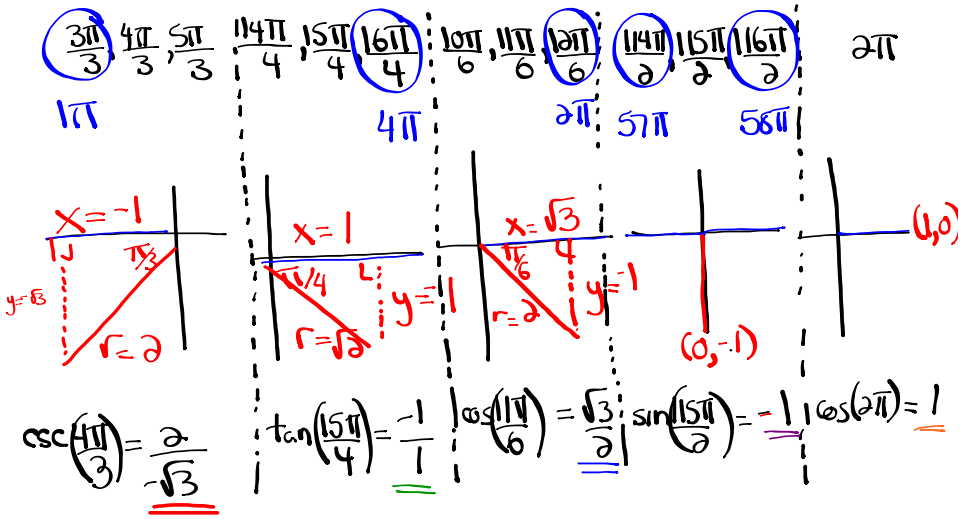
Questions from Homework

$-\frac{13\pi}{6}+\frac{24\pi}{6}=\frac{11\pi}{6}$ $\quad -14\pi+16\pi=2\pi$

⑤ $\csc^2\left(\frac{4\pi}{3}\right)\tan\left(\frac{15\pi}{4}\right)+\cos\left(-\frac{13\pi}{6}\right)-\sin\left(\frac{115\pi}{2}\right)+\cos(-14\pi)$

$\csc^2\left(\frac{4\pi}{3}\right)\tan\left(\frac{15\pi}{4}\right)+\cos\left(\frac{11\pi}{6}\right)-\sin\left(\frac{115\pi}{2}\right)+\cos(2\pi)$



$\csc^2\left(\frac{4\pi}{3}\right)\tan\left(\frac{15\pi}{4}\right)+\cos\left(\frac{11\pi}{6}\right)-\sin\left(\frac{115\pi}{2}\right)+\cos(2\pi)$

$\left(-\frac{2}{\sqrt{3}}\right)^2(-1)+\left(\frac{\sqrt{3}}{2}\right)-(-1)+(1)$

$\left(\frac{4}{3}\right)(-1)+\frac{\sqrt{3}}{2}+1+1$

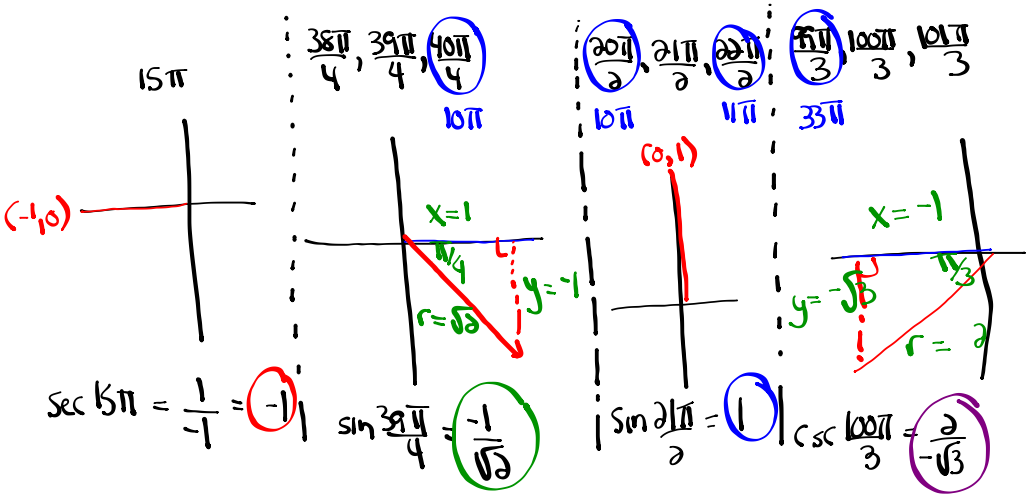
$-\frac{4}{3}+\frac{\sqrt{3}}{2}+\frac{2}{1}$

$-\frac{8}{6}+\frac{3\sqrt{3}}{6}+\frac{12}{6}$

$\boxed{\frac{4+3\sqrt{3}}{6}}$ or $\frac{3\sqrt{3}+4}{6}$

Questions from Homework

⑥ $\sec 15\pi + \sqrt{2}\sin\frac{39\pi}{4}\sin\frac{21\pi}{2} - \csc^2\frac{100\pi}{3}$



$\sec 15\pi = \frac{1}{-1} = -1$

$\sin\frac{39\pi}{4} = \frac{-1}{\sqrt{2}}$

$\sin\frac{21\pi}{2} = 1$

$\csc\frac{100\pi}{3} = \frac{2}{-\sqrt{3}}$

$\sec 15\pi + \sqrt{2}\sin\frac{39\pi}{4}\sin\frac{21\pi}{2} - \csc^2\frac{100\pi}{3}$

$(-1) + \sqrt{2}\left(\frac{-1}{\sqrt{2}}\right)(1) - \left(\frac{2}{-\sqrt{3}}\right)^2$

$-1 + \sqrt{2}\left(\frac{-1}{\sqrt{2}}\right)(1) - \frac{4}{3}$

$-1 - \frac{\sqrt{2}}{\sqrt{2}} - \frac{4}{3}$

$-1 - 1 - \frac{4}{3}$

$-\frac{3}{3} - \frac{3}{3} - \frac{4}{3}$

$\frac{-10}{3}$

Questions from Homework

8 $\dfrac{\tan\left(-\frac{15\pi}{4}\right)+\sec\left(\frac{301\pi}{3}\right)}{\csc\left(\frac{25\pi}{2}\right)+\cot\left(-\frac{31\pi}{6}\right)}$

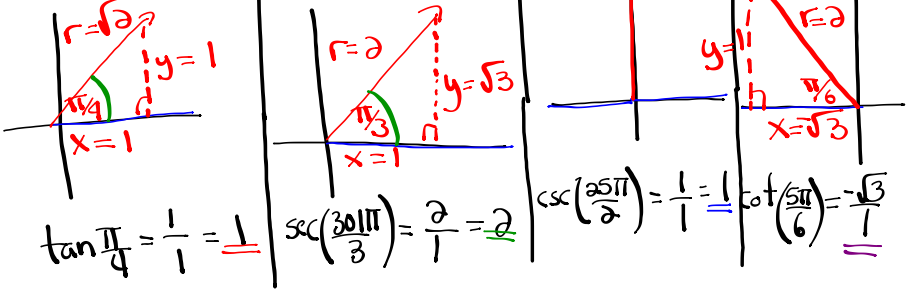
$-\frac{15\pi}{4}+\frac{16\pi}{4}=\frac{\pi}{4}$

$-\frac{31\pi}{6}+\frac{36\pi}{6}=\frac{5\pi}{6}$

$\dfrac{\tan\left(\frac{\pi}{4}\right)+\sec\left(\frac{301\pi}{3}\right)}{\csc\left(\frac{25\pi}{2}\right)+\cot\left(\frac{5\pi}{6}\right)}$

$\frac{0\pi}{4}, \frac{1\pi}{4}, \frac{2\pi}{4}$ | $\frac{300\pi}{3}, \frac{301\pi}{3}, \frac{302\pi}{3}$ | $\frac{24\pi}{2}, \frac{25\pi}{2}, \frac{26\pi}{2}$ | $\frac{4\pi}{6}, \frac{5\pi}{6}, \frac{6\pi}{6}$

0 | $\pi/2$ | 100π | 12π | 13π | 1π



$\tan\frac{\pi}{4}=\frac{1}{1}=1$ $\quad \sec\left(\frac{301\pi}{3}\right)=\frac{2}{1}=2$ $\quad \csc\left(\frac{25\pi}{2}\right)=\frac{1}{1}=1$ $\quad \cot\left(\frac{5\pi}{6}\right)=\frac{-\sqrt{3}}{1}$

$\dfrac{\tan\left(\frac{\pi}{4}\right)+\sec\left(\frac{301\pi}{3}\right)}{\csc\left(\frac{25\pi}{2}\right)+\cot\left(\frac{5\pi}{6}\right)}$

$\dfrac{1+2}{1+(-\sqrt{3})}$

$\dfrac{3}{(1-\sqrt{3})}\cdot\dfrac{(1+\sqrt{3})}{(1+\sqrt{3})}$

$\dfrac{3+3\sqrt{3}}{1+\sqrt{3}-\sqrt{3}-3}$

$\dfrac{3+3\sqrt{3}}{-2}$ or $-\dfrac{3-3\sqrt{3}}{2}$

Introduction to Trigonometric Equations

trigonometric equation

- an equation involving trigonometric ratios

Focus on...

- algebraically solving first-degree and second-degree trigonometric equations in radians and in degrees
- verifying that a specific value is a solution to a trigonometric equation
- identifying exact and approximate solutions of a trigonometric equation in a restricted domain
- determining the general solution of a trigonometric equation

Did You Know?

In equations, mathematicians often use the notation $\cos^2 \theta$. This means the same as $(\cos \theta)^2$.

Let's start with basic LINEAR trigonometric equations...

...Pre-Calculus 110

Solve: $\sin\theta = 0.9659\ ,\ -360° < \theta < 720°$ (Degrees)

- Reference angle?
- Which quadrants?
- Any co-terminal angles acceptable?

- If the domain is in degrees, give solutions in degrees.
- If the domain is in radians, give solutions in radians.

$\sin\theta = 0.9659$ use positive for $\bar{\theta}$ where is $\sin\theta > 0$ (positive)

$\bar{\theta} = \sin^{-1}(0.9659)$

$\bar{\theta} = 75°$

Q1	Q2
$\theta = \bar{\theta}$	$\theta = 180° - \bar{\theta}$
$\theta = 75°$	$\theta = 180° - 75° = 105°$
$\theta = 75° - 360° = -285°$	$\theta = 105° - 360° = -255°$
$\theta = 75° + 360° = 435°$	$\theta = 105° + 360° = 465°$

Solve: $\sec\theta = \frac{-1.3054}{1}, \; -2\pi \le \theta \le 2\pi$ (Radians) (Approx. Value)

-6.28 6.28

$\cos\theta = \frac{1}{-1.3054}$ where is $\cos < 0$ (negative)

$\cos\theta = -0.7660$

$\bar{\theta} = \cos^{-1}(0.7660)$ ← use positive for $\bar{\theta}$

$\bar{\theta} = 0.7$ rads.

Q2	Q3
$\theta = \pi - \bar{\theta}$	$\theta = \pi + \bar{\theta}$
$\theta = 3.14 - 0.7 = 2.44$	$\theta = 3.14 + 0.7 = 3.84$
$\theta = 2.44 - 6.28 = -3.84$	$\theta = 3.84 - 6.28 = -2.44$

Ex. $\sqrt{2}\cos\theta + 1 = 0, \; -360° \le \theta \le 720°$ (Degrees) (Exact Value)

$\frac{\sqrt{2}\cos\theta}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$

$\cos\theta = \frac{-1}{\sqrt{2}}$

where is $\cos < 0$ (negative)

S | A
T | C

use positive for $\bar{\theta}$

$\bar{\theta} = \cos^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\bar{\theta} = 45°$

Q2	Q3
$\theta = 180° - \bar{\theta}$	$\theta = 180° + \bar{\theta}$
$\theta = 180° - 45° = 135°$	$\theta = 180° + 45° = 225°$
$\theta = 135° - 360° = -225°$	$\theta = 225° - 360° = -135°$
$\theta = 135° + 360° = 495°$	$\theta = 225° + 360° = 585°$

(Radians)

Ex. $\sin x + 1 = 0, \quad -2\pi \le x \le 4\pi$ (Exact Value)

$\sin x = -1$

$\bar{x} = \frac{3\pi}{2}$ (Unit Circle)

$x = \frac{3\pi}{2}$

$x = \frac{3\pi}{2} - \frac{2\pi}{1}$ | $x = \frac{3\pi}{2} + \frac{2\pi}{1}$

$x = \frac{3\pi}{2} - \frac{4\pi}{2}$ | $x = \frac{3\pi}{2} + \frac{4\pi}{2}$

$x = -\frac{\pi}{2}$ | $x = \frac{7\pi}{2}$

Solutions:

$x = -\frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{2}$

① $\sin\theta = -\frac{\sqrt{3}}{2}$ where is $\sin\theta < 0$ (S \| A / T \| C)

$\bar{\theta} = \sin^{-1}\left(\frac{\sqrt{3}}{2}\right)$ use positive

$\bar{\theta} = 60°$

Q3	Q4
$\theta = 180° + 60° = 240°$	$\theta = 360° - 60° = 300°$
$240° \pm 360°n, n \in \mathbb{N}$	$300° \pm 360°n, n \in \mathbb{N}$

② $\cos\theta = -\frac{1}{2}$ where is $\cos\theta < 0$ (S \| A / T \| C)

$\bar{\theta} = \cos^{-1}\left(\frac{1}{2}\right)$

$\bar{\theta} = 60°$

Q2	Q3
$\theta = 180° - 60° = 120°$	$\theta = 180° + 60° = 240°$
$120° \pm 360°n, n \in \mathbb{N}$	$240° \pm 360°n, n \in \mathbb{N}$

③ $\tan\theta = -\frac{\sqrt{3}}{3}$ where is $\tan\theta < 0$ (S \| A / T \| C)

$\tan\theta = -\frac{1}{\sqrt{3}}$

$\bar{\theta} = \tan^{-1}\left(\frac{1}{\sqrt{3}}\right)$

$\bar{\theta} = 30°$

Q2	Q4
$\theta = 180° - 30° = 150°$	$\theta = 360° - 30° = 330°$
$150° \pm 360°n, n \in \mathbb{N}$	$330° \pm 360°n, n \in \mathbb{N}$

④ $\tan\theta = \frac{\sqrt{3}}{1}$ Where is $\tan\theta > 0$ (S \| A / T \| C)

$\bar{\theta} = \tan^{-1}\left(\frac{\sqrt{3}}{1}\right)$

$\bar{\theta} = 60°$

Q1	Q4
$\theta = 60°$	$\theta = 360° - 60° = 300°$
$60° \pm 360°n, n \in \mathbb{N}$	$300° \pm 360°n, n \in \mathbb{N}$

⑥ $\sin\theta = -1$

* Unit Circle

$\theta = 270°$ → $270° \pm 360°n, n \in \mathbb{N}$

⑬ $5\sin\theta - 4 = 0$ where is $\sin\theta > 0$ (S \| A / T \| C)

$\sin\theta = \frac{4}{5}$

$\sin\theta = 0.8$ (approx. value)

$\bar{\theta} = \sin^{-1}(0.8)$

$\bar{\theta} = 53.1°$

Q1	Q2
$\theta = 53.1°$	$\theta = 180° - 53.1 = 126.9°$
$53.1° \pm 360°n, n \in \mathbb{N}$	$126.9° \pm 360°n, n \in \mathbb{N}$

Your Turn

Solve each trigonometric equation in the specified domain.

a) $3\cos\theta - 1 = \cos\theta + 1,\ -2\pi \le \theta \le 2\pi$

b) $4\sec x + 8 = 0,\ 0° \le x < 360°$

a) $3\cos\theta - 1 = \cos\theta + 1$, $-2\pi \le \theta \le 2\pi$ (Radians)

$3\cos\theta - \cos\theta = 1 + 1$

$2\cos\theta = 2$

$\cos\theta = 1$

(Unit Circle)

$\theta = 0, 2\pi, -2\pi$

$\theta = 0 - 2\pi = -2\pi$

b) $4\sec x + 8 = 0$ $0° \le x < 360°$ (Degrees)

$\frac{4\sec x}{4} = \frac{-8}{4}$

$\sec x = \frac{-2}{1}$

$\cos x = -\frac{1}{2}$ where is $\cos x < 0$ (Negative)

$\bar{x} = 60°$

Q2	Q3
$\theta = 180° - \bar{\theta}$	$\theta = 180° + \bar{\theta}$
$\theta = 180° - 60°$	$\theta = 180° + 60°$
$\theta = 120°$	$\theta = 240°$

Questions from Homework

Back

③ $\tan\theta = -\frac{\sqrt{3}}{3} = -\frac{1}{\sqrt{3}}$ Where is $\tan\theta < 0$ (Negative)

(Triangle) $\bar{\theta} = 30°$

Q2	Q4
$\theta = 180° - \bar{\theta}$	$\theta = 360° - \bar{\theta}$
$\theta = 180° - 30° = 150°$	$\theta = 360° - 30° = 330°$
$150° \pm 360°n, n \in N$	$330° \pm 360°n, n \in N$

Back

⑫ $\tan\theta =$ undefined $\tan\theta = \frac{o}{a} = \frac{y}{x}$

(Unit Circle)

$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$

Front

⑫ $5\sin\theta - 4 = 0$ (Approximate Value)

$\frac{5\sin\theta}{5} = \frac{4}{5}$

$\sin\theta = 0.8$

$\bar{\theta} = \sin^{-1}(0.8)$

$\bar{\theta} = 53.1°$

Where is $\sin\theta > 0$ (Positive)

Q1	Q2
$\theta = \bar{\theta}$	$\theta = 180° - \bar{\theta}$
$\theta = 53.1°$	$\theta = 180° - 53.1°$
	$\theta = 126.9°$
$53.1° \pm 360°n, n \in N$	$126.9° \pm 360°n, n \in N$

Factoring trinomials:

① Hard Trinomial

$2x^2+7x+6$ $\quad 3 \times 4 = 12$, $3+4=7$

$(x+\frac{3}{2})(x+\frac{4}{2})$

$(2x+3)(x+2)$

② Simple trinomial

x^2+7x+6 $\quad 6 \times 1 = 6$, $6+1=7$

$(x+1)(x+6)$

Let's move onto QUADRATIC trigonometric equations...

...Pre-Calculus 110

- What strategies can we use to solve quadratic equations?
- Quadratic trigonometric equations will ultimately become TWO linear trigonometric equations.

exponent = 2 (Factor)

- Common
- diff. of squares
- trinomial

(Radians)

Solve: $2\sin^2 x + \sin x = 1, \quad 0 \le x \le 4\pi$

$2\sin^2 x + \sin x - 1 = 0$

$-1 \times 2 = -2$

$-1 + 2 = 1$

$(\sin x + 2)(\sin x - 1) = 0$

$(\sin x + 1)(2\sin x - 1) = 0$

$\sin x + 1 = 0$	$2\sin x - 1 = 0$
$\sin x = -1$	$\sin x = \frac{1}{2}$
(unit circle)	(Special triangle)
$x = \frac{3\pi}{2}$	$\bar{x} = \sin^{-1}\left(\frac{1}{2}\right)$
$x = \frac{3\pi}{2} + \frac{2\pi}{1}$	$\bar{x} = \frac{\pi}{6}$
$x = \frac{3\pi}{2} + \frac{4\pi}{2} = \frac{7\pi}{2}$	

S | A
T | C

where is $\sin x > 0$

Q1	Q2
$\theta = \frac{\pi}{6}$	$\theta = \pi - \frac{\pi}{6}$
	$\theta = \frac{5\pi}{6}$
$\theta = \frac{\pi}{6} + 2\pi$	$\theta = \frac{5\pi}{6} + 2\pi$
$\theta = \frac{13\pi}{6}$	$\theta = \frac{17\pi}{6}$

Ex. $\cos^2\theta - \frac{1}{2}\cos\theta = 0, \ -2\pi \le \theta \le 4\pi$ (Common Factor)

$\cos\theta\left(\cos\theta - \frac{1}{2}\right) = 0$

$\cos\theta = 0$	$\cos\theta - \frac{1}{2} = 0$
(unit circle)	$\cos\theta = \frac{1}{2}$
$\theta = \frac{\pi}{2}, \frac{3\pi}{2}$	(special triangle)
$\theta = \frac{5\pi}{2}, \frac{7\pi}{2}$ (Add 2π)	$\bar{\theta} = \cos^{-1}\left(\frac{1}{2}\right)$
$\theta = \frac{-3\pi}{2}, \frac{-\pi}{2}$ (Sub 2π)	$\bar{\theta} = \frac{\pi}{3}$

S | A
T | C

where is $\cos\theta > 0$

Q1	Q4
$\theta = \frac{\pi}{3}$	$\theta = \frac{2\pi}{1} - \frac{\pi}{3} = \frac{5\pi}{3}$
$\theta = \frac{7\pi}{3}$	$\theta = \frac{11\pi}{3}$
$\theta = -\frac{5\pi}{3}$	$\theta = \frac{-\pi}{3}$

$\theta = -\frac{5\pi}{3}, -\frac{3\pi}{2}, -\frac{\pi}{2}, -\frac{\pi}{3}, \frac{\pi}{3}, \frac{\pi}{2}, \frac{3\pi}{2}, \frac{7\pi}{3}, \frac{5\pi}{2}, \frac{7\pi}{2}, \frac{11\pi}{3}$

Ex. $6\sin^2 x - \sin x = 2, -2\pi \le \theta \le 4\pi$ (Hard Trinomial)

Your Turn

Solve for θ.

$\cos^2\theta - \cos\theta - 2 = 0, 0° \leq \theta < 360°$ (Simple Trinomial)

Give solutions as exact values where possible. Otherwise, give approximate measures to the nearest thousandth of a degree.

General Solution of a Trigonometric Equation

Assume degrees.

Solve: $3\cos^2\theta - \cos\theta = 2;\ \theta \in \mathbb{R}$ (Hard Trinomial)

$3\cos^2\theta - \cos\theta - 2 = 0$

$-3 \times 2 = -6$

$-3 + 2 = -1$

$(\cos\theta - 3)(\cos\theta + 2) = 0$

$\div 3 \quad \div 3$

$(\cos\theta - 1)(3\cos\theta + 2) = 0$

$\cos\theta - 1 = 0$

$\cos\theta = 1$

(Unit Circle)

$\theta = 0°, 360°$

$\theta = 0° \pm 360°n,\ n \in \mathbb{N}$

$3\cos\theta + 2 = 0$

$\cos\theta = -\frac{2}{3}$

(Approximate Value)

$\bar{\theta} = \cos^{-1}\left(\frac{2}{3}\right)$

$\bar{\theta} = 48.2°$

where is $\cos\theta < 0$

S	A
T	C

Q2	Q3
$\theta = 180° - 48.2°$	$\theta = 180° + 48.2°$
$\theta = 131.8°$	$\theta = 228.2°$
$\theta = 131.8° \pm 360°n,\ n \in \mathbb{N}$	$\theta = 228.2° \pm 360°n,\ n \in \mathbb{N}$

(All of the angles)

Determine the general solution for $\sin^2 x - 1 = 0$ over the real numbers if x is measured in radians.

$x^2 - 4$

$(x+2)(x-2)$

$\sin^2 x - 1 = 0$ (Difference of Squares)

$(\sin x + 1)(\sin x - 1) = 0$

$\sin x + 1 = 0$	$\sin x - 1 = 0$
$\sin x = -1$	$\sin x = 1$
(Unit Circle)	(Unit Circle)
$x = \frac{3\pi}{2}$	$x = \frac{\pi}{2}$
$\frac{3\pi}{2} \pm 2\pi n, n \in \mathbb{N}$	$\frac{\pi}{2} \pm 2\pi n, n \in \mathbb{N}$

Did You Know?

$2n$, where $n \in \mathrm{I}$, represents all even integers.

$2n + 1$, where $n \in \mathrm{I}$, is an expression for all odd integers.

✓ $x = \frac{\pi}{2} + 2\pi n$, where $n \in \mathrm{I}$

✓ $x = \frac{3\pi}{2} + 2\pi n$, where $n \in \mathrm{I}$

or

$x = \frac{\pi}{2} + \pi n$, where $n \in \mathrm{I}$

or

$(2n + 1)\left(\frac{\pi}{2}\right)$, $n \in \mathrm{I}$

(Difference of squares)

Determine the general solution for $\cos^2 x - 1 = 0$, where the domain is real numbers measured in degrees.

Unit Review...

What topics have we covered??

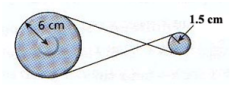
- Radian Measure
- Co-terminal angles
- Principal Angles
- Angular Velocity (Open Response)
- The Unit Circle
- Trig Expressions (Open Response)
- Trig Equations (Open Response)

Chapter 4 Review:

Pages 215 - 217

$\frac{\sqrt{2}}{2} = \frac{1}{\sqrt{2}}$ or $\frac{\sqrt{3}}{3} = \frac{1}{\sqrt{3}}$

If the belt in the pulley system below travels 30 cm, what is the angle of rotation of the smaller pulley?



[A] $\frac{\pi}{9}$ radians [B] 20°

[C] 20 radians [D] 5°

Given:
$a = 30cm$
$r = 1.5$

Small wheel
$\theta = \frac{a}{r} = \frac{30cm}{1.5cm} = 20 rads$

Nibbles the hamster is running at 0.02 m/s on an exercise wheel of radius 8 cm. What is the angular velocity of this wheel?
[A] 0.15 rad/minute [B] 240 rad/minute [C] 0.25 rad/minute [D] 15 radians/minute

Given:
$a = 0.02m$
$r = 8cm = 0.08m$

(i) Find θ
$\theta = \frac{a}{r} = \frac{0.02m}{0.08cm} = 0.25 rads$

(ii) Find V_a
$V_a = \frac{\theta}{t} = \frac{0.25 rads}{1 sec} = \frac{15 rads}{60 sec} = 15 rads/min$

Solve: $2(1-\sin\theta)^2 + \sin\theta = 2(3-4\sin^2\theta),\ -360° \le \theta \le 720°$

$2(1-\sin\theta)^2 + \sin\theta = 2(3-4\sin^2\theta)$

$2(1-\sin\theta)(1-\sin\theta) + \sin\theta = 6 - 8\sin^2\theta$

$2(1 - \sin\theta - \sin\theta + \sin^2\theta) + \sin\theta = 6 - 8\sin^2\theta$

$2 - 2\sin\theta - 2\sin\theta + 2\sin^2\theta + \sin\theta = 6 - 8\sin^2\theta$

$2 - 4\sin\theta + 2\sin^2\theta + \sin\theta - 6 + 8\sin^2\theta = 0$

$10\sin^2\theta - 3\sin\theta - 4 = 0$ (Hard Trinomial)

$-8 \times 5 = -40$
$-8 + 5 = -3$

$(\sin\theta + \frac{5}{10})(\sin\theta - \frac{8}{10}) = 0$

$(\sin\theta + \frac{1}{2})(\sin\theta - \frac{4}{5}) = 0$

$(2\sin\theta + 1)(5\sin\theta - 4) = 0$

$2\sin\theta + 1 = 0$
$\sin\theta = -\frac{1}{2}$
(Triangle)
$\bar{\theta} = \sin^{-1}(\frac{1}{2})$
$\bar{\theta} = 30°$
where is $\sin\theta < 0$ S | A / T | C

$5\sin\theta - 4 = 0$
$\sin\theta = \frac{4}{5}$
(Approx Value)
$\bar{\theta} = \sin^{-1}(\frac{4}{5})$
$\bar{\theta} = 53.1°$
where is $\sin\theta > 0$ S | A / T | C

Q3	Q4	Q1	Q2
$\theta = 180° + 30°$	$\theta = 360° - 30°$	$\theta = 53°$	$\theta = 180° - 53°$
$\theta = 210°$	$\theta = 330°$	$\theta = 413°$	$\theta = 127°$
$\theta = -510°$	$\theta = 690°$	$\theta = -307$	$\theta = 487°$
$\theta = -150°$	$\theta = -30°$		$\theta = -233°$

Solve: $6\sin^2\theta - 3\sin\theta = 0\ ,\ 0 \le \theta \le 360°$

[A] $0°, 30°, 180°, 330°, 360°$ [B] $0°, 30°, 180°, 150°, 360°$

[C] $30°, 90°, 120°, 270°$ [D] $0°, 180°, 210°, 330°, 360°$

$6\sin^2\theta - 3\sin\theta = 0$, $0° \le \theta \le 360°$ (Common Factor)

$3\sin\theta(2\sin\theta - 1) = 0$

$\sin^2\theta = (\sin\theta)(\sin\theta)$

S | A
T | C

$\frac{3\sin\theta}{3} = \frac{0}{3}$ | $2\sin\theta - 1 = 0$

$\sin\theta = 0$ | $2\sin\theta = 1$ | where is $\sin\theta > 0$

(Unit Circle) | $\sin\theta = \frac{1}{2}$

$\theta = 0°, 180°, 360°$ | $\bar{\theta} = \sin^{-1}\left(\frac{1}{2}\right)$

$\bar{\theta} = 30°$

Q1	Q2
$\theta = 30°$	$\theta = 180° - 30°$
	$\theta = 150°$

If $\csc\theta < 0$ and $\tan\theta > 0$, then ~~which of the~~ following could be a possible measure of angle θ?

[A] $\frac{11\pi}{6}$ (Q4) [B] $\frac{4\pi}{3}$ (Q3) [C] $\frac{3\pi}{4}$ (Q2) [D] $\frac{\pi}{2}$

between Q1 + Q2

$\csc\theta < 0$ & $\tan\theta > 0$

S | A

T | C

$\frac{3\pi}{3}, \frac{4\pi}{3}, \frac{5\pi}{3}$

π

What is the principal angle of $-\frac{25\pi}{4}$?

[A] $\frac{3\pi}{4}$ [B] $\frac{\pi}{4}$ [C] $-\frac{\pi}{4}$ [D] $\frac{7\pi}{4}$

① $-\frac{25\pi}{4} \times \frac{1}{2\pi} = -\frac{25\pi}{8\pi} = -3\frac{1}{8}\pi$

② $-3\frac{1}{8}\pi + 3 = -\frac{1}{8}\pi$

③ $-\frac{1}{8}\pi \cdot 2\pi = -\frac{2\pi}{8} = -\frac{\pi}{4}$ Answer must be positive

④ $-\frac{\pi}{4} + 2\pi$

$-\frac{\pi}{4} + \frac{8\pi}{4} = \frac{7\pi}{4}$

$-\frac{25\pi}{4} + 8\pi$

$-\frac{25\pi}{4} + \frac{32\pi}{4} = \frac{7\pi}{4}$

Chapter 4 Review

① a) $\cos\theta = \frac{\sqrt{3}}{2}$, $0° < \theta < 360°$

(triangle)

$\bar{\theta} = \cos^{-1}\left(\frac{\sqrt{3}}{2}\right)$

$\bar{\theta} = 30°$

where is $\cos\theta > 0$

S	A ✓
T	C ✓

Q1	Q4
$\theta = 30°$	$\theta = 360° - 30° = 330°$

b) $\sin\theta = -\frac{\sqrt{2}}{2}$, $0 < \theta < 2\pi$

$\sin\theta = -\frac{1}{\sqrt{2}}$

(Triangle)

$\bar{\theta} = \sin^{-1}\left(\frac{1}{\sqrt{2}}\right)$

$\bar{\theta} = \frac{\pi}{4}$

where is $\sin\theta < 0$

S	A
T ✓	C ✓

Q3	Q4
$\theta = \pi + \frac{\pi}{4} = \frac{5\pi}{4}$	$\theta = 2\pi - \frac{\pi}{4} = \frac{7\pi}{4}$

c) $\cot\theta$ = undefined, $0° \le \theta \le 720°$

(unit circle)

$\theta = 0°, 180°, 360°$

$\theta = 540°, 720°$ (Add 360°)

d) $2\sin\theta - 1 = 0$, $-2\pi \le \theta \le 2\pi$

$\sin\theta = \frac{1}{2}$

(Triangle)

$\bar{\theta} = \sin^{-1}\left(\frac{1}{2}\right)$

$\bar{\theta} = \frac{\pi}{6}$

where is $\sin\theta > 0$

S ✓	A
T	C

Q1	Q2
$\theta = \frac{\pi}{6}$	$\theta = \pi - \frac{\pi}{6} = \frac{5\pi}{6}$
$\theta = \frac{\pi}{6} - 2\pi = -\frac{11\pi}{6}$	$\theta = \frac{5\pi}{6} - 2\pi = -\frac{7\pi}{6}$

Ch. 4 Review

① e) $\cos^2\theta + \frac{1}{2}\cos\theta = 0$, $0° \le \theta \le 360°$

$\cos\theta(\cos\theta + \frac{1}{2}) = 0$

$\cos\theta = 0$ (Unit Circle) $\theta = 90°, 270°$

$\cos\theta + \frac{1}{2} = 0$

$\cos\theta = -\frac{1}{2}$ (Triangle) where is $\cos\theta < 0$

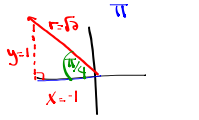
$\bar{\theta} = \cos^{-1}(\frac{1}{2})$

$\bar{\theta} = 60°$

Q2	Q3
$\theta = 180° - 60°$	$\theta = 180° + 60°$
$\theta = 120°$	$\theta = 240°$

$\frac{2\pi}{4}, \frac{3\pi}{4}, \frac{4\pi}{4}$

π



$\sin\frac{3\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

② a) $\frac{3}{1 - 2\sin\frac{3\pi}{4}}$

$\frac{3}{1 - 2(\frac{\sqrt{2}}{2})}$

$\frac{3}{(1-\sqrt{2})}\frac{(1+\sqrt{2})}{(1+\sqrt{2})}$ Rationalize

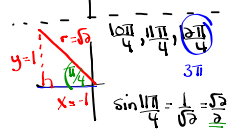
$\frac{3+3\sqrt{2}}{1+\sqrt{2}-\sqrt{2}-2}$

$\frac{3+3\sqrt{2}}{-1}$ or $-3-3\sqrt{2}$

② d) $\frac{2\cos 3\pi + \sin\frac{11\pi}{4}}{\cos^2\frac{\pi}{6}}$

$\cos 3\pi = -1$

$\frac{2(-1) + \frac{\sqrt{2}}{2}}{(\frac{\sqrt{3}}{2})^2}$

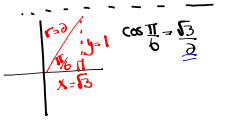


$\frac{9\pi}{4}, \frac{11\pi}{4}, \frac{8\pi}{4}$

3π

$\sin\frac{11\pi}{4} = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

$\frac{-2 + \frac{\sqrt{2}}{2}}{\frac{3}{4}}$



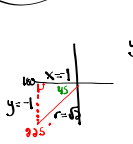
$\cos\frac{\pi}{6} = \frac{\sqrt{3}}{2}$

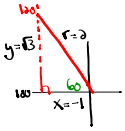
$(\frac{-4}{2} + \frac{\sqrt{2}}{2}) \cdot \frac{4}{3}$ ← multiply by reciprocal

$(\frac{-4+\sqrt{2}}{2}) \cdot \frac{4}{3}$

$\frac{-16 + 4\sqrt{2}}{6} \rightarrow \frac{-8 + 2\sqrt{2}}{3}$ reduce

② b) $\frac{\sin^2 225°}{8\sin 120°}$





$\sin 225° = \frac{-1}{\sqrt{2}}$

$\sin 120° = \frac{\sqrt{3}}{2}$

$\frac{(\frac{-1}{\sqrt{2}})^2}{8(\frac{\sqrt{3}}{2})}$

$\frac{\frac{1}{2}}{4\sqrt{3}}$

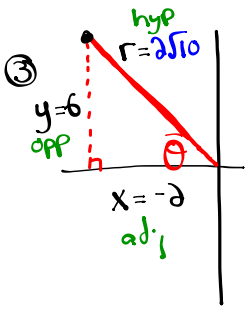
$\frac{1}{2} \times \frac{1}{4\sqrt{3}}$

$\frac{1}{8\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$

$\frac{\sqrt{3}}{24}$

Ch. 4 Review

③



① Find r:

$x^2+y^2=r^2$

$(-2)^2+(6)^2=r^2$

$4+36=r^2$

$40=r^2$

$\sqrt{40}=r$

$\sqrt{2\cdot2\cdot2\cdot5}=r$

$2\sqrt{10}=r$

② $\sin\theta=\frac{6}{2\sqrt{10}}=\frac{3}{\sqrt{10}}=\frac{3\sqrt{10}}{10}$

$\cos\theta=\frac{-2}{2\sqrt{10}}=\frac{-1}{\sqrt{10}}=\frac{-\sqrt{10}}{10}$

$\tan\theta=\frac{6}{-2}=-3$

$\csc\theta=\frac{\sqrt{10}}{3}$

$\sec\theta=-\sqrt{10}$

$\cot\theta=-\frac{1}{3}$

} Reciprocal Ratios

④ $\sec\theta=-\frac{\sqrt{11}}{3}$ (hyp / adj), $\cos\theta<0$, and $\tan\theta>0$

Given:

$r=\text{hyp}=\sqrt{11}$

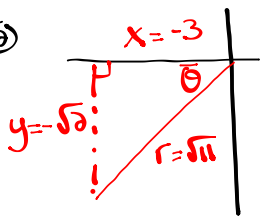
$x=\text{adj}=-3$

① • $\cos\theta<0$
• $\tan\theta>0$

~~S~~	~~A~~
~~T~~	C

θ is in Q3

②



$x^2+y^2=r^2$

$(-3)^2+y^2=(\sqrt{11})^2$

$9+y^2=11$

$y^2=2$

$y=\pm\sqrt{2}$

$y=-\sqrt{2}$ Q3

③ $\sin\theta=\frac{-\sqrt{2}}{\sqrt{11}}=-\frac{\sqrt{22}}{11}$

$\cos\theta=\frac{-3}{\sqrt{11}}=-\frac{3\sqrt{11}}{11}$

$\tan\theta=\frac{-\sqrt{2}}{-3}=\frac{\sqrt{2}}{3}$

$\csc\theta=-\frac{\sqrt{11}}{\sqrt{2}}=-\frac{\sqrt{22}}{2}$

$\cot\theta=\frac{3}{\sqrt{2}}=\frac{3\sqrt{2}}{2}$

Ch. 4 Review

⑤ a) $\frac{2\pi}{9}$

$A_c = \frac{2\pi}{9} + 2\pi$ | $A_c = \frac{2\pi}{9} - 2\pi$

$= \frac{2\pi}{9} + \frac{18\pi}{9}$ | $= \frac{2\pi}{9} - \frac{18\pi}{9}$

$= \frac{20\pi}{9}$ | $= -\frac{16\pi}{9}$

b) -900°

$A_c = -900^\circ + 1080^\circ$ | $A_c = -900^\circ - 360^\circ$

$= 180^\circ$ | $= -1260^\circ$

c) 300°

$A_c = 300^\circ + 360^\circ$ | $A_c = 300^\circ - 360^\circ$

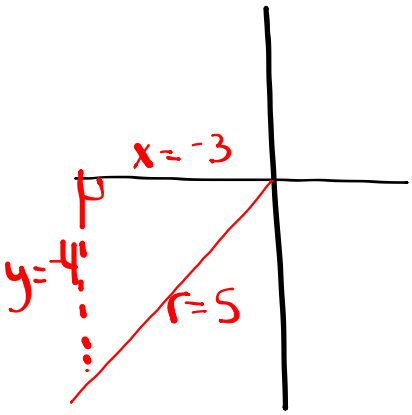
$= 660^\circ$ | $= -60^\circ$

$\cot\theta = \frac{3}{4}$, $180° \le \theta \le 270°$ (Q3)

$\sec\theta = \frac{h}{a} = \frac{r}{x} = \frac{5}{-3} = -\frac{5}{3}$

$x = 3$

$y = 4$



Attachments

Worksheet - Sketching Angles in Radians.doc