Higher Derivatives

Since the derivative of a function f is itself a function f', we can take its derivative $(f')'$. The result is a function called the second derivative of f and is denoted by f''

In Leibniz notation we write:

$$f''(x) = \frac{d^2y}{dx^2}$$

Find $y'' = f''(x) = \dfrac{d^2y}{dx^2}$

Examples

$y = x^6$

$y' = 6x^5$

$\boxed{y'' = 30x^4}$

$f(x) = 5x^2 + \sqrt{x} = 5x^2 + x^{1/2}$

$f'(x) = 10x + \frac{1}{2}x^{-1/2}$

$f''(x) = 10 - \frac{1}{4}x^{-3/2} = \boxed{10 - \frac{1}{4\sqrt{x^3}}}$

$f(x) = (2 - x^2)^{10}$

$f'(x) = 10(2-x^2)^9(-2x) = (-20x)(2-x^2)^9$

$f''(x) = -20x(9)(2-x^2)^8(-2x) - 20(2-x^2)^9$

$= 360x^2(2-x^2)^8 - 20(2-x^2)^9$

$= 20(2-x^2)^8\left[18x^2 - (2-x^2)\right]$

$\boxed{= 20(2-x^2)^8(19x^2-2)}$

Since the first derivative of a function can be interpreted either as the slope of a tangent line or as a rate of change, the second derivative can be interpreted as *the rate of change of the slope of the tangent line.*
This idea will be pursued later where the second derivative gives valuable information about the shape of a graph, or when dealing with acceleration.

Higher derivatives can also be defined. The third derivative is the derivative of the second derivative.

$$y''' = f'''(x) = \frac{d^3y}{dx^3}$$

$y^{(4)} \rightarrow$ 4th Derivative

$y^{(5)} \rightarrow$ 5th "

Find the first five derivatives of

$$y = x^4 + 2x^3 - 5x^2 + 3x - 6$$

$y' = 4x^3 + 6x^2 - 10x + 3$

$y'' = 12x^2 + 12x - 10$

$y''' = 24x + 12$

$y^{(4)} = 24$

$y^{(5)} = 0$

Homework

Exercise 2.8
Omit 6 and 7

① c) $f(t) = 2t - \frac{1}{t+1} = 2t - (t+1)^{-1}$

$F'(x) = 2 + 1(t+1)^{-2}(1) = 2 + (t+1)^{-2}$

$f''(x) = -2(t+1)^{-3}(1)$

$= \boxed{\frac{-2}{(t+1)^3}}$

g) $y = \sqrt{x^2+1} = (x^2+1)^{1/2}$

$y' = \frac{1}{2}(x^2+1)^{-1/2}(2x) = x(x^2+1)^{-1/2} = \frac{x}{\sqrt{x^2+1}}$

$y'' = 1(x^2+1)^{-1/2} + x\left(-\frac{1}{2}(x^2+1)^{-3/2}(2x)\right)$

$y'' = (x^2+1)^{-1/2} - x^2(x^2+1)^{-3/2}$

$y'' = (x^2+1)^{-3/2}\left[(x^2+1)^1 - x^2\right]$

$y'' = \frac{1}{(x^2+1)^{3/2}} = \frac{1}{\sqrt{(x^2+1)^3}}$

$-\frac{1}{2} - \left(-\frac{3}{2}\right)$
$-\frac{1}{2} + \frac{3}{2}$
$\frac{2}{2}$
1

① g) $f(x) = \sqrt{x^2+1} = (x^2+1)^{1/2}$

$$f'(x) = \frac{1}{2}(x^2+1)^{-1/2}(2x)$$

$$= x(x^2+1)^{-1/2}$$

$$f''(x) = x\left(\frac{-1}{2}\right)(x^2+1)^{-3/2}(2x) + 1(x^2+1)^{-1/2}$$

$$= -x^2(x^2+1)^{-3/2} + (x^2+1)^{-1/2}$$

$$= (x^2+1)^{-3/2}\left[-x^2 + (x^2+1)\right]$$

$$= (x^2+1)^{-3/2}(1)$$

$$= \frac{1}{\sqrt{(x^2+1)^3}}$$