

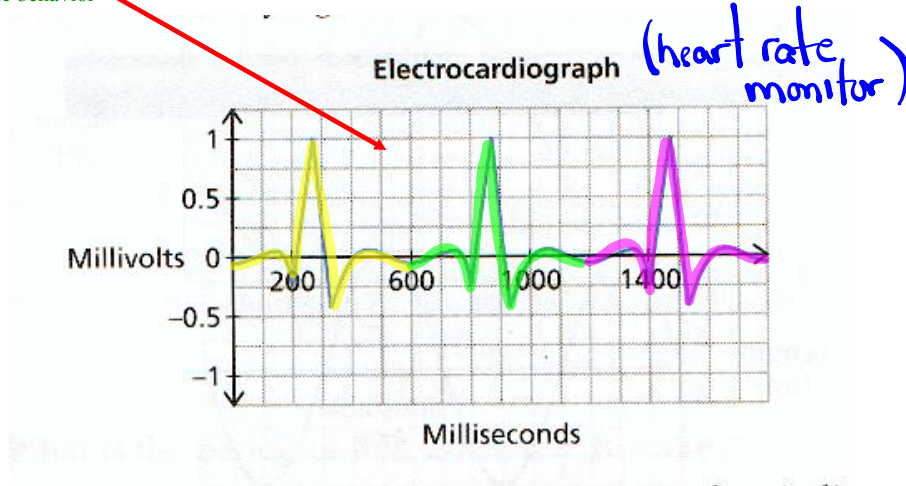
Sinusoidal Relations (Trig Graphs)

$y = \sin x$
 $y = \cos x$

Periodic Function: A function for which the dependent variable takes on the same set of values over and over again as the independent variable changes.

(a function that repeats)

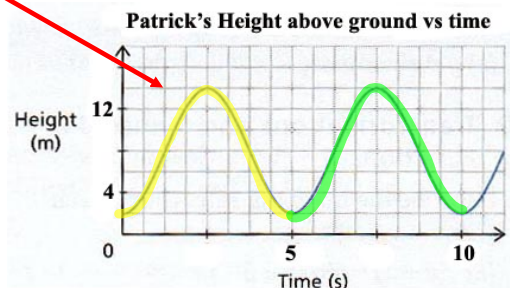
Example of periodic behavior



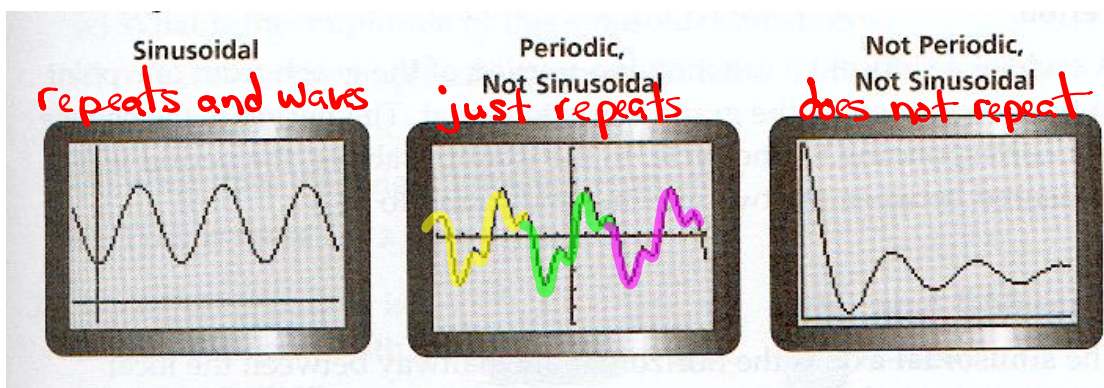
Sinusoidal Function: A periodic function that looks like waves, where any portion of the curve can be translated onto another portion of the curve.

(Repeats and looks like a smooth wave).

Example of sinusoidal behavior



These illustrations should summarize periodic and sinusoidal...

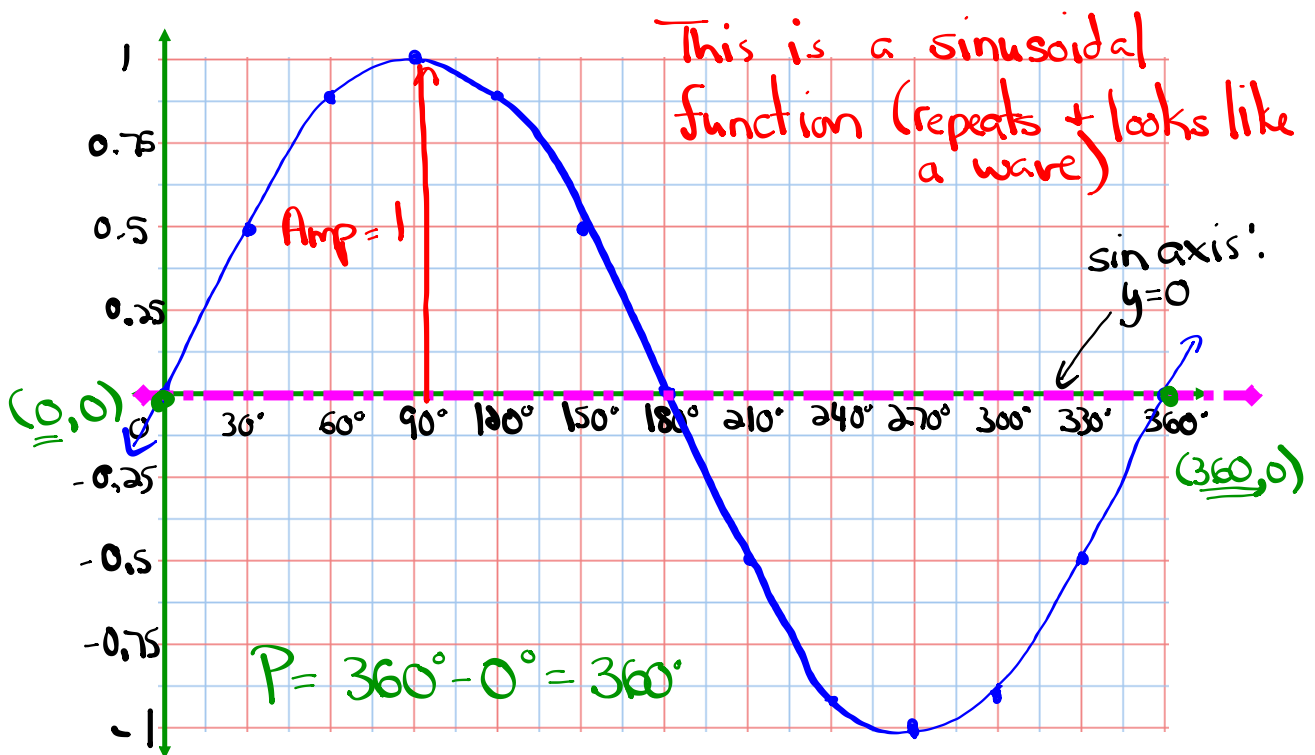


Let's examine the graph of $y = \sin \theta$

$$y = \sin x$$

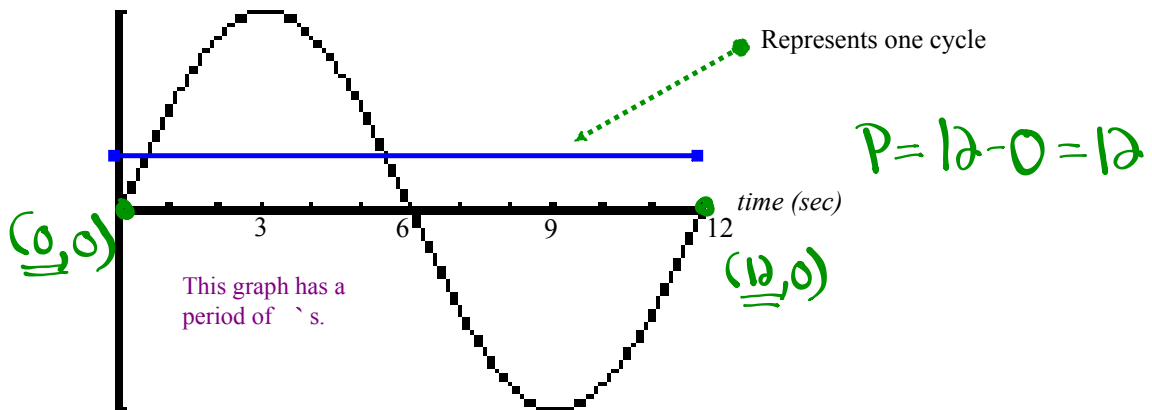
θ	0	30	60	90	120	150	180	210	240	270	300	330	360
y	0	0.5	0.87	1	0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0

Now plot the above points...

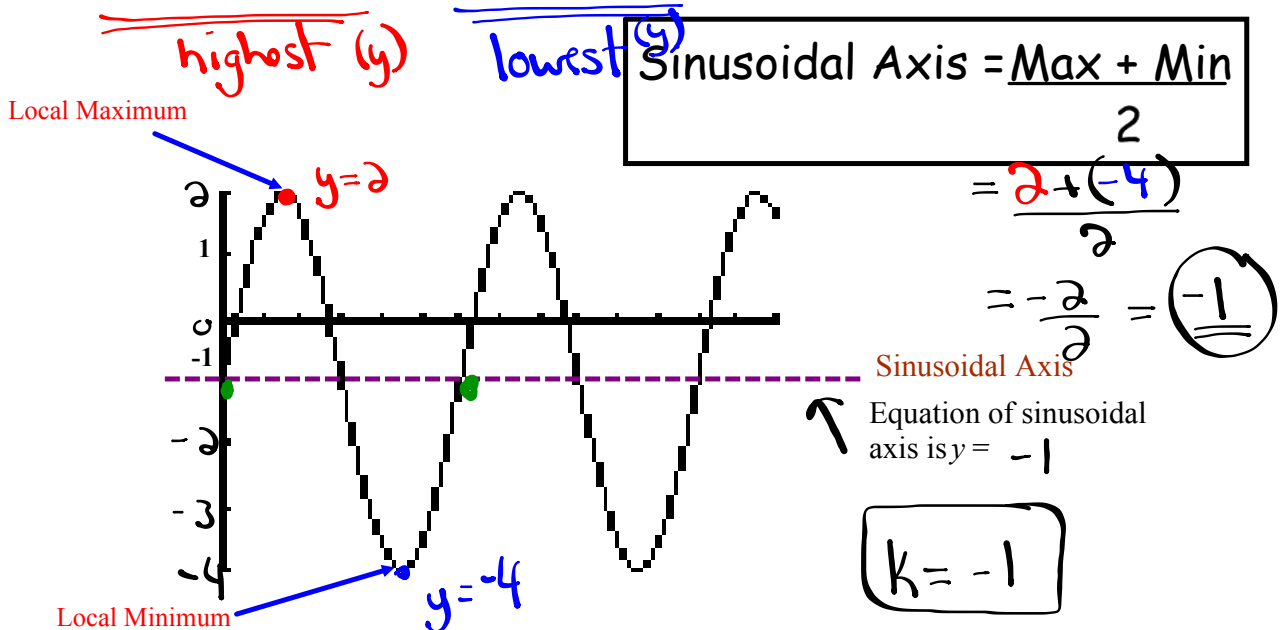


Vocabulary of Sinusoidal Functions

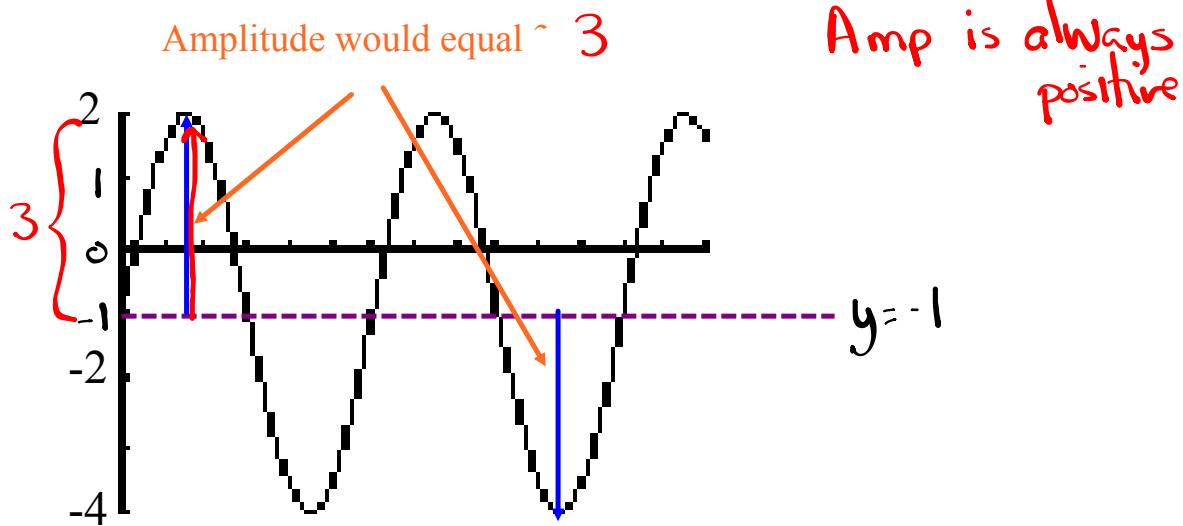
I. **Period:** The change in x corresponding to one cycle. *(one repetition)*



II. **Sinusoidal Axis:** The horizontal line halfway between the local maximum and local minimum.

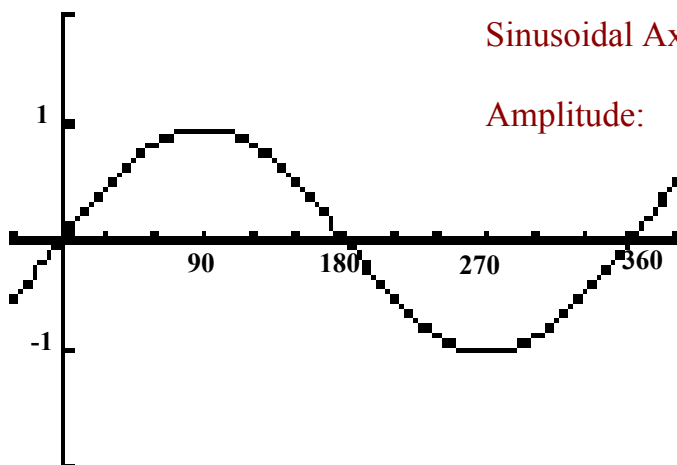


III. **Amplitude:** The vertical distance from the sinusoidal axis to a local maximum or local minimum. *Amplitude = |a|*



Summarize...

Here is the graph of $y = \sin \theta$



Period :

Sinusoidal Axis:

Amplitude:

What about $y = \cos \theta$?

$y = \cos x$

Complete the table of values and sketch below

θ	θ	30	60	90	120	150	180	210	240	270	300	330	360
y		0.87	0.5	0	-0.5	-0.87	-1	-0.87	-0.5	0	0.5	0.87	1



Is this a sinusoidal function? *Yes (repeats + looks like waves)*

What about the period, sinusoidal axis, and amplitude?

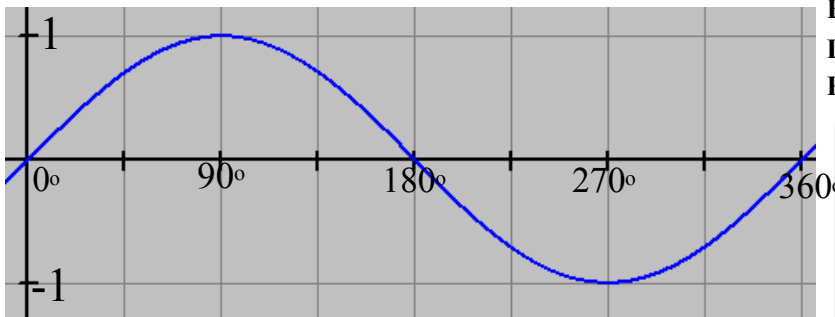
$Period = 360^\circ - 0^\circ = 360^\circ$

$sinusoidal\ axis = \frac{max + min}{2} = \frac{1 + (-1)}{2} = \frac{0}{2} = 0 \ (y=0)$

$Amplitude = 1$

Basic Trig Graphs

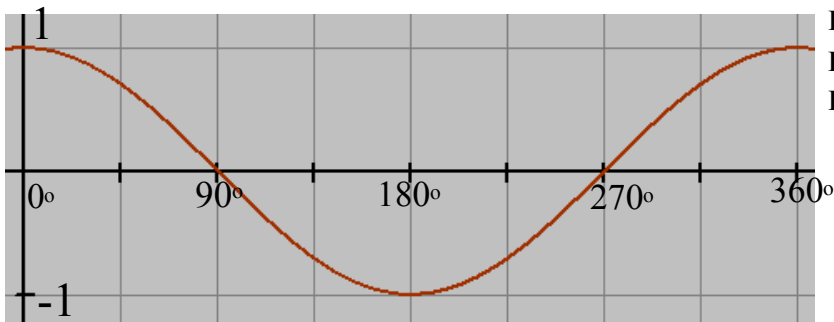
$$y = \sin \theta$$



Period = 360°
 Amplitude = 1
 Eq'n of Sinusoidal Axis: $y = 0$
 Domain: $\{\theta \in \mathbf{R}\}$
 Range: $\{-1 \leq y \leq 1\}$

θ	y
0°	0
90°	1
180°	0
270°	-1
360°	0

$$y = \cos \theta$$



Period = 360°
 Amplitude = 1
 Eq'n of Sinusoidal Axis: $y = 0$
 Domain: $\{\theta \in \mathbf{R}\}$
 Range: $\{-1 \leq y \leq 1\}$

θ	y
0°	1
90°	0
180°	-1
270°	0
360°	1

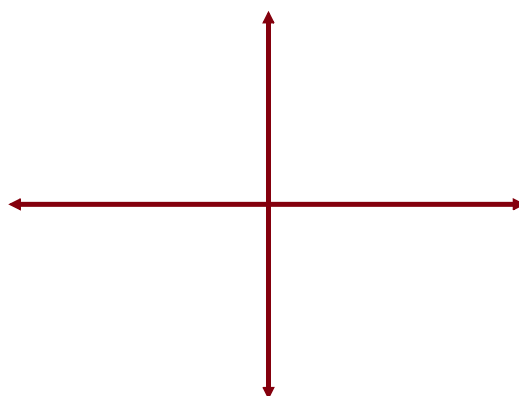
Transformations of the Sinusoidal Function

Recall...

$$y = -2(x-3)^2 + 4$$

Vertex \Rightarrow

Sketch \Rightarrow



Now, let's look at a sinusoidal function...

$$y = -2 \sin[3(\theta - 60^\circ)] - 1$$

Equations in Standard Form

$$y = a \sin[b(x - \underline{h})] + \underline{k}$$

$$y = a \sin[b(x - \underline{c})] + \underline{d} \quad \text{or} \quad y = a \cos[b(x - h)] + k$$

$|a|$ = **Amplitude** → influences how tall the sine curve is.

$$b = \frac{360^\circ}{P} \rightarrow \text{influences how often the pattern repeats. } b = \frac{2\pi}{P}$$

Period

h = **Horizontal Translation** → Influences how far to the left or the right that the graph will shift. **(Phase Shift)**

- If h is positive → Shift Left
 - If h is negative → Shift Right
- } **Inside brackets**

k = **Vertical Translation** → influences how far up and down the graph will shift.

- If k is positive → Shift Up
- If k is negative → Shift Down
- equal to the sinusoidal axis:

↳ equation of sinusoidal axis: $y = k$

Example:

$$2y + 5 = -6 \sin\left(\frac{1}{3}x - 30^\circ\right) - 3 \quad \text{(Subtract 5 from both sides)}$$

$$\frac{2y}{2} = \frac{-6 \sin\left(\frac{1}{3}x - 30^\circ\right) - 8}{2} \quad \text{(Divide by 2)}$$

$$y = -3 \sin\left(\frac{1}{3}x - 30^\circ\right) - 4 \quad \text{(Factor out a } \frac{1}{3}\text{)}$$

$$y = \underline{-3} \sin\left(\underline{\frac{1}{3}}(x - \underline{90^\circ})\right) - \underline{4}$$

$$a = -3 \quad b = \frac{1}{3} \quad h = 90^\circ \quad k = -4$$

$$\text{Amp} = 3 \quad P = \frac{360^\circ}{\frac{1}{3}} = 1080^\circ \quad \text{equation of sinusoidal axis: } y = -4$$

Homework

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$$\text{ex: } 2y - 5 = -4\cos[3x - 90^\circ] - 7$$

$$\frac{\partial y}{\partial} = \frac{-4\cos[3x - 90^\circ] - 2}{\partial}$$

$$y = -2\cos[3x - 90^\circ] - 1 \quad \text{Factor}$$

$$y = -2\cos[3(x - 30^\circ)] - 1$$

$$y = -2\cos[3(x - 30^\circ)] - 1$$

$a = -2$ (Amp = 2) \rightarrow A vertical stretch by a factor of 2
A vertical reflection in the x-axis

$b = 3 \rightarrow$ A horizontal stretch by a factor of $\frac{1}{3}$

$$P = \frac{360^\circ}{b} = \frac{360^\circ}{3} = 120^\circ$$

$h = 30^\circ \rightarrow$ shift 30° right (Phase Shift = 30°)

$k = -1 \rightarrow$ Shift 1 unit down

Equation of sinusoidal axis: $y = -1$

$$b = \frac{360^\circ}{P} \text{ or } \frac{2\pi}{P}$$

$$P = \frac{360^\circ}{b} \text{ or } \frac{2\pi}{b}$$

$$h) \frac{1}{2}(y+2) = 3\cos(x-90^\circ) + 0 \quad (a+k)$$

$$y+2 = 6\cos(x-90^\circ)$$

$$y = \underline{6}\cos(x-\underline{90^\circ}) - \underline{2}$$

$$a = 6$$

$$h = 90^\circ$$

equation of sinusoidal axis: $y = -2$

$$b = 1$$

$$k = -2$$

$$P = \frac{360^\circ}{b} = \frac{360^\circ}{1} = 360^\circ$$

Sketching Sinusoidal Functions using Transformations

Development of a standard form for sinusoidal functions...

$$\text{Standard Form} \longrightarrow f(\theta) = a \sin k(\theta - c) + d$$

1. Reflection: If $a < 0$ the graph will be reflected in the x -axis.
2. Amplitude: The amplitude of the graph will be equal to $|a|$.
3. Period: The period of the graph will be equal to $\frac{360^\circ}{k}$
4. Horizontal Phase Shift: The graph will shift " c " units to the right. (Think Opposite)
5. Vertical Translation: The graph will shift " d " units up.

$$\text{Mapping Notation: } (x, y) \rightarrow \left(\frac{1}{k} \theta + c, ay + d \right)$$

Transformations of Sinusoidal Functions



Example: $f(\theta) = -2 \sin 3(\theta + 30^\circ) - 2$

Domain	
Range	
Reflection	
Amplitude	
Horizontal Phase Shift	
Vertical Translation	
Period	

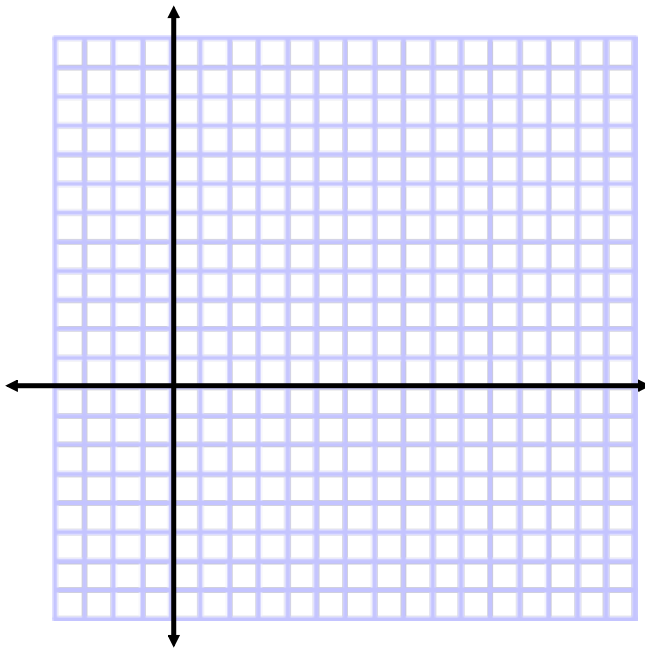
EXAMPLE #1

Now let's sketch a graph of $f(\theta) = -2 \sin 3(\theta + 30^\circ) - 2$

" THINK: RST "

Sketching using transformations:

- *Apply the reflections and stretches first*
- *Apply phase shift and vertical translation second*

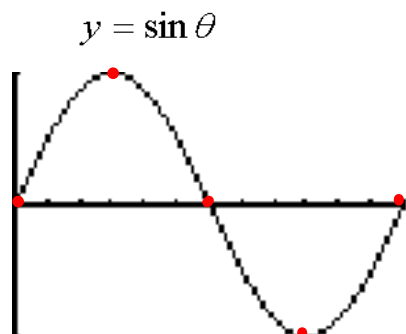


DOMAIN	
RANGE	
AMPLITUDE	
PERIOD	
PHASE SHIFT	
VERTICAL TRANSLATION	
EQUATION OF SINUSOIDAL AXIS	

Check our graph using a graphing calculator

This time we will graph the same function using a mapping:

$$f(\theta) = -2 \sin 3(\theta + 30^\circ) - 2$$

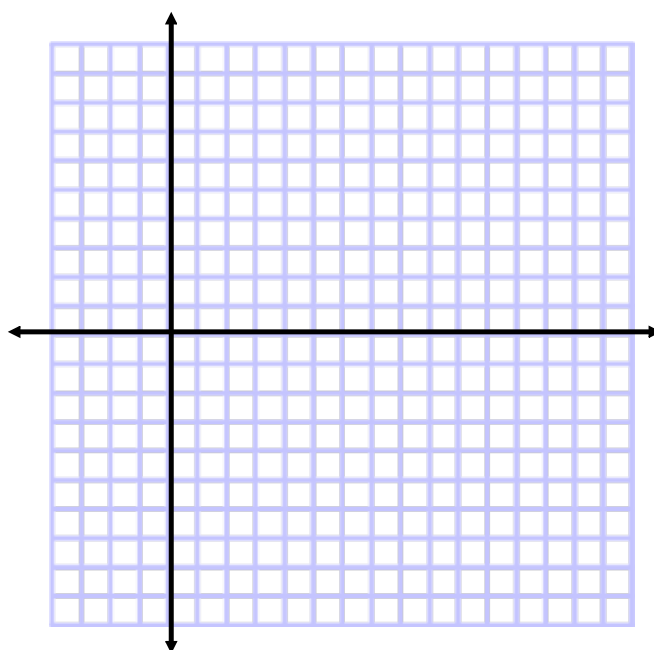


Mapping:

θ	y
0	
90	
180	
270	
360	

New points after mapping \longrightarrow

θ	y



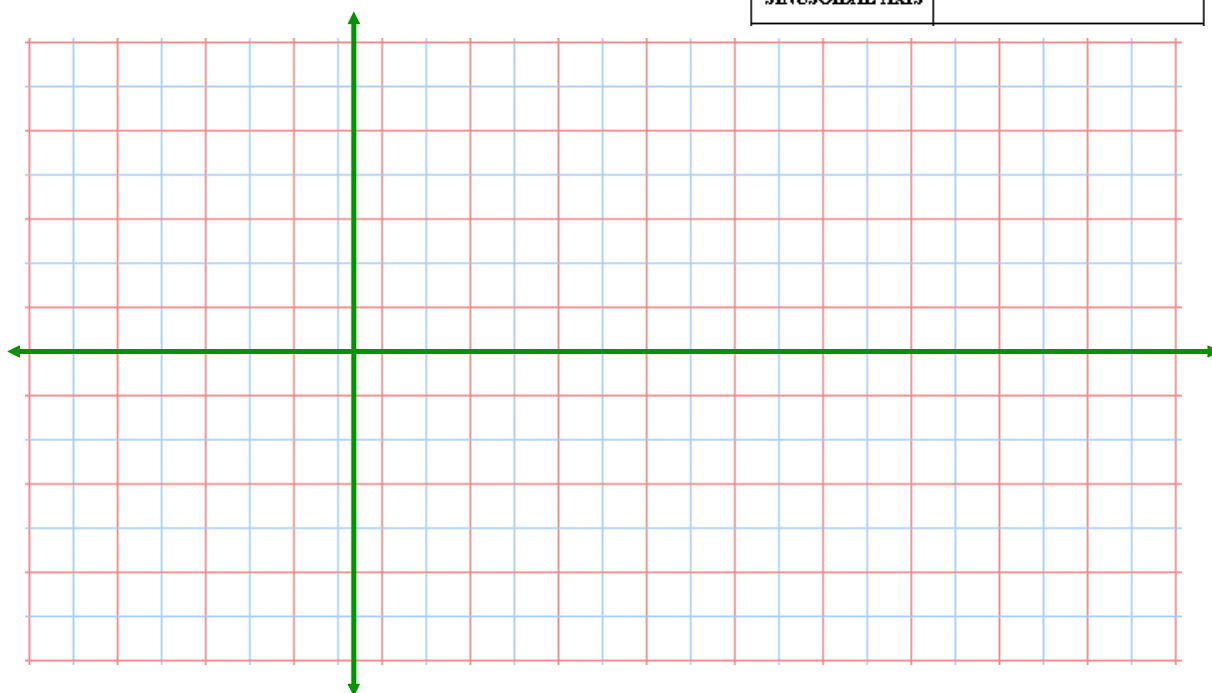
EXAMPLE #2

Now let's sketch a graph of $y = 3 \cos[2(\theta - 135^\circ)] + 2$

Sketching using transformations:

- *Apply the reflections and stretches first*
- *Apply phase shift and vertical translation second*

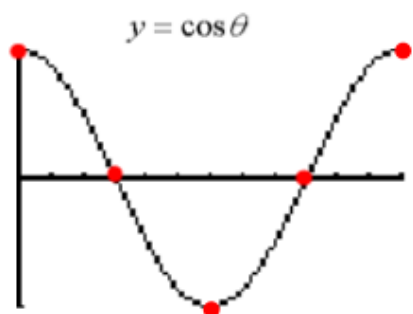
DOMAIN	
RANGE	
AMPLITUDE	
PERIOD	
PHASE SHIFT	
VERTICAL TRANSLATION	
EQUATION OF SINUSOIDAL AXIS	



Check our graph using a graphing calculator

This time we will graph the same function using a mapping:

$$y = 3 \cos[2(\theta - 135^\circ)] + 2$$

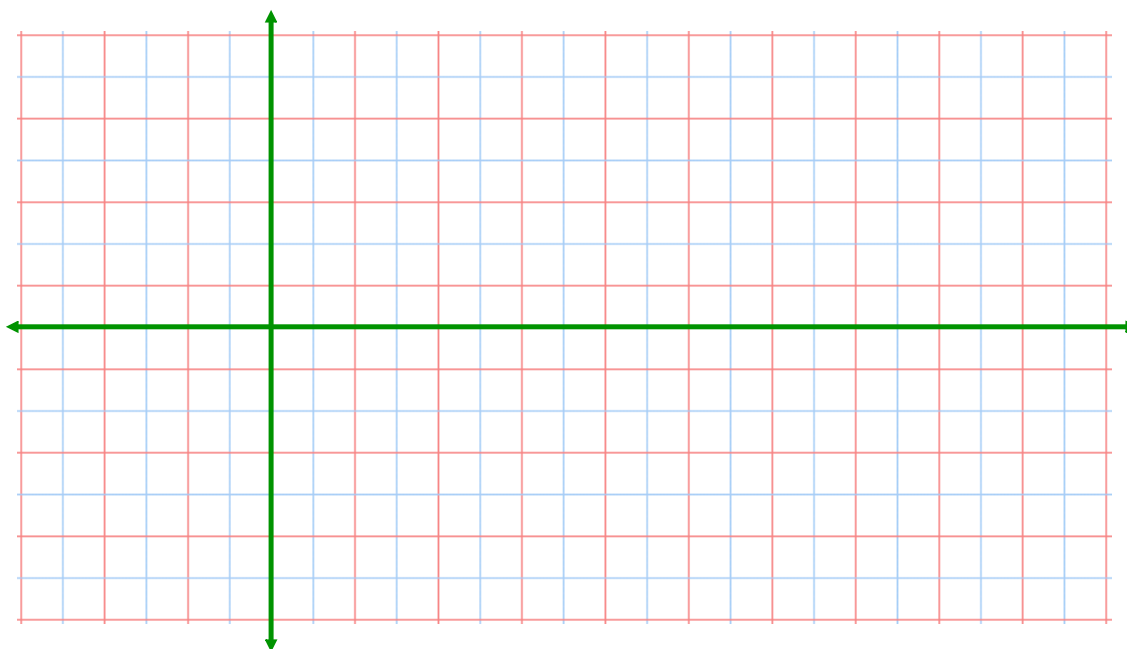


Mapping:

θ	y
0	
90	
180	
270	
360	

New points after mapping \longrightarrow

θ	y



Attachments

worksheet-sketching in radian measure.doc

Worksheet - Finding the Equation.doc

Worksheet - Sketching Trigonometric Functions.doc

Worksheet Solns - Sketching Sinusoidal Relations.doc

Worksheet - Sketching Sinusoidal relations (sept06).pdf

Bonus Soln - Fox Population.doc

Worksheet Solns - Applications of Sinusoidal Relations.doc

Review - Practice Test for Sinusoidal Functions.doc

Review - Trigonometric Functions(3)(4).doc