# Confidence Intervals

## Inferential Statistics

- Used to estimate characteristics of a population from characteristics of a sample.

  ie. By choosing a sample and obtaining statistics such as the mean and the standard deviation, you can infer information about the mean and standard deviation of the entire population.

Notation:

| | Sample $S$ | Population $P$ |
|---|---|---|
| MEAN | $\bar{x}$ | $\mu$ |
| STANDARD DEVIATION | $s$ | $\sigma$ |

- Suppose that you are attempting to use a sample mean obtained from a single sample to represent a plausible value of the population mean. A single sample mean is called a *point estimate* because this single number is used as a plausible value for the population mean.

- Suppose that instead of reporting a *point estimate* as the most credible value of a population mean, you report an interval of reasonable values based on the sample data. This interval estimator of the population mean is called the *confidence interval.*

- Associated with each confidence interval is a *confidence level*. This level indicates the level of assurance you have that the resulting confidence interval encloses the unknown population mean.

# Confidence Interval

Formula:

$$\bar{x} \pm Z \frac{\sigma}{\sqrt{n}}$$

Sample mean

Margin of error

# Confidence Interval

Formula:

$$\bar{x} \pm Z \frac{\sigma}{\sqrt{n}}$$

Sample mean

standard deviation

# of items in the sample

Margin of error

To get the margin of error you first need to decide what confidence level you want to use.

90%, 95% or 99%

$$\bar{x} \pm \underbrace{Z \frac{\sigma}{\sqrt{n}}}_{\text{Margin of error}}$$

To get the margin of error you need to decide what confidence level you want to use.

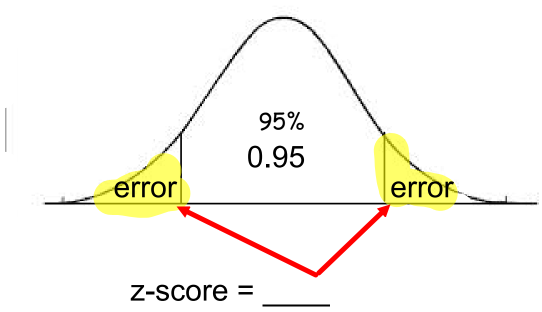
90%, 95% or 99%

90% , z = 1.645

95%, z = 1.96

99% z = 2.56

95% confidence level



Examples:

90% , z = 1.645
95%, z = 1.96
99% z = 2.56

1. Mary collects a sample of size 47 from a known population with a population mean of 230 and a population standard deviation of 23. She finds that the sample mean is 236.

(a) Determine the 90% confidence interval for this sample.

$$\bar{x} \pm Z \frac{\sigma}{\sqrt{n}}$$

$$236 \pm 1.645 \frac{(23)}{\sqrt{47}}$$

$$236 \pm 1.645\,(3.35)$$

$$236 \pm 5.51$$

.36 - 5.51 | 236 + 5.51
230.49 | 241.51

**Confidence Interval:** $230.49 \leq \mu \leq 241.51$

1. Mary collects a sample of size 47 from a known population with a population mean of 230 and a population standard deviation of 23. She finds that the sample mean is 236.

(b) Determine the 95% confidence interval for this sample.

90% , z = 1.645
95%, z = 1.96
99% z = 2.56

$$\bar{x} \pm Z \frac{\sigma}{\sqrt{n}}$$

$$236 \pm 1.96 \frac{(23)}{\sqrt{47}}$$

$$236 \pm 1.96\ (3.35)$$

$$236 \pm 6.57$$

236 - 6.57 = 229.43

236 + 6.57 = 242.57

$$229.43 \le \mu \le 242.57$$

1. Mary collects a sample of size 47 from a known population with a population mean of 230 and a population standard deviation of 23. She finds that the sample mean is 236.

90% , z = 1.645
95%, z = 1.96
99% z = 2.56

(c) Determine the 99% confidence interval for this sample.

$$\bar{x} \pm Z \frac{\sigma}{\sqrt{n}}$$

$$236 \pm 2.56 \frac{(23)}{\sqrt{47}}$$

$$236 \pm 2.56\,(3.35)$$

$$236 \pm 8.58$$

$236 - 8.58$

227.42

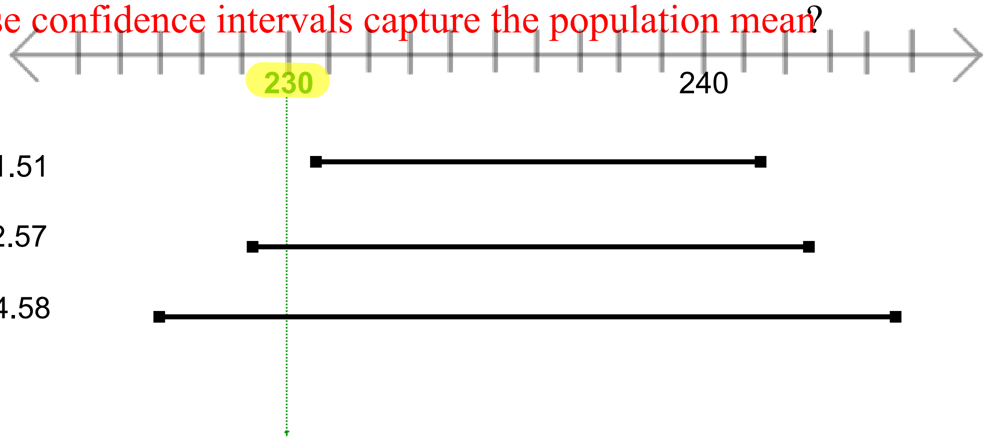
$236 + 8.58$

244.58

$$227.42 \leq \mu \leq 244.58$$

1. Mary collects a sample of size 47 from a known population with a population mean of 230 and a population standard deviation of 23. She finds that the sample mean is 236.

(d) Did all of these confidence intervals capture the population mean?




90% a) $230.49 \le \mu \le 241.51$

95% b) $229.43 \le \mu \le 242.57$

99% c) $227.42 \le \mu \le 244.58$

95% Confidence Interval : $10.3 \leq \mu \leq 12.1$

Translation:

This means that the method that produced this interval from 10.3 to 12.1 has a 0.95 probability of enclosing the population mean.

It DOES NOT mean that there is 0.95 probability that the population mean falls within the interval 10.3 to 12.1.

Check out page 209

Mathematical Modeling Book 2

Questions:

Pg.206 #21

Pg. 211 #'s 28, 29, 31, 32, 33