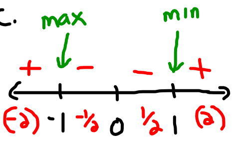
③ Asymptotes: None

④ Intervals of Inc/Dec.

$f'(x) = 15x^2(x^2-1)$

$f'(x) = 15x^2(x-1)(x+1)$

CV: $x = 0, \pm 1$



Inc on $(-\infty,-1)$ + $(1,\infty)$

Dec on $(-1, 1)$

⑤ Max/Mins:

$f(-1) = 3(-1)^5 - 5(-1)^3$ (-1,2) max

$= -3 + 5$

$= 2$

$f(1) = 3(1)^5 - 5(1)^3$ (1,-2) min

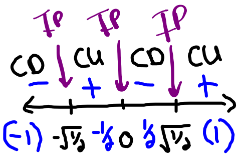
$= 3 - 5$

$= -2$

⑥ Concavity:

$f''(x) = 30x(2x^2-1)$

CV: $x = 0, \pm\sqrt{1/2}$

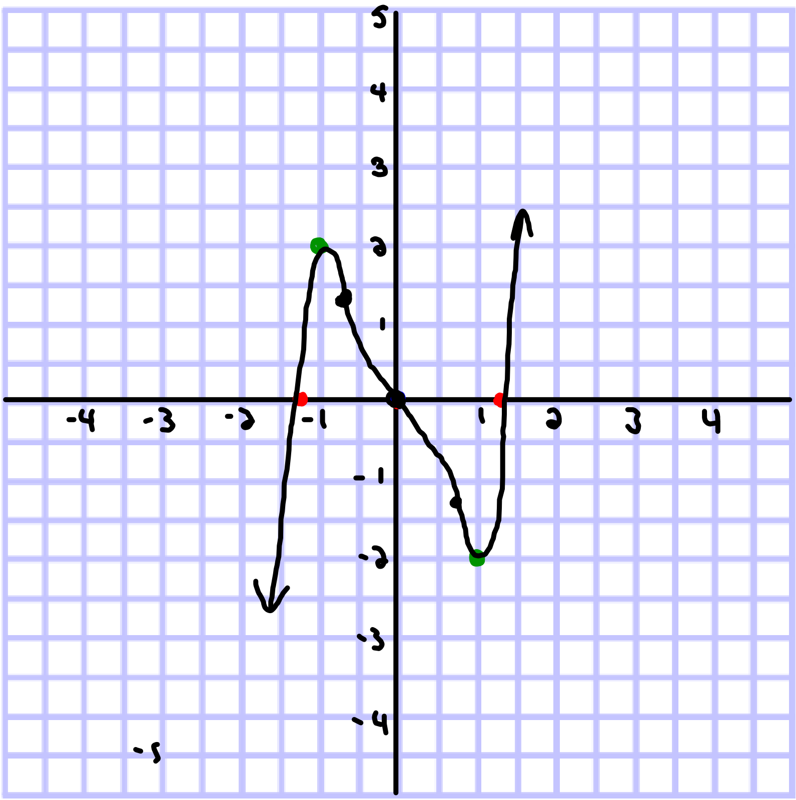


⑦ Inflection Points:

$f(-\sqrt{1/2}) \approx -0.53 + 1.767 \approx 1.238$ (-0.707, 1.238)

$f(0) = 0$ (0,0)

$f(\sqrt{1/2}) \approx 0.53 - 1.767 \approx -1.238$ (0.707, -1.238)


5
4
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-4
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4

homework

Examine the function $f(x)=\frac{x^2}{x-7}$ with respect to…

- Intercepts
- Symmetry
- Asymptotes
- Intervals of Increase or Decrease
- Local Maximum and Minimum values
- Concavity and Points of Inflection
- Sketch the Curve

① x-int (y=0)

$f(x)=\frac{x^2}{x-7}$

$(x-7)\ 0=\frac{x^2}{\cancel{x-7}}\ \cancel{(x-7)}$

$0=x^2$

$0=x$

(0,0)

② y-int (x=0)

$f(x)=\frac{x^2}{x-7}$

$f(0)=\frac{(0)^2}{(0)-7}=\frac{0}{-7}=0$

$y=0$

(0,0)

③ Symmetry:

$f(x)=\frac{x^2}{x-7}$

$f(-x)=\frac{(-x)^2}{(-x)-7}$

$f(-x)=\frac{x^2}{-x-7}$

No symmetry

④ VA: (denom = 0)

$x-7=0$

$x=7$

$\lim_{x\to 7^-}\frac{x^2}{x-7}=\frac{(+)}{(-)}=-\infty$

(6.9)

$\lim_{x\to 7^+}\frac{x^2}{x-7}=\frac{(+)}{(+)}=+\infty$

(x=7.1)

⑤ SA:

$$\begin{array}{r} x+7 \\ x-7\,\overline{)\,x^2} \\ -(x^2-7x) \\ \hline 7x \\ -(7x-49) \\ \hline 49\ R \end{array}$$

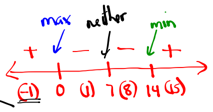
$y=x+7$

$m=\frac{1}{1}\ \frac{\text{rise}}{\text{run}}$

b = 7 yint /

⑥ Intervals of Inc/Dec:

$F'(x)=\frac{x(x-14)}{(x-7)^2}$



CV: $x=0$ | $x-14=0$, $x=14$ | $(x-7)^2=0$, $x-7=0$, $x=7$

Increasing on $(-\infty,0)+(14,\infty)$

$x<0+x>14$

Decreasing on $(0,14)$

$0<x<14$

⑦ Local max/min

$f(x)=\frac{x^2}{x-7}$

when $x=0$

$f(0)=\frac{(0)^2}{(0)-7}=\frac{0}{-7}=0$

(0,0)

local max @ (0,0)

When $x=14$

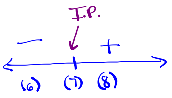
$f(14)=\frac{(14)^2}{(14)-7}=\frac{196}{7}=28$

(14,28)

local min @ (14,28)

⑧ Intervals of Concavity:

$F''(x)=\frac{98}{(x-7)^3}$



CV: $98\neq 0$ | $(x-7)^3=0$, $x-7=0$, $x=7$

Concave down on $(-\infty,7)$

$x<7$

Concave up on $(7,\infty)$

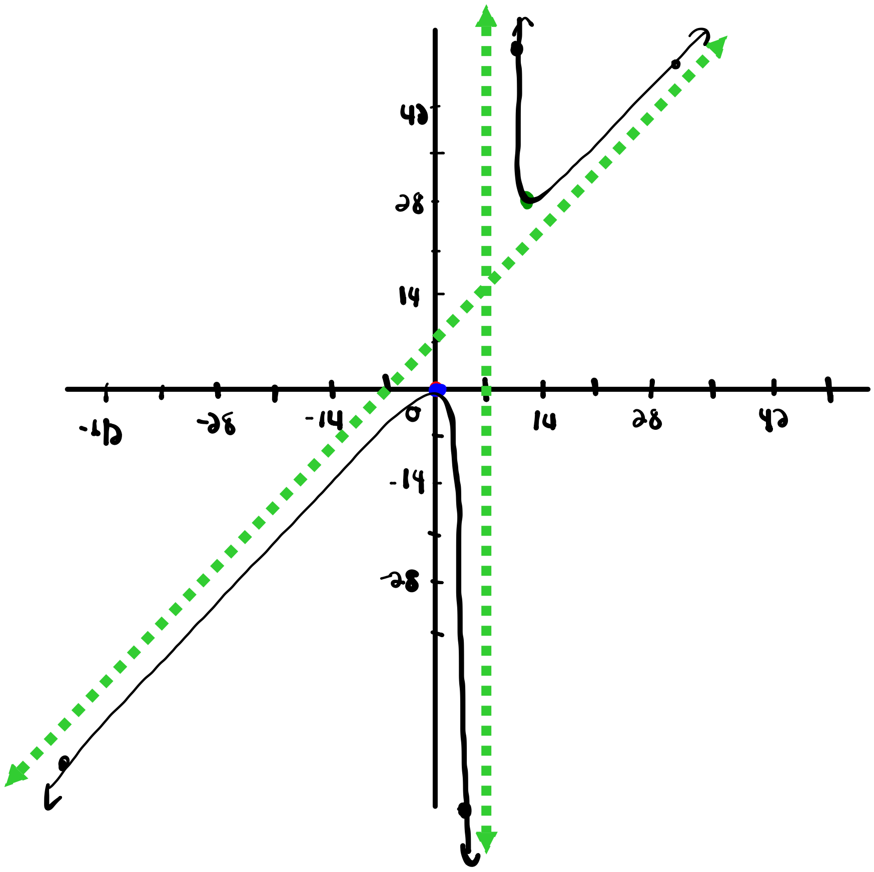
$x>7$

⑨ I.P. (x=7)

$f(x)=\frac{x^2}{x-7}$

$f(7)=\frac{(7)^2}{(7)-7}=\frac{49}{0}=\text{DNE}$

x=7 is the vertical asymptote.


42
28
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42

homework

Examine the function $f(x)=\frac{x^2}{1-x^2}$ with respect to…

- Intercepts
- Symmetry
- Asymptotes
- Intervals of Increase or Decrease
- Local Maximum and Minimum values
- Concavity and Points of Inflection
- Sketch the Curve

$f(x)=\frac{x^2}{1-x^2}$ | $f'(x)=\frac{2x}{(1-x^2)^2}$ | $f''(x)=\frac{2+6x^2}{(1-x^2)^3}=\frac{2(3x^2+1)}{(1-x^2)^3}$

① x-int (y=0)

$f(x)=\frac{x^2}{1-x^2}$

$(1-x^2)0=\frac{x^2}{\cancel{1-x^2}}\cancel{(1-x^2)}$

$0=x^2$

$0=x$

(0,0)

② y-int (x=0)

$f(x)=\frac{x^2}{1-x^2}$

$f(0)=\frac{(0)^2}{1-(0)^2}=\frac{0}{1}=0$

(0,0)

③ Symmetry:

$f(x)=\frac{x^2}{1-x^2}$

$f(-x)=\frac{(-x)^2}{1-(-x)^2}$

$f(x)=\frac{x^2}{1-x^2}$

Even

④ VA: (set denom =0)

$1-x^2=0$

$(1-x)(1+x)=0$

$1-x=0$ | $1+x=0$

$\boxed{1=x}$ | $\boxed{x=-1}$

$\lim\limits_{x\to-1^-}\frac{x^2}{(1-x)(1+x)}=\frac{(+)}{(+)(-)}=-\infty$

(x=-1.1)

$\lim\limits_{x\to-1^+}\frac{x^2}{(1-x)(1+x)}=\frac{(+)}{(+)(+)}=+\infty$

(x=-0.9)

$\lim\limits_{x\to1^-}\frac{(+)}{(+)(+)}=+\infty$

(x=0.9)

$\lim\limits_{x\to1^+}\frac{(+)}{(-)(+)}=-\infty$

(x=1.1)

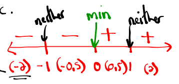
⑤ HA:

$\lim\limits_{x\to\infty}\frac{x^2}{1-x^2}=\frac{1}{-1}$

$\boxed{y=-1}$

⑥ Intervals of Inc/Dec:

$f'(x)=\frac{2x}{(1-x^2)^2}$



CV: $2x=0$

$x=0$

$(1-x^2)^2=0$

$1-x^2=0$

$1=x^2$

$\pm1=x$

Increasing on $(0,\infty)$

$x>0$

Decreasing on $(-\infty,0)$

$x<0$

⑦ Local max/min:

$f(x)=\frac{x^2}{1-x^2}$

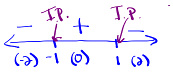
when $x=0$

$f(0)=\frac{(0)^2}{1-(0)^2}=0$

local min @ (0,0)

⑧ Intervals of Concavity:

$f''(x)=\frac{2(3x^2+1)}{(1-x^2)^3}$



CV: $2(3x^2+1)=0$

$3x^2+1=0$

$3x^2=-1$

$x^2=-\frac{1}{3}$

Not Possible

$(1-x^2)^3=0$

$1-x^2=0$

$1=x^2$

$\pm1=x$

Concave up on $(-1,1)$

$-1<x<1$

Concave down on $(-\infty,-1)$

$+(1,\infty)$

⑨ IP

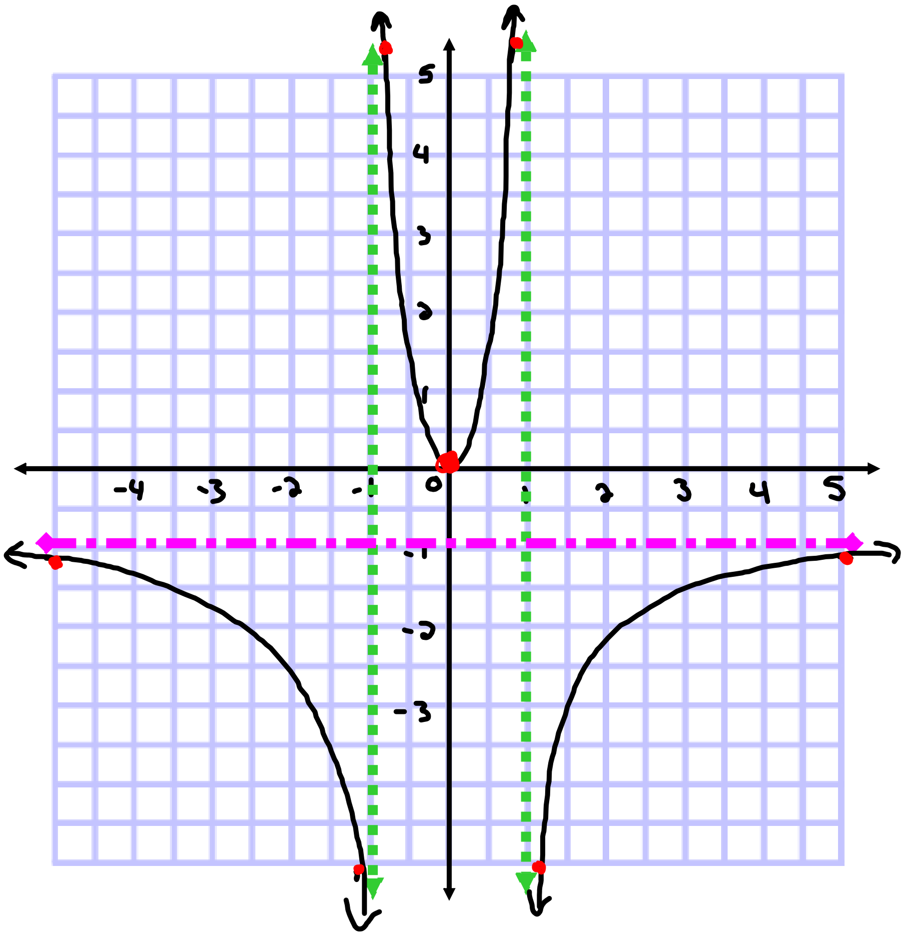
$f(x)=\frac{x^2}{1-x^2}$

when $x=1$

$f(1)=(1)^2$

when $x=-1$

$f(-1)=(-1)^2$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