Developing Trigonometric Functions from Properties...

Develop a trigonometric function that fits the following description...

- **Models a sine function**
- **Period is 120°**
- **Graph is reflected in x-axis**
- **Wave has a range of -8** $\leq y \leq 2$
- **Graph has a phase shift of 60° right**
- **Graph has a vertical translation of 3 units down**

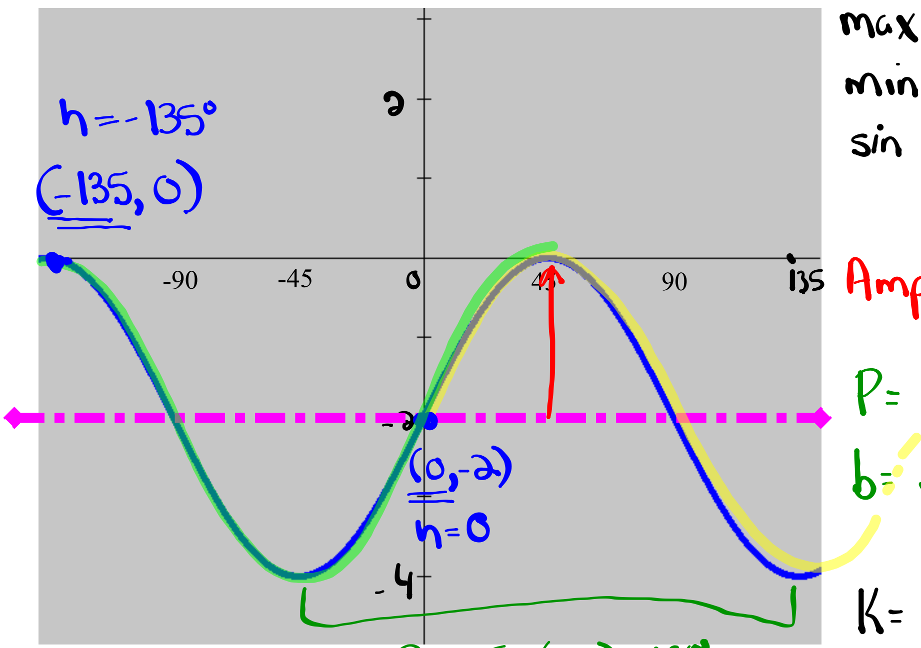
...Now we must learn how to identify all of the above information from a graph.

Developing the Equation of a Sinusoidal Function

STEPS:

1) Identify & label the sinusoidal axis.
2) Determine the amplitude, period & vertical translation.
3) Pick a trig function & determine the corresponding phase shift.

- the choices are: positive sine, positive cosine, negative sine, negative cosine



P = 135-(-45) = 180°

max = 0

min = -4

$\text{sin axis} = \frac{0+(-4)}{2} = -2$

Amp = 2 $a = \pm 2$

P = 180°

$b = \frac{360°}{180°} = 2$

k = -2

$y = \sin x \ (h = 0°)$

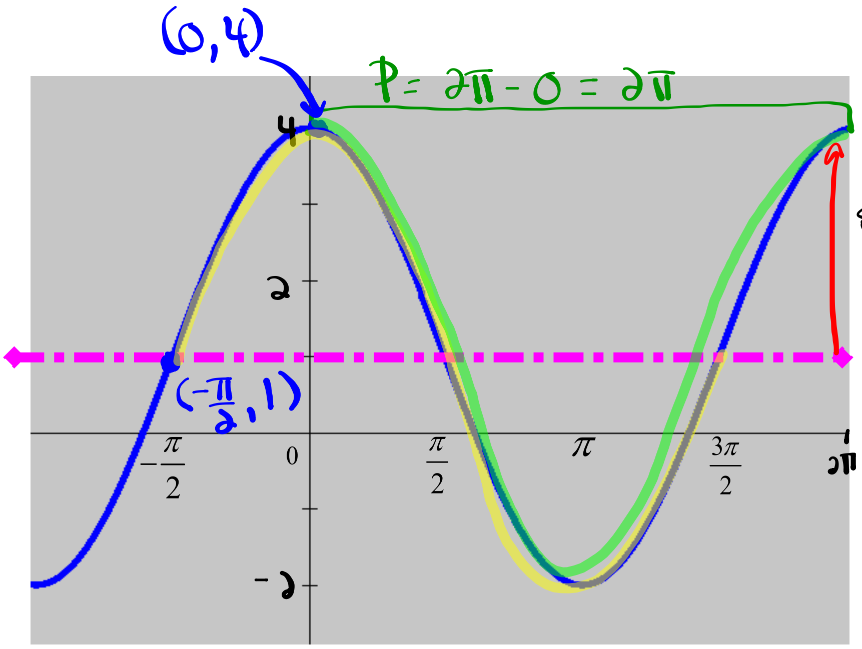
$y = 2\sin[2(x-0°)] - 2$

$y = \cos x \ (h = -135)$

$y = 2\cos[2(x+135°)] - 2$

Finding an Equation from a Graph:

Determine a sine and a cosine equation for this graph



max = 4

min = −2

sin axis = $\frac{4+(-2)}{2} = 1$

Amp = 3 a = ±3

P = 2π

b = 1

k = 1

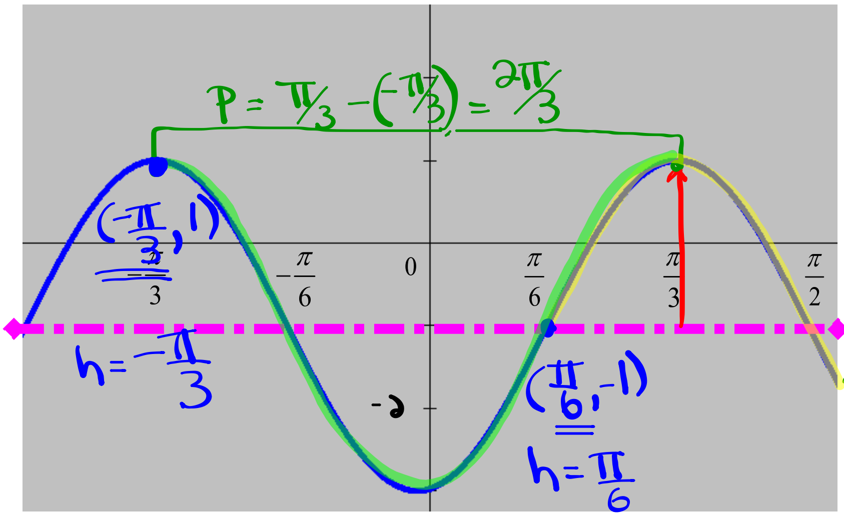
$y = \sin x \; (h = -\frac{\pi}{2})$

$y = 3\sin[1(x + \frac{\pi}{2})] + 1$

$y = \cos x \; (h = 0)$

$y = 3\cos[1(x - 0)] + 1$

Determine a sine and a cosine equation for this graph $2\pi \div \frac{2\pi}{3}$



max = 1

min = -3

sin axis: $\frac{1+(-3)}{2} = -1$

Amp = 2 a = ±2

$P = \frac{2\pi}{3}$

$b = 2\pi \times \frac{3}{2\pi} = 3$

k = -1

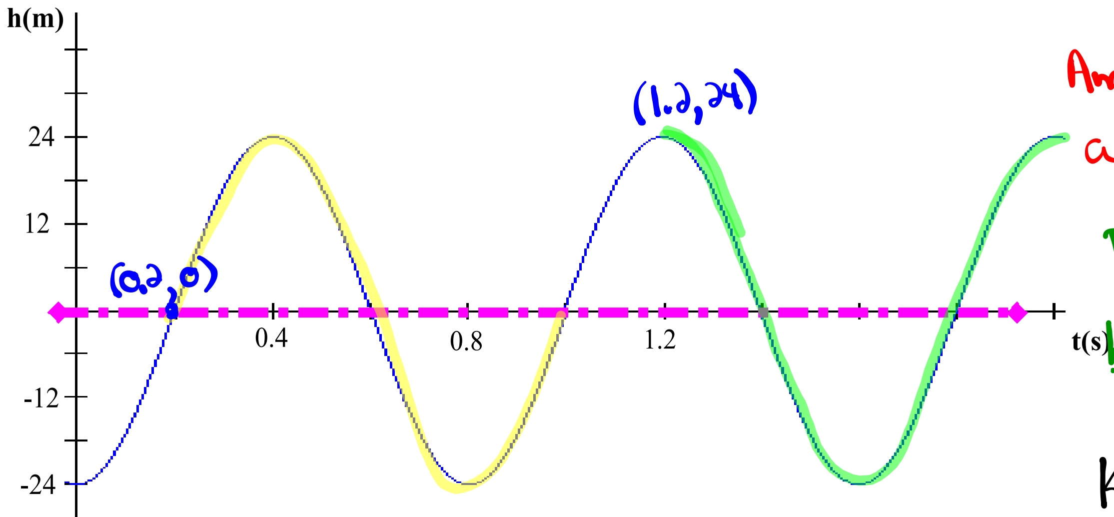
$y = \sin x \; (h = \frac{\pi}{6})$

$y = 2\sin[3(x - \frac{\pi}{6})] - 1$

$y = \cos x \; (h = -\frac{\pi}{3})$

$y = 2\cos[3(x + \frac{\pi}{3})] - 1$

Determine a sine and a cosine equation for this graph



Amp = 24

a = ± 24

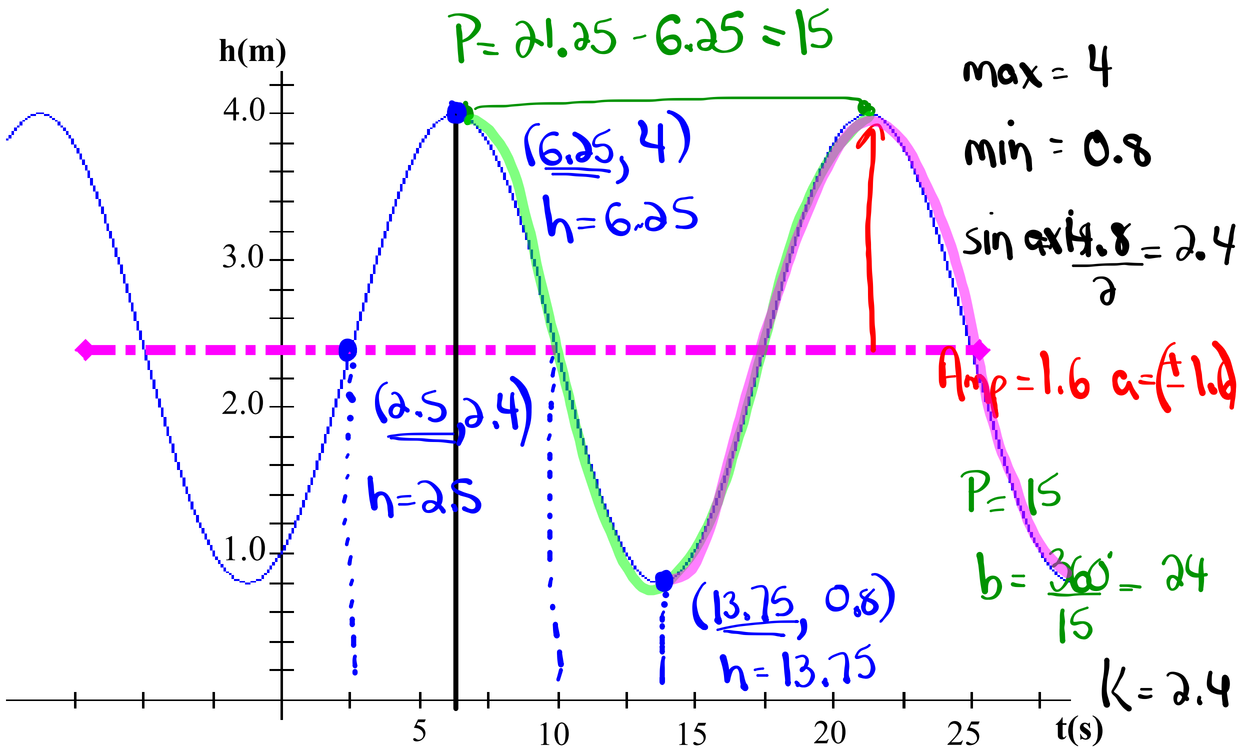
P = 0.8

$b = \frac{360}{0.8} = 450$

k = 0

$y = 24\sin[450(x - 0.2)]$ | $y = 24\cos[450(x - 1.2)]$

Find 4 equations to describe the graph.



$$y = 1.6 \sin[24(x - 2.5)] + 2.4$$

$$y = 1.6 \cos[24(x - 6.25)] + 2.4$$

$$y = -1.6 \sin[24(x - 10)] + 2.4$$

$$y = -1.6 \cos[24(x - 13.75)] + 2.4$$

EXTRA PRACTICE...

Worksheet: #28 a) - f)

Applications of Sinusoidal Relations

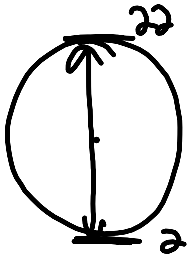
Strategy: (1) Translate ALL key pieces of information from the problem.
(2) Draw a sketch with ALL key points identified.
(3) Develop an equation that models the problem.
(4) Answer the question(s) being asked.

CHECK??? Do the numbers make sense?

* Radius = Amp.

* Count by $\frac{P}{4}$ on x-axis

* min + diameter = max
  min + radius = sin axis



r=10
min=2
max=?

* From max to min or min to max is half the period

Ex: max @ 10s P=10s
min @ 15s
max @ 20s

Applications of Sinusoidal Functions

A carnival Ferris wheel with a radius of 14 m makes one complete revolution every 16 seconds. The bottom of the wheel is 1.5 m above the ground. If a person is at the top of the wheel when a stop watch is started, determine how high above the ground that person will be after 1 minute and 7 seconds? Sketch one period of this function.

Amp = 14   P = 16   min = 1.5   k = 15.5

a = ± 14 $b = \frac{360}{16} = 22.5$ max = min + diameter = 1.5 + 28 = 29.5   h = 0

y = cos x

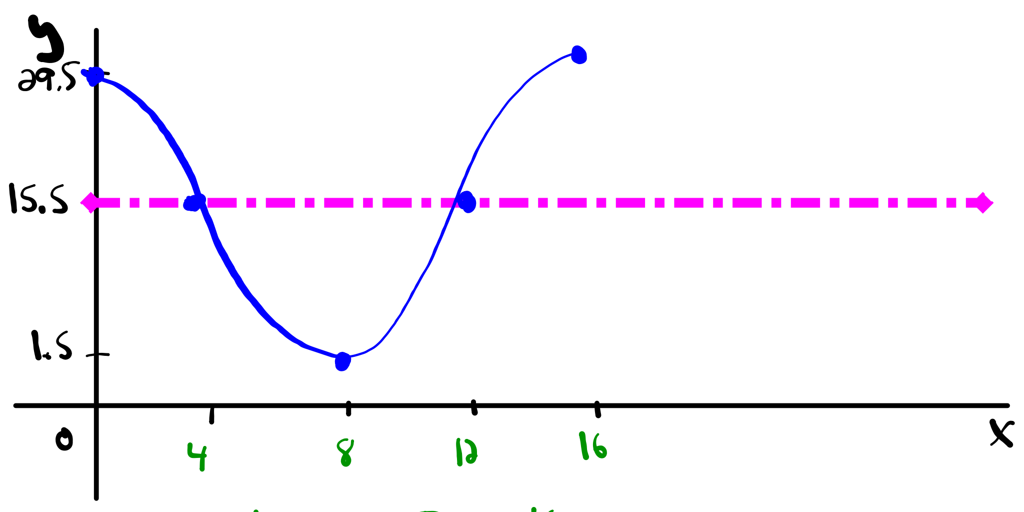
sin axis = min + radius = 1.5 + 14 = 15.5

equation: $y = 14\cos[22.5(x)] + 15.5$

x = 1 min and 7 sec = 67 sec → Find y

$y = 14\cos[22.5(67)] + 15.5$

y = 20.86 m



Count by $\frac{P}{4} = \frac{16}{4} = 4$s

Ocean Tides

The alternating half-daily cycles of the rise and fall of the ocean are called tides. Tides in one section of the Bay of Fundy caused the water level to rise 6.5m above mean sea-level and to drop 6.5m below. The tide completes one cycle every 12 h. Assuming the height of water with respect to mean sea-level to be modelled by a sine function,

(a) draw the graph for a the motion of the tides for one complete day;
(b) find an equation for the graph in (a).

Amp = 6.5
$a = \pm 6.5$

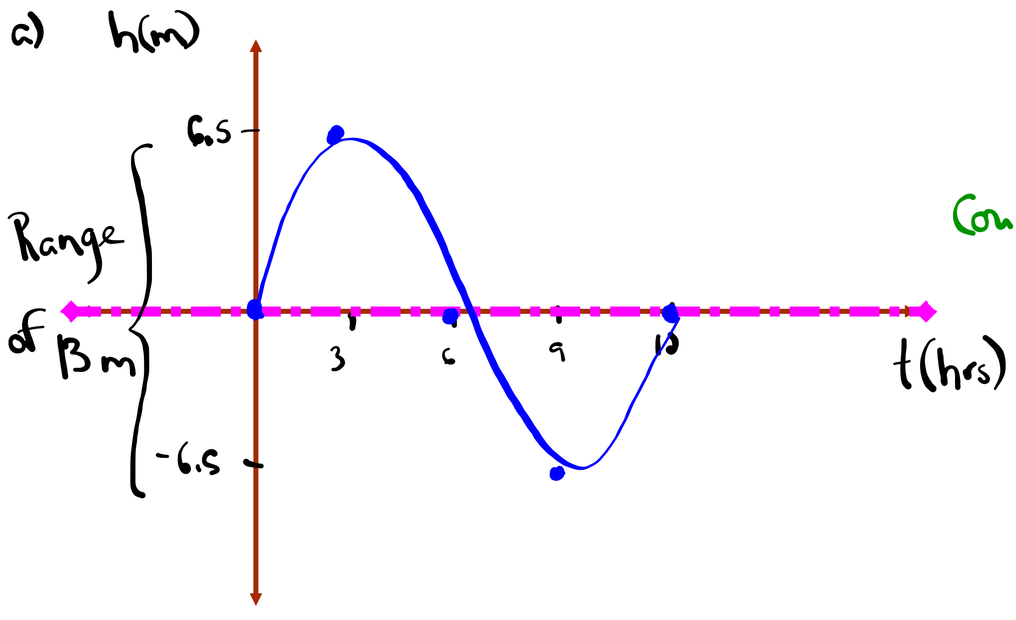
max = 6.5
min = -6.5

sin axis = $\frac{6.5 + (-6.5)}{2} = 0$
k = 0

P = 12 h
$b = \frac{360}{12} = 30$

h = 0

a)

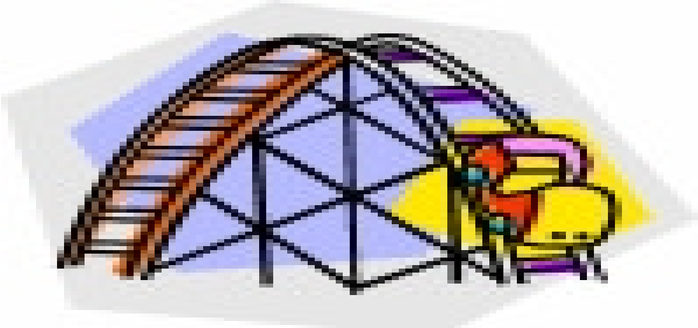


Count by $\frac{12}{4} = 3$ on x axis

b) $y = 6.5\sin[30(x - 0)] + 0$

$y = 6.5\sin[30(x)]$

Roller Coaster



John climbs on a roller coaster at Six Flags Amusement Park. An observer starts a stopwatch and observes that John is at a maximum height of 12 m at t = 13.2 s. At t = 14.6 s, John reaches a minimum height of 4 m.

a) Sketch a graph of the function.

max @ 13.2
min @ 14.6

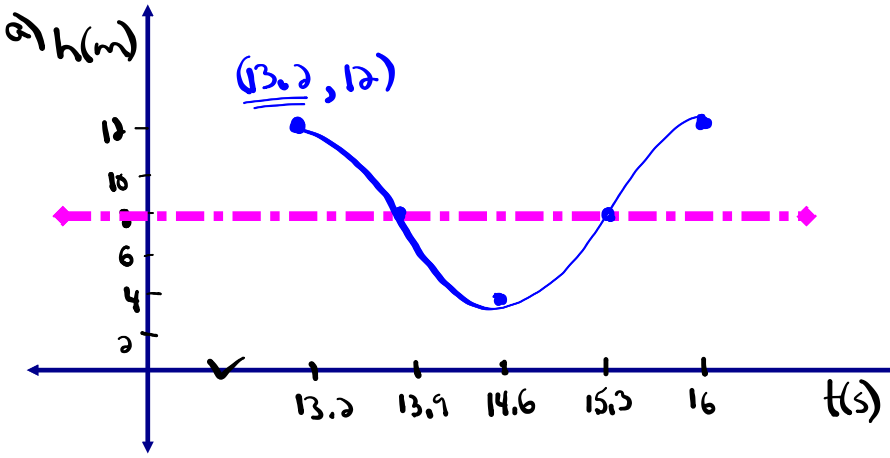
max = 12m
min = 4m

b) Find an equation that expresses John's height in terms of time.

c) How high is John above the ground at t = 20.8 s?

sin axis = $\frac{16}{2}$ = 8

a)



K = 8

Amp = 4

a = ± 4

P = 2(1.4) = 2.8

b = $\frac{360}{2.8}$ = 128.57

h = 13.2

count by $\frac{2.8}{4}$ = 0.7

b) $y = 4\cos[128.57(x - 13.2)] + 8$

c) when x = 20.8 s

$y = 4\cos[128.57(20.8 - 13.2)] + 8$

$\boxed{y = 7.1\text{ m}}$

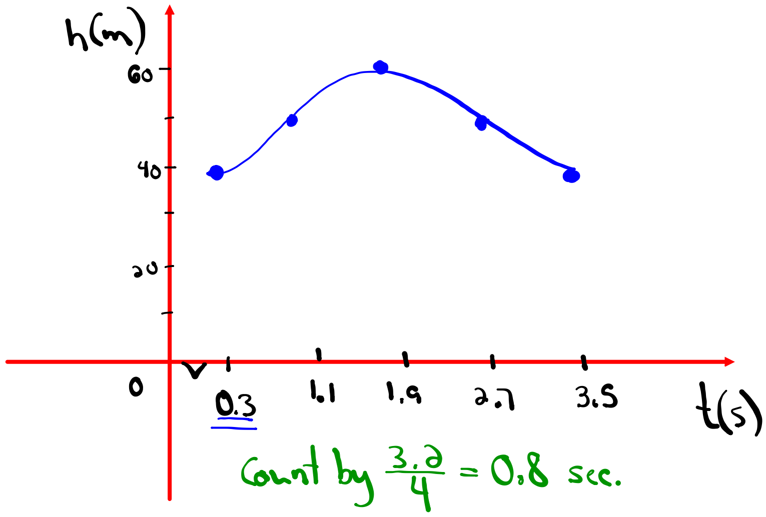
Spring Problem

A weight attached to a long spring is being bounced up and down by an electric motor. As it bounces, its distance from the floor varies periodically with time. You start a stopwatch. When the stopwatch reads 0.3 seconds, the weight reaches its first low point 40 cm above the ground. The next high point, 60 cm above the ground, occurs at 1.9 seconds.

a) Sketch a graph of the function. From max to min took $1.9 - 0.3 = 1.6$ s

b) Write an equation expressing the distance above the ground in terms of the numbers of seconds the stopwatch reads. $y = -10\cos[112.5(x-0.3)] + 50$

c) When is the mass 43.75m high?



Count by $\frac{3.2}{4} = 0.8$ sec.

max = 60
min = 40
k = 50
Amp = 10
a = ± 10
$P = 2(1.6) = 3.2$
$b = \frac{360}{3.2} = 112.5$
h = 0.3

c) (i) $y = -10\cos[112.5(x-0.3)] + 50$

when $x = 17.2$

$y = -10\cos[112.5(17.2-0.3)] + 50$

$y = 51.95$ m

(ii) $y = -10\cos[112.5(x-0.3)] + 50$

when $y = 43.75$m

$43.75 = -10\cos[112.5(x-0.3)] + 50$ (−50 both sides)

$\frac{-6.25}{-10} = \frac{-10\cos[112.5(x-0.3)]}{-10}$

$0.625 = \cos[112.5(x-0.3)]$

$\cos^{-1}(0.625) = 112.5(x-0.3)$

$\frac{51.32}{112.5} = \frac{112.5(x-0.3)}{112.5}$

$0.46 = x - 0.3$ (+0.3 both sides)

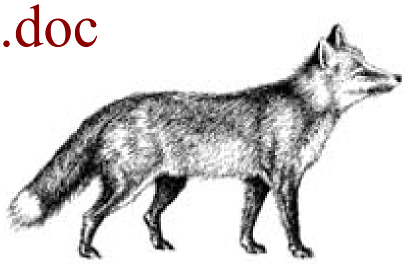
$\boxed{0.76\text{s} = x}$

Biology!

Naturalists find that the populations of some animals varies periodically with time. Records started being taken at t = 0 years. A minimum number, 200 foxes, occurred when t = 2.9 years. The next maximum, 800 foxes, occurred at t = 5.1 years.

Give two different times at which the fox population is 625.

Bonus Soln - Fox Population.doc



Warm Up

1. Determine the range of the trigonometric function $\frac{1}{5}(y+2)=\sin(2\theta+60^\circ)$

[A] $1 \leq y \leq 3$ [B] $-7 \leq y \leq 3$ [C] $-3 \leq y \leq 7$ [D] $-5 \leq y \leq 5$

$5\cdot \quad \frac{1}{5}(y+2) = \sin[2(\theta+30^\circ)] + 0 \quad \cdot 5 \qquad k=-2$

$y+2 \; {}^{-2} = 5\sin[2(\theta+30^\circ)] + 0 \; {}^{-2} \qquad Amp = 5$

$y = 5\sin[2(\theta+30^\circ)] - 2$

$max = -2+5 = 3$

$min = -2-5 = -7$

2. The graph of y = cos x is transformed to a new image according to the mapping rule $(x,y)\rightarrow\left(\frac{2}{3}x+30^\circ, 5y-2\right)$
What is the period of this transformation?

HSF

[A] 240° [B] 540° [C] 72° [D] 1.5°

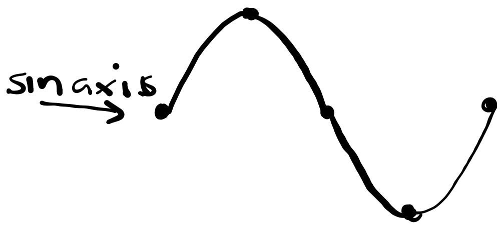
$a=5 \qquad b=\frac{3}{2} \qquad h=30^\circ \qquad k=-2$

$P = 360^\circ \div \frac{3}{2}$

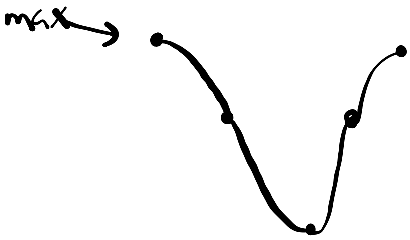
$= 360^\circ \times \frac{2}{3}$

$= \frac{720^\circ}{3}$

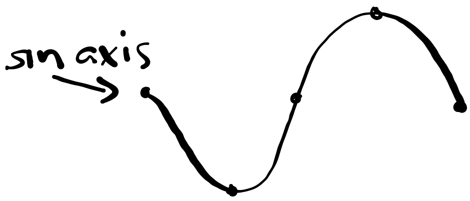
$= 240^\circ$



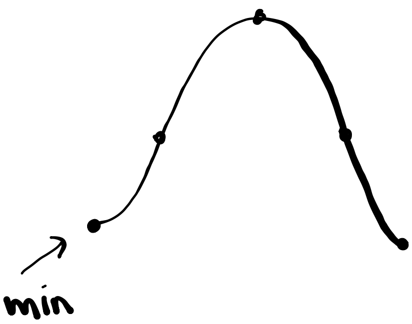
$y = \sin x$



$y = \cos x$



$y = -\sin x$



$y = -\cos x$

Applications of Sinusoidal Functions: Worksheet

Let's look at the detailed solutions...

Worksheet Solns - Applications of Sinusoidal Relations.doc

Check-Up...

Tarzan is swinging back and forth on his grapevine. As he swings, he goes back and forth across the riverbank, going alternately over land and water. Jane decides to model mathematically his motion and starts her stopwatch. Let t be the number of seconds the stopwatch reads and let y be the number of feet Tarzan is from the riverbank. Assume that y varies sinusoidally with t, and that y is positive when
→ Tarzan is over water and negative when he is over land. Jane finds that when t = 2.8 seconds, Tarzan is at one end of his swing, 23 feet from the riverbank, over the water. She finds when t = 6.3 seconds he reaches the other end of his swing and is situated 17 feet from the riverbank, however this time over land.

(a) Where was Tarzan when Jane started the stopwatch? at time zero (t=0)

(b) Provide three instances when Tarzan was located at a position 14 feet from the riverbank, over the water. (y = 14)

Given:
max: 23 ft
min: -17 ft
sin axis: $y = \frac{23+(-17)}{2}$
$y = 3$

$d = 3$
$a = 20$
$P = 2(6.3-2.8) = 7$
$b = \frac{360}{7} = 51.43$
$c = 2.8$

$$y = 20\cos[51.43(t-2.8)]+3$$

a) $y = 20\cos[51.43(0-2.8)]+3$
$y = 20\cos[51.43(-2.8)]+3$
$y = -13.18$ ft

Tarzan was 13.18 ft from the riverbank, over land.

b) $14 - 3 = 20\cos[51.43(t-2.8)]+3 - 3$
$\frac{11}{20} = \frac{20\cos(51.43t-144)}{20}$
$0.55 = \cos(51.43t-144)$
$\cos^{-1}(0.55) = 51.43t - 144$
$56.63 + 144 = 51.43t - 144 + 144$
$\frac{200.63}{51.43} = \frac{51.43t}{51.43}$
$3.9\text{s} = t$
$3.9 + 7 = 10.9\text{s}$
$3.9 + 14 = 17.9\text{s}$

Tarzan is 14 ft over water @ 3.9s, 10.9s and 17.9s

Solve a Trigonometric Equation in Radians

Determine the general solutions for the trigonometric equation $16 = 6\cos\frac{\pi}{6}x + 14$. Express your answers to the nearest hundredth.

Model Electric Power

The electricity coming from power plants into your house is alternating current (AC). This means that the direction of current flowing in a circuit is constantly switching back and forth. In Canada, the current makes 60 complete cycles each second.

The voltage can be modelled as a function of time using the sine function $V = 170 \sin 120\pi t$.

a) What is the period of the current in Canada?

b) Graph the voltage function over two cycles. Explain what the scales on the axes represent.

c) Suppose you want to switch on a heat lamp for an outdoor patio. If the heat lamp requires 110 V to start up, determine the time required for the voltage to first reach 110 V.

Did You Know?

The number of cycles per second of a periodic phenomenon is called the frequency. The hertz (Hz) is the SI unit of frequency. In Canada, the frequency standard for AC is 60 Hz.

Voltages are expressed as root mean square (RMS) voltage. RMS is the square root of the mean of the squares of the values. The RMS voltage is given by $\frac{\text{peak voltage}}{\sqrt{2}}$. What is the RMS voltage for Canada?

What about graphs of other trigonometric functions ???

Graph the Tangent Function

Graph the function $y = \tan\theta$ for $-2\pi \leq \theta \leq 2\pi$. Describe its characteristics.

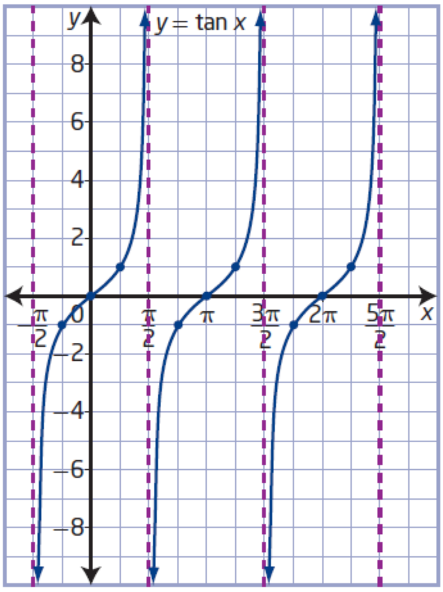
Angle Measure	0°	45°	90°	135°	180°	225°	270°	315°	360°
y-coordinate on Tangent Line									

Key Ideas

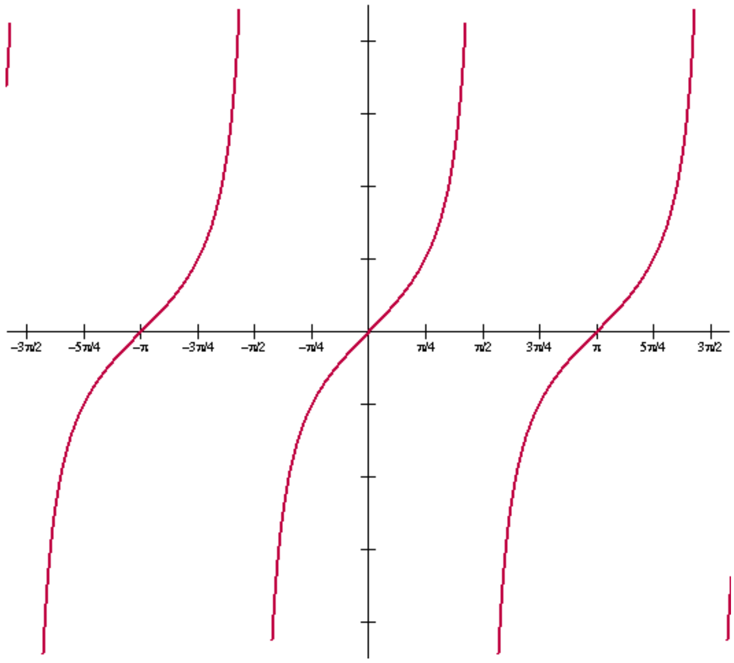
- You can use asymptotes and three points to sketch one cycle of a tangent function. To graph $y = \tan x$, draw one asymptote; draw the points where $y = -1$, $y = 0$, and $y = 1$; and then draw another asymptote.

How can you determine the location of the asymptotes for the function $y = \tan x$?

- The tangent function $y = \tan x$ has the following characteristics:
  - The period is π.
  - The graph has no maximum or minimum values.
  - The range is $\{y \mid y \in R\}$.
  - Vertical asymptotes occur at $x = \frac{\pi}{2} + n\pi$, $n \in I$.
  - The domain is $\left\{x \mid x \neq \frac{\pi}{2} + n\pi, x \in R, n \in I\right\}$.
  - The x-intercepts occur at $x = n\pi$, $n \in I$.
  - The y-intercept is 0.



y = tan θ



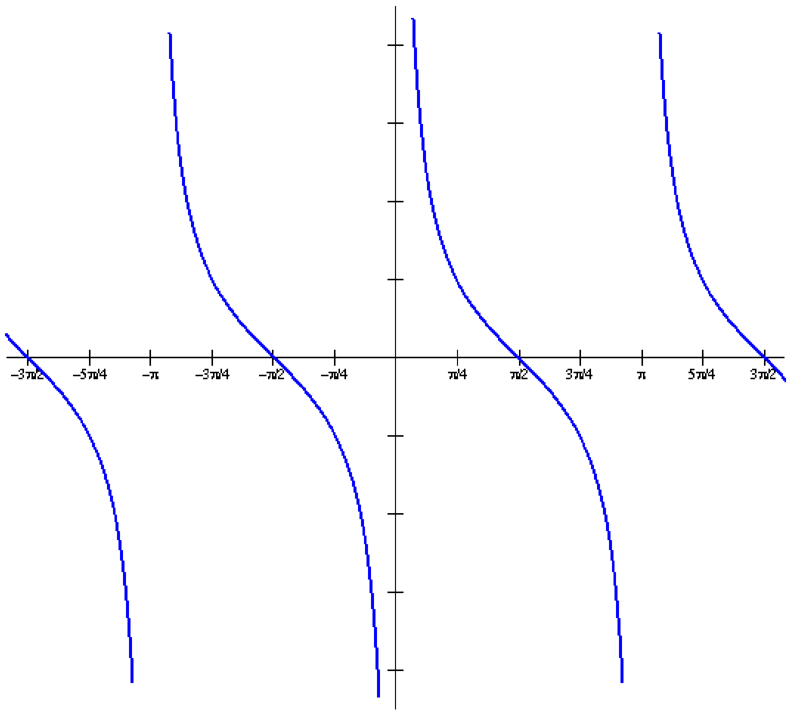
What would the graph of **cot θ** look like?

REMEMBER:

$$\tan x = \frac{1}{\cot x}$$

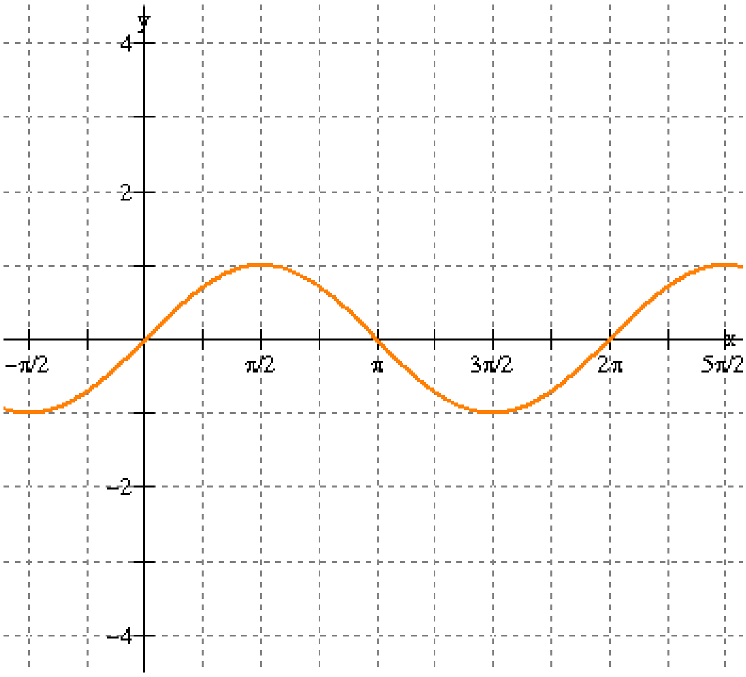
where tan x = 0,
cot x is undefined

y = cot θ



Graphs of Other Trigonometric Functions

y = sin θ



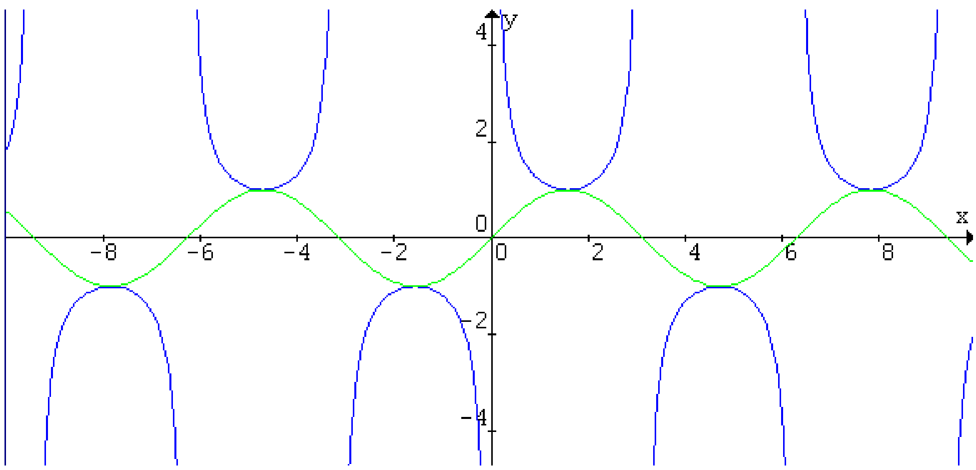
What would the graph of **csc θ** look like?

REMEMBER:

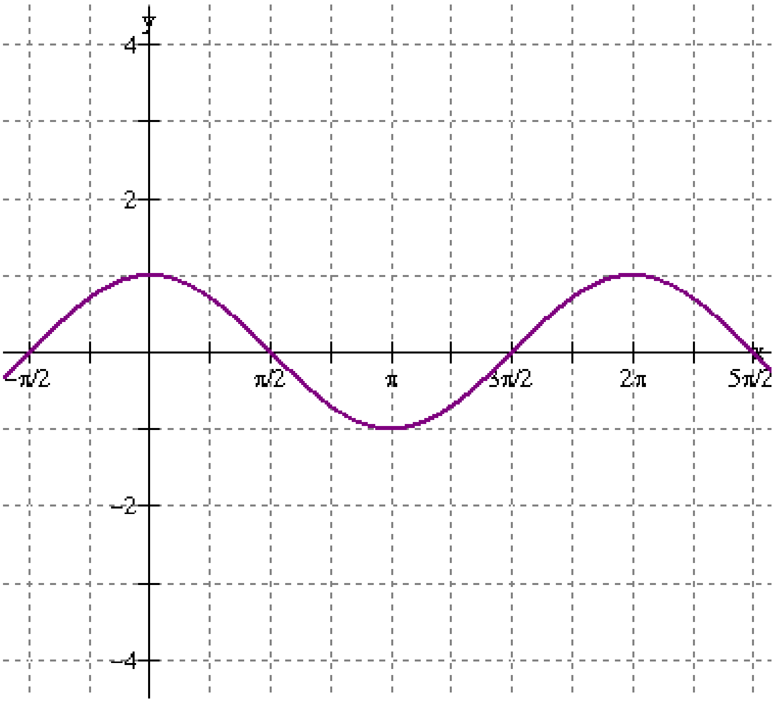
$$\csc\theta = \frac{1}{\sin\theta}$$

where sin x = 0,
csc x is undefined

y = sin x y = csc x



y = cos θ



What would the graph of **sec θ** look like?

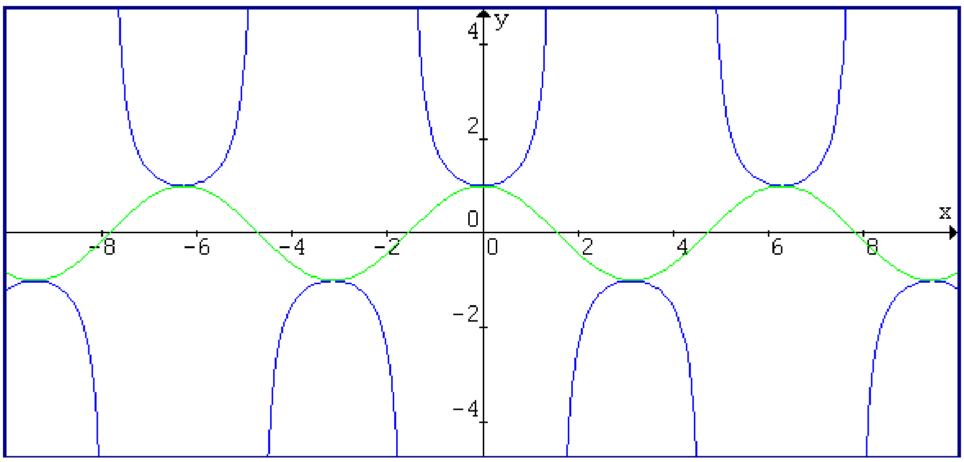
REMEMBER:

$$\sec\theta = \frac{1}{\cos\theta}$$

where cos x = 0,
sec x is undefined

y = cos x

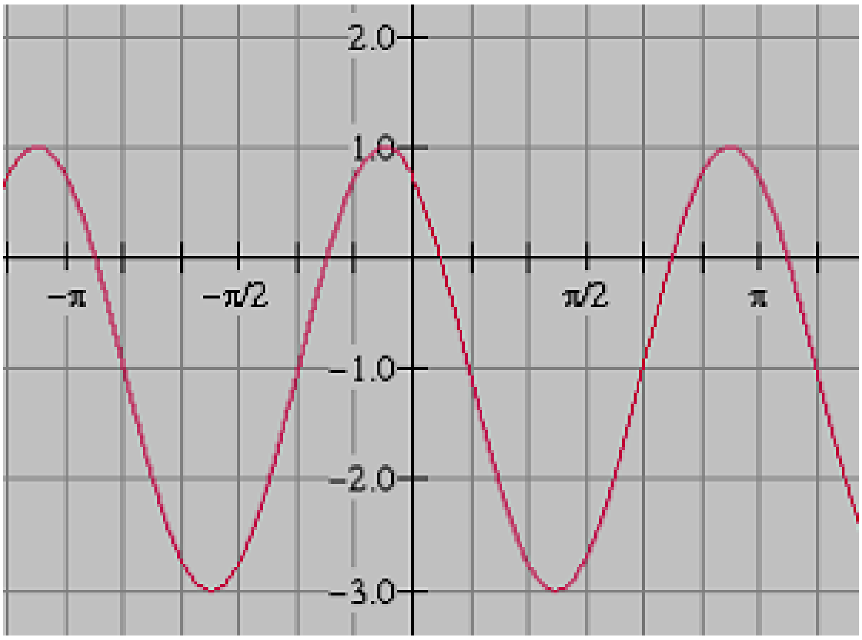
y = sec x



REVIEW - Sketching Trigonometric Functions

- sinusoidal functions
  - properties: domain/range, amplitude, period, phase shift, vertical translation, eq'n of sinusoidal axis, mapping notation.
  - sketching equation in standard form.
- finding the function (both a sine/cosine) given a graph
- solving trigonometric equations where period is not 360
- applications of sinusoidal functions.
  - sketch
  - develop a function
  - use function to answer question
- sketches of all SIX trigonometric ratios

Write both a cosine and sine function to describe the graph shown



Complete the chart shown below and sketch one full cycle of this function

$$-\frac{1}{2}(y+2)=\sin\left(3\theta+\frac{\pi}{8}\right)-2$$

DOMAIN	
RANGE	
AMPLITUDE	
PERIOD	
PHASE SHIFT	
VERTICAL TRANSLATION	
EQUATION OF SINUSOIDAL AXIS	

The Canadian National Historic Windpower Centre, at Etzikom, Alberta, has various styles of windmills on display. The tip of the blade of one windmill reaches its minimum height of 8 m above the ground at a time of 2 s. Its maximum height is 22 m above the ground. The tip of the blade rotates 12 times per minute.

a) Write a sine or a cosine function to model the rotation of the tip of the blade.

b) What is the height of the tip of the blade after 4 s?

c) For how long is the tip of the blade above a height of 17 m in the first 10 s?

PRACTICE TIME...

Review - Practice Test for Sinusoidal Functions.doc

Practice Test Solutions

Part A: Multiple Choice

1. A
2. D
3. A
4. C
5. B
6. D
7. A
8. D
9. B
10. A
11. A (second hand)
12. C
13. A
14. C
15. D
16. D
17. B
18. D
19. A
20. A

Part B: Open Response

1. $-\frac{5}{4}$

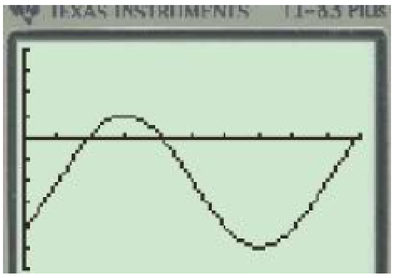
2. (i)

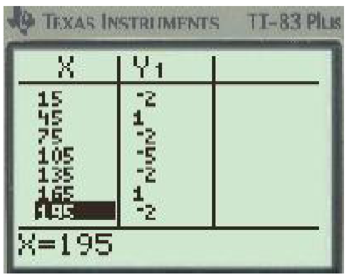
$$y = 3\sin\frac{3}{2}(x - 160°) - 6$$

$$y = 3\cos\frac{3}{2}(x + 20°) - 6$$

(ii) $(x, y) \to \left(\frac{2}{3}x + 160°, 3y - 6\right)$

3.





4. 10.28 m

MORE PRACTICE???

Review - Trigonometric Functions.doc

SOLUTIONS

1. (a) 39° (b) 53°

2. (a) −2 (b) $\frac{7-2\sqrt{3}}{4}$

3. (a) II (b) II

4. (a) −1.2799 (b) −1.0864
 c) 1.2690 (d) 39°
 (e) −5 (f) 25°

5. $\sin\theta = \frac{-\sqrt{5}}{5}$ $\csc\theta = -\sqrt{5}$

 $\cos\theta = \frac{-2\sqrt{5}}{5}$ $\sec\theta = \frac{\sqrt{5}}{2}$

 $\tan\theta = \frac{1}{2}$ $\cot\theta = 2$

6. $\frac{-\sqrt{10}}{2}$

8. Amp = 3
Period = 180°
V.T. = Up 2
P.S. = none
Domain: $0° \le \theta \le 360°$

(b) Amp = 2
Period = 120°
V.T. = Down 2
P.S. = 60° left
Domain: $\theta \in R$

(c) Amp = 2
Period = 720°
V.T. = Up 5
P.S. = none
Domain: $-90 \le \theta \le 360°$
Range: $-3 \le y \le 7$

(d) Amp = 6
Period = 360°
V.T. = None
P.S. = 90° right
Domain: $\theta \in R$
Range: $-6 \le y \le 6$

10. 11.9 m

11. 46.2 cm

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Worksheet - Sketching Trigonometric Functions.doc

Worksheet Solns - Sketching Sinusoidal Relations.doc

Worksheet - Sketching Sinusoidal relations (sept06).pdf

Bonus Soln - Fox Population.doc

Worksheet Solns - Applications of Sinusoidal Relations.doc

Review - Practice Test for Sinusoidal Functions.doc

Review - Trigonometric Functions(3)(4).doc

Sketching Sinusoidal Functions #2.pdf

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Sketching Sinusoidal Functions #3 (Solutions).doc

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