Questions from Homework

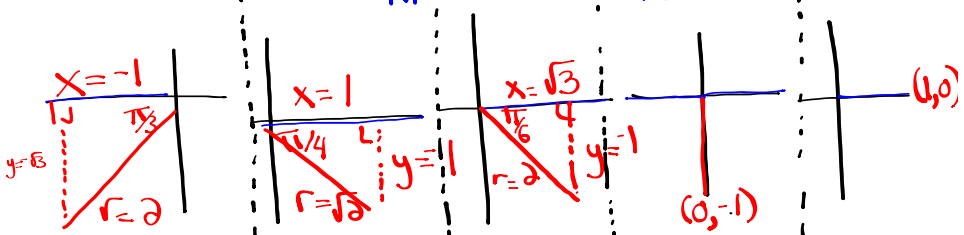
$-\frac{13\pi}{6}+\frac{24\pi}{6}=\frac{11\pi}{6}$ $\quad -14\pi+16\pi=2\pi$

⑤ $\csc^2\left(\frac{4\pi}{3}\right)\tan\left(\frac{15\pi}{4}\right)+\cos\left(-\frac{13\pi}{6}\right)-\sin\left(\frac{115\pi}{2}\right)+\cos(-14\pi)$

$\csc^2\left(\frac{4\pi}{3}\right)\tan\left(\frac{15\pi}{4}\right)+\cos\left(\frac{11\pi}{6}\right)-\sin\left(\frac{115\pi}{2}\right)+\cos(2\pi)$

$\frac{3\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}, \frac{14\pi}{4}, \frac{15\pi}{4}, \frac{16\pi}{4}, \frac{10\pi}{6}, \frac{11\pi}{6}, \frac{12\pi}{6}, \frac{14\pi}{2}, \frac{15\pi}{2}, \frac{16\pi}{2}, 2\pi$

$1\pi \quad 4\pi \quad 2\pi \quad 57\pi \quad 58\pi$



$\csc\left(\frac{4\pi}{3}\right)=\frac{2}{-\sqrt{3}}$ $\quad \tan\left(\frac{15\pi}{4}\right)=\frac{-1}{1}$ $\quad \cos\left(\frac{11\pi}{6}\right)=\frac{\sqrt{3}}{2}$ $\quad \sin\left(\frac{15\pi}{2}\right)=-1$ $\quad \cos(2\pi)=1$

$\csc^2\left(\frac{4\pi}{3}\right)\tan\left(\frac{15\pi}{4}\right)+\cos\left(\frac{11\pi}{6}\right)-\sin\left(\frac{115\pi}{2}\right)+\cos(2\pi)$

$\left(-\frac{2}{\sqrt{3}}\right)^2(-1)+\left(\frac{\sqrt{3}}{2}\right)-(-1)+(1)$

$\left(\frac{4}{3}\right)(-1)+\frac{\sqrt{3}}{2}+1+1$

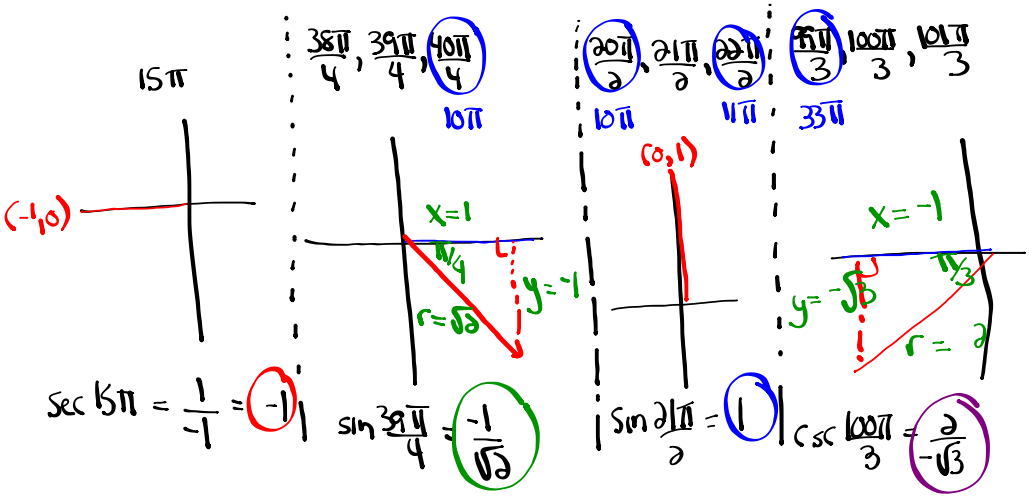
$-\frac{4}{3}+\frac{\sqrt{3}}{2}+\frac{2}{1}$

$-\frac{8}{6}+\frac{3\sqrt{3}}{6}+\frac{12}{6}$

$\boxed{\frac{4+3\sqrt{3}}{6}}$ or $\frac{3\sqrt{3}+4}{6}$

Questions from Homework

⑥ $\sec 15\pi + \sqrt{2}\sin\frac{39\pi}{4}\sin\frac{21\pi}{2} - \csc^2\frac{100\pi}{3}$



$\sec 15\pi = \frac{1}{-1} = -1$

$\sin\frac{39\pi}{4} = \frac{-1}{\sqrt{2}}$

$\sin\frac{21\pi}{2} = 1$

$\csc\frac{100\pi}{3} = \frac{2}{-\sqrt{3}}$

$\sec 15\pi + \sqrt{2}\sin\frac{39\pi}{4}\sin\frac{21\pi}{2} - \csc^2\frac{100\pi}{3}$

$(-1) + \sqrt{2}\left(\frac{-1}{\sqrt{2}}\right)(1) - \left(\frac{2}{-\sqrt{3}}\right)^2$

$-1 + \sqrt{2}\left(\frac{-1}{\sqrt{2}}\right)(1) - \frac{4}{3}$

$-1 - \frac{\sqrt{2}}{\sqrt{2}} - \frac{4}{3}$

$-1 - 1 - \frac{4}{3}$

$-\frac{3}{3} - \frac{3}{3} - \frac{4}{3}$

$\frac{-10}{3}$

Questions from Homework

8 $\dfrac{\tan\left(-\frac{15\pi}{4}\right)+\sec\left(\frac{301\pi}{3}\right)}{\csc\left(\frac{25\pi}{2}\right)+\cot\left(-\frac{31\pi}{6}\right)}$

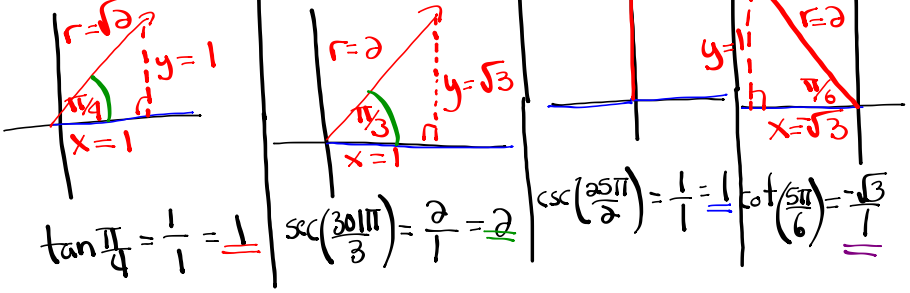
$-\frac{15\pi}{4}+\frac{16\pi}{4}=\frac{\pi}{4}$

$-\frac{31\pi}{6}+\frac{36\pi}{6}=\frac{5\pi}{6}$

$\dfrac{\tan\left(\frac{\pi}{4}\right)+\sec\left(\frac{301\pi}{3}\right)}{\csc\left(\frac{25\pi}{2}\right)+\cot\left(\frac{5\pi}{6}\right)}$

$\frac{0\pi}{4}, \frac{1\pi}{4}, \frac{2\pi}{4}$ | $\frac{300\pi}{3}, \frac{301\pi}{3}, \frac{302\pi}{3}$ | $\frac{24\pi}{2}, \frac{25\pi}{2}, \frac{26\pi}{2}$ | $\frac{4\pi}{6}, \frac{5\pi}{6}, \frac{6\pi}{6}$

0 | $\pi/2$ | 100π | 12π | 13π | 1π



$\tan\frac{\pi}{4}=\frac{1}{1}=1$ $\quad \sec\left(\frac{301\pi}{3}\right)=\frac{2}{1}=2$ $\quad \csc\left(\frac{25\pi}{2}\right)=\frac{1}{1}=1$ $\quad \cot\left(\frac{5\pi}{6}\right)=\frac{-\sqrt{3}}{1}$

$\dfrac{\tan\left(\frac{\pi}{4}\right)+\sec\left(\frac{301\pi}{3}\right)}{\csc\left(\frac{25\pi}{2}\right)+\cot\left(\frac{5\pi}{6}\right)}$

$\dfrac{1+2}{1+(-\sqrt{3})}$

$\dfrac{3}{(1-\sqrt{3})}\cdot\dfrac{(1+\sqrt{3})}{(1+\sqrt{3})}$

$\dfrac{3+3\sqrt{3}}{1+\sqrt{3}-\sqrt{3}-3}$

$\dfrac{3+3\sqrt{3}}{-2}$ or $-\dfrac{3-3\sqrt{3}}{2}$

Introduction to Trigonometric Equations

trigonometric equation

- an equation involving trigonometric ratios

Focus on...

- algebraically solving first-degree and second-degree trigonometric equations in radians and in degrees
- verifying that a specific value is a solution to a trigonometric equation
- identifying exact and approximate solutions of a trigonometric equation in a restricted domain
- determining the general solution of a trigonometric equation

Did You Know?

In equations, mathematicians often use the notation $\cos^2 \theta$. This means the same as $(\cos \theta)^2$.

Let's start with basic LINEAR trigonometric equations...

...Pre-Calculus 110

Solve: $\sin\theta = 0.9659\ ,\ -360° < \theta < 720°$ (Degrees)

- Reference angle?
- Which quadrants?
- Any co-terminal angles acceptable?

- If the domain is in degrees, give solutions in degrees.
- If the domain is in radians, give solutions in radians.

$\sin\theta = 0.9659$ — use positive for $\bar{\theta}$ — where is $\sin\theta > 0$ (positive)

$\bar{\theta} = \sin^{-1}(0.9659)$

$\bar{\theta} = 75°$

Q1	Q2
$\theta = \bar{\theta}$	$\theta = 180° - \bar{\theta}$
$\theta = 75°$	$\theta = 180° - 75° = 105°$
$\theta = 75° - 360° = -285°$	$\theta = 105° - 360° = -255°$
$\theta = 75° + 360° = 435°$	$\theta = 105° + 360° = 465°$

Solve: $\sec\theta = \frac{-1.3054}{1}$, $-2\pi \le \theta \le 2\pi$ (Radians) -6.28 6.28 (Approx. Value)

$\cos\theta = \frac{1}{-1.3054}$ where is cos < 0 (negative)

$\cos\theta = -0.7660$

$\bar{\theta} = \cos^{-1}(0.7660)$ use positive for $\bar{\theta}$

$\bar{\theta} = 0.7$ rads.

Q2	Q3
$\theta = \pi - \bar{\theta}$	$\theta = \pi + \bar{\theta}$
$\theta = 3.14 - 0.7 = 2.44$	$\theta = 3.14 + 0.7 = 3.84$
$\theta = 2.44 - 6.28 = -3.84$	$\theta = 3.84 - 6.28 = -2.44$

Ex. $\sqrt{2}\cos\theta + 1 = 0, \; -360° \le \theta \le 720°$ (Degrees) (Exact Value)

$\frac{\sqrt{2}\cos\theta}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$

$\cos\theta = \frac{-1}{\sqrt{2}}$

use positive for $\bar{\theta}$

$\bar{\theta} = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\bar{\theta} = 45°$

~~S~~ A
~~T~~ C

where is cos < 0 (negative)

Q2	Q3
$\theta = 180° - \bar{\theta}$	$\theta = 180° + \bar{\theta}$
$\theta = 180° - 45° = 135°$	$\theta = 180° + 45° = 225°$
$\theta = 135° - 360° = -225°$	$\theta = 225° - 360° = -135°$
$\theta = 135° + 360° = 495°$	$\theta = 225° + 360° = 585°$

(Radians)

Ex. $\sin x + 1 = 0, \; -2\pi \le x \le 4\pi$ (Exact Value)

$\sin x = -1$

$\bar{x} = \frac{3\pi}{2}$ (Unit Circle)

$x = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} - \frac{2\pi}{1}$ | $x = \frac{3\pi}{2} + \frac{2\pi}{1}$

$x = \frac{3\pi}{2} - \frac{4\pi}{2}$ | $x = \frac{3\pi}{2} + \frac{4\pi}{2}$

$x = -\frac{\pi}{2}$ | $x = \frac{7\pi}{2}$

Solutions:

$x = -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}$

① $\sin\theta = -\frac{\sqrt{3}}{2}$ where is $\sin\theta < 0$ (S | A / T | C)

$\bar{\theta} = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ use positive

$\bar{\theta} = 60°$

Q3	Q4
$\theta = 180° + 60° = 240°$	$\theta = 360° - 60° = 300°$
$240° \pm 360°n, n \in \mathbb{N}$	$300° \pm 360°n, n \in \mathbb{N}$

② $\cos\theta = -\frac{1}{2}$ where is $\cos\theta < 0$ (S | A / T | C)

$\bar{\theta} = \cos^{-1}\left(\frac{1}{2}\right)$

$\bar{\theta} = 60°$

Q2	Q3
$\theta = 180° - 60° = 120°$	$\theta = 180° + 60° = 240°$
$120° \pm 360°n, n \in \mathbb{N}$	$240° \pm 360°n, n \in \mathbb{N}$

③ $\tan\theta = -\frac{\sqrt{3}}{3}$ where is $\tan\theta < 0$ (S | A / T | C)

$\tan\theta = -\frac{1}{\sqrt{3}}$

$\bar{\theta} = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\bar{\theta} = 30°$

Q2	Q4
$\theta = 180° - 30° = 150°$	$\theta = 360° - 30° = 330°$
$150° \pm 360°n, n \in \mathbb{N}$	$330° \pm 360°n, n \in \mathbb{N}$

④ $\tan\theta = \frac{\sqrt{3}}{1}$ Where is $\tan\theta > 0$ (S | A / T | C)

$\bar{\theta} = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$\bar{\theta} = 60°$

Q1	Q4
$\theta = 60°$	$\theta = 360° - 60° = 300°$
$60° \pm 360°n, n \in \mathbb{N}$	$300° \pm 360°n, n \in \mathbb{N}$

⑥ $\sin\theta = -1$

* Unit Circle

$\theta = 270° \rightarrow 270° \pm 360°n, n \in \mathbb{N}$

⑬ $5\sin\theta - 4 = 0$ where is $\sin\theta > 0$ (S | A / T | C)

$\sin\theta = \frac{4}{5}$

$\sin\theta = 0.8$ (approx. value)

$\bar{\theta} = \sin^{-1}(0.8)$

$\bar{\theta} = 53.1°$

Q1	Q2
$\theta = 53.1°$	$\theta = 180° - 53.1 = 126.9°$
$53.1° \pm 360°n, n \in \mathbb{N}$	$126.9° \pm 360°n, n \in \mathbb{N}$

Your Turn

Solve each trigonometric equation in the specified domain.

a) $3\cos\theta - 1 = \cos\theta + 1,\ -2\pi \le \theta \le 2\pi$

b) $4\sec x + 8 = 0,\ 0° \le x < 360°$

a) $3\cos\theta - 1 = \cos\theta + 1$, $-2\pi \le \theta \le 2\pi$ (Radians)

$3\cos\theta - \cos\theta = 1 + 1$

$2\cos\theta = 2$

$\cos\theta = 1$

(Unit Circle)

$\theta = 0, 2\pi, -2\pi$

$\theta = 0 - 2\pi = -2\pi$

b) $4\sec x + 8 = 0$ $0° \le x < 360°$ (Degrees)

$\frac{4\sec x}{4} = \frac{-8}{4}$

$\sec x = \frac{-2}{1}$

* $\cos x = -\frac{1}{2}$ where is $\cos x < 0$ (Negative)

$\bar{x} = 60°$

Q2	Q3
$\theta = 180° - \bar{\theta}$	$\theta = 180° + \bar{\theta}$
$\theta = 180° - 60°$	$\theta = 180° + 60°$
$\theta = 120°$	$\theta = 240°$

Questions from Homework

Back

③ $\tan\theta = -\frac{\sqrt{3}}{3} = -\frac{1}{\sqrt{3}}$ Where is $\tan\theta < 0$ (Negative)

(Triangle) $\bar{\theta} = 30°$

Q2	Q4
$\theta = 180° - \bar{\theta}$	$\theta = 360° - \bar{\theta}$
$\theta = 180° - 30° = 150°$	$\theta = 360° - 30° = 330°$
$150° \pm 360°n, n \in N$	$330° \pm 360°n, n \in N$

Back

⑫ $\tan\theta =$ undefined $\tan\theta = \frac{o}{a} = \frac{y}{x}$

(Unit Circle)

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

Front

⑫ $5\sin\theta - 4 = 0$ (Approximate Value)

$\frac{5\sin\theta}{5} = \frac{4}{5}$

$\sin\theta = 0.8$

$\bar{\theta} = \sin^{-1}(0.8)$

$\bar{\theta} = 53.1°$

Where is $\sin\theta > 0$ (Positive)

Q1	Q2
$\theta = \bar{\theta}$	$\theta = 180° - \bar{\theta}$
$\theta = 53.1°$	$\theta = 180° - 53.1°$
	$\theta = 126.9°$
$53.1° \pm 360°n, n \in N$	$126.9° \pm 360°n, n \in N$

Factoring trinomials:

① Hard Trinomial

$2x^2+7x+6$ $\quad 3 \times 4 = 12$, $3+4=7$

$(x+\frac{3}{2})(x+\frac{4}{2})$

$(2x+3)(x+2)$

② Simple trinomial

x^2+7x+6 $\quad 6 \times 1 = 6$, $6+1=7$

$(x+1)(x+6)$

Let's move onto QUADRATIC trigonometric equations...

...Pre-Calculus 110

- What strategies can we use to solve quadratic equations?
- Quadratic trigonometric equations will ultimately become TWO linear trigonometric equations.

exponent = 2 (Factor)

- Common
- diff. of squares
- trinomial

(Radians)

Solve: $2\sin^2 x + \sin x = 1, \quad 0 \le x \le 4\pi$

$2\sin^2 x + \sin x - 1 = 0$

$-1 \times 2 = -2$

$-1 + 2 = 1$

$(\sin x + 2)(\sin x - 1) = 0$

$(\sin x + 1)(2\sin x - 1) = 0$

$\sin x + 1 = 0$	$2\sin x - 1 = 0$
$\sin x = -1$	$\sin x = \frac{1}{2}$
(unit circle)	(Special triangle)
$x = \frac{3\pi}{2}$	$\bar{x} = \sin^{-1}\left(\frac{1}{2}\right)$
$x = \frac{3\pi}{2} + \frac{2\pi}{1}$	$\bar{x} = \frac{\pi}{6}$
$x = \frac{3\pi}{2} + \frac{4\pi}{2} = \frac{7\pi}{2}$	

S | A

T | C

where is $\sin x > 0$

Q1	Q2
$\theta = \frac{\pi}{6}$	$\theta = \pi - \frac{\pi}{6}$
	$\theta = \frac{5\pi}{6}$
$\theta = \frac{\pi}{6} + 2\pi$	$\theta = \frac{5\pi}{6} + 2\pi$
$\theta = \frac{13\pi}{6}$	$\theta = \frac{17\pi}{6}$

Ex. $\cos^2\theta - \frac{1}{2}\cos\theta = 0, \ -2\pi \le \theta \le 4\pi$ (Common Factor)

$\cos\theta\left(\cos\theta - \frac{1}{2}\right) = 0$

$\cos\theta = 0$	$\cos\theta - \frac{1}{2} = 0$
(unit circle)	$\cos\theta = \frac{1}{2}$
$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$	(special triangle)
$\theta = \frac{5\pi}{2}, \frac{7\pi}{2}$ (Add 2π)	$\bar{\theta} = \cos^{-1}\left(\frac{1}{2}\right)$
$\theta = \frac{-3\pi}{2}, \frac{-\pi}{2}$ (Sub 2π)	$\bar{\theta} = \frac{\pi}{3}$

S | A
T | C

where is $\cos\theta > 0$

Q1	Q4
$\theta = \frac{\pi}{3}$	$\theta = \frac{2\pi}{1} - \frac{\pi}{3} = \frac{5\pi}{3}$
$\theta = \frac{7\pi}{3}$	$\theta = \frac{11\pi}{3}$
$\theta = -\frac{5\pi}{3}$	$\theta = \frac{-\pi}{3}$

$\theta = -\frac{5\pi}{3}, -\frac{3\pi}{2}, -\frac{\pi}{2}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{3}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{3}$

Ex. $6\sin^2 x - \sin x = 2, -2\pi \le \theta \le 4\pi$ (Hard Trinomial)

Your Turn

Solve for θ.

$\cos^2\theta - \cos\theta - 2 = 0,\ 0° \leq \theta < 360°$ (Simple Trinomial)

Give solutions as exact values where possible. Otherwise, give approximate measures to the nearest thousandth of a degree.

General Solution of a Trigonometric Equation

$Solve: \quad 3\cos^2\theta - \cos\theta = 2; \ \theta \in \mathbb{R}$ (Hard Trinomial)

(All of the angles)

Determine the general solution for $\sin^2 x - 1 = 0$ over the real numbers if x is measured in radians.

$x^2 - 4$

$(x+2)(x-2)$

$\sin^2 x - 1 = 0$ (Difference of Squares)

$(\sin x + 1)(\sin x - 1) = 0$

$\sin x + 1 = 0$	$\sin x - 1 = 0$
$\sin x = -1$	$\sin x = 1$
(Unit Circle)	(Unit Circle)
$x = \frac{3\pi}{2}$	$x = \frac{\pi}{2}$
$\frac{3\pi}{2} \pm 2\pi n, n \in N$	$\frac{\pi}{2} \pm 2\pi n, n \in N$

Did You Know?

$2n$, where $n \in I$, represents all even integers.

$2n + 1$, where $n \in I$, is an expression for all odd integers.

✓ $x = \frac{\pi}{2} + 2\pi n$, where $n \in I$

✓ $x = \frac{3\pi}{2} + 2\pi n$, where $n \in I$

or

$x = \frac{\pi}{2} + \pi n$, where $n \in I$

or

$(2n + 1)\left(\frac{\pi}{2}\right)$, $n \in I$

(Difference of squares)

Determine the general solution for $\cos^2 x - 1 = 0$, where the domain is real numbers measured in degrees.

Unit Review...

What topics have we covered??

- Radian Measure
- Co-terminal angles
- Principal Angles
- Angular Velocity (Open Response)
- The Unit Circle
- Trig Expressions (Open Response)
- Trig Equations (Open Response)

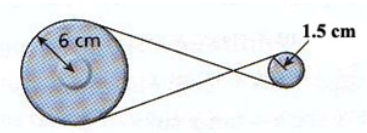
Chapter 4 Review:

Pages 215 - 217

Check-Up problem...

Solve: $\sin x \sec x + 2\sin x = 0 \ , \ x \in R$ (x is measured in radians)

If the belt in the pulley system below travels 30 cm, what is the angle of rotation of the smaller pulley?



[A] $\frac{\pi}{9}$ radians [B] 20°

[C] 20 radians [D] 5°

Nibbles the hamster is running at 0.02 m/s on an exercise wheel of radius 8 cm. What is the angular velocity of this wheel?

[A] 0.15 rad/minute [B] 240 rad/minute [C] 0.25 rad/minute [D] 15 radians/minute

Solve: $2(1-\sin\theta)^2+\sin\theta=2(3-4\sin^2\theta)\,,\ -360°\le\theta\le 720°$

$2(1-\sin\theta)^2+\sin\theta=2(3-4\sin^2\theta)$

$2(1-\sin\theta)(1-\sin\theta)+\sin\theta=6-8\sin^2\theta$

$2(1-\sin\theta-\sin\theta+\sin^2\theta)+\sin\theta=6-8\sin^2\theta$

$2(1-2\sin\theta+\sin^2\theta)+\sin\theta=6-8\sin^2\theta$

$2-4\sin\theta+2\sin^2\theta+\sin\theta-6+8\sin^2\theta=0$

$10\sin^2\theta-3\sin\theta-4=0$ (Hard Trinomial)

$\_\,x\,\_\_=-40$

$\_+\_\ =-3$

Solve: $6\sin^2\theta - 3\sin\theta = 0$, $0 \le \theta \le 360°$

[A] $0°, 30°, 180°, 330°, 360°$ [B] $0°, 30°, 180°, 150°, 360°$

[C] $30°, 90°, 120°, 270°$ [D] $0°, 180°, 210°, 330°, 360°$

$6\sin^2\theta - 3\sin\theta = 0$, $0° \le \theta \le 360°$ (Common Factor)

$3\sin\theta(2\sin\theta - 1) = 0$

$\sin^2\theta = (\sin\theta)(\sin\theta)$

S | A
T | C

$\frac{3\sin\theta}{3} = \frac{0}{3}$

$\sin\theta = 0$

(Unit Circle)

$\theta = 0°, 180°, 360°$

$2\sin\theta - 1 = 0$

$\frac{2\sin\theta}{2} = \frac{1}{2}$

$\sin\theta = \frac{1}{2}$

$\bar{\theta} = \sin^{-1}\left(\frac{1}{2}\right)$

$\bar{\theta} = 30°$

where is $\sin\theta > 0$

Q1	Q2
$\theta = 30°$	$\theta = 180° - 30°$
	$\theta = 150°$

If $\csc\theta < 0$ and $\tan\theta > 0$, then which of the following could be a possible measure of angle θ?

[A] $\frac{11\pi}{6}$ [B] $\frac{4\pi}{3}$ [C] $\frac{3\pi}{4}$ [D] $\frac{\pi}{2}$

What is the principal angle of $-\frac{25\pi}{4}$?

[A] $\frac{3\pi}{4}$ [B] $\frac{\pi}{4}$ [C] $-\frac{\pi}{4}$ [D] $\frac{7\pi}{4}$

Attachments

Worksheet - Sketching Angles in Radians.doc