Arithmetic
(common difference "d")

$$t_n = a + (n-1)d$$

$$S_n = \frac{n}{2}(2a + (n-1)d)$$

$$S_n = \frac{n}{2}(a + t_n)$$

Geometric
(Common Ratio "r")

$$t_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

* $$\boxed{S_n = \frac{a}{1-r}}$$

$-1 < r < 1$

Ex 1.7

① b) $1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \ldots$

$a = 1$

$r = \frac{-2}{3}$

$S_\infty = \frac{a}{1-r}$

$= \frac{1}{1-(\frac{-2}{3})} \longrightarrow 1 + \frac{2}{3}$

$\frac{3}{3} + \frac{2}{3}$

$= \frac{1}{5/3}$

$= 1 \times \frac{3}{5}$

$= \boxed{\frac{3}{5}}$

c) $\frac{1}{4} - \frac{5}{16} + \frac{25}{64} - \frac{125}{256} + \ldots$

$a = \frac{1}{4}$

$r = \frac{-5}{4}$

Divergent (r is too small)

② b) $\sum_{n=1}^{\infty} \left(-\frac{2}{5}\right)^n = \frac{-2}{5} + \frac{4}{25} - \frac{8}{125} + \frac{16}{625} - \ldots$

$a = \frac{-2}{5}$

$r = \frac{4}{25} \div \frac{-2}{5}$

$= \frac{\cancel{4}^{2}}{\cancel{25}_{5}} \times \frac{-\cancel{5}^{1}}{\cancel{2}_{1}}$

$= \frac{-2}{5}$

$S_n = \frac{-2/5}{1-(-2/5)}$

$= \frac{\frac{-2}{5}}{7/5}$

$= \frac{-2}{\cancel{5}} \times \frac{\cancel{5}}{7}$

$= \boxed{\frac{-2}{7}}$

② b) $\sum_{n=1}^{\infty} \left(-\frac{2}{5}\right)^n = -\frac{2}{5} + \frac{4}{25} - \frac{8}{125} + \dots$

$t_1 = \left(\frac{-2}{5}\right)^1 = \frac{(-2)^1}{(5)^1} = \frac{-2}{5}$

$t_2 = \left(\frac{-2}{5}\right)^2 = \frac{(-2)^2}{(5)^2} = \frac{4}{25}$

$t_3 = \left(\frac{-2}{5}\right)^3 = \frac{(-2)^3}{(5)^3} = \frac{-8}{125}$

$r = \frac{4}{25} \div -\frac{2}{5}$

$= \frac{4}{25} \times -\frac{5}{2}$

$= \frac{-20}{50} = -\frac{2}{5}$

$S_n = \frac{a}{1-r}$

$S_n = \frac{-2/5}{1-(-2/5)}$

$S_n = \frac{-2/5}{1+2/5}$

$S_n = \frac{-2/5}{5/5+2/5} = \frac{-2}{5} \div \frac{7}{5} = \frac{-2}{5} \times \frac{5}{7}$

$= \frac{-10}{35} = \frac{-2}{7}$

① g) $0.1 + 0.05 + 0.025 + 0.0125 + \ldots$

$0.5 \quad 0.5 \quad 0.5$

$a = 0.1$

$r = 0.5$

$$S_n = \frac{a}{1-r} = \frac{0.1}{1-0.5} = \frac{0.1}{0.5} = \boxed{0.2}$$

$$= \frac{2}{10}$$

$$= \frac{1}{5}$$

② b) $\displaystyle\sum_{n=1}^{\infty} \left(-\frac{2}{5}\right)^n = -\frac{2}{5} + \frac{4}{25} - \frac{8}{125} + \ldots$

$t_1 = \left(-\frac{2}{5}\right)^1 = -\frac{2}{5}$

$t_2 = \left(-\frac{2}{5}\right)^2 = \frac{4}{25}$

$t_3 = \left(-\frac{2}{5}\right)^3 = -\frac{8}{125}$

$r = \frac{4}{25} \div -\frac{2}{5}$

$r = \frac{4}{25} \times -\frac{5}{2} = -\frac{20}{50} = -\frac{2}{5}$

$$S_n = \frac{-\frac{2}{5}}{1-\left(-\frac{2}{5}\right)}$$

$$S_n = \frac{-2/5}{1+2/5}$$

$$S_n = \frac{-2/5}{5/5+2/5}$$

$$S_n = -\frac{2}{5} \div \frac{7}{5}$$

$$S_n = -\frac{2}{5} \times \frac{5}{7} = -\frac{10}{35} = \boxed{-\frac{2}{7}}$$

Series and Sequence Review

① Given: $80000, \_\_\_, \_\_\_, \_\_\_, \underline{117128}$

$a = 80000$

$t_5 = 117128$

$n = 5$

$r = ?$

$$t_n = ar^{n-1}$$

$$117128 = 80000r^{5-1}$$

$$\frac{117128}{80000} = \frac{\cancel{80000}r^4}{\cancel{80000}}$$

$$(1.4641)^{1/4} = (r^4)^{1/4}$$

$$\boxed{1.1 = r}$$

$$\begin{aligned} \text{AROI} &= 100(r-1) \\ &= 100(1.1-1) \\ &= 100(0.1) \\ &= 10\% \end{aligned}$$

Series and Sequence Review

② Given:

$a = 110$

$d = 80$

$t_5 = ?$

$t_n = a + (n-1)d$

$t_5 = 110 + (5-1)(80)$

$t_5 = 110 + 320$ $\boxed{\$430}$

③ $\underline{60}$, 57, 54.150, 51.4425, . . .

a) Geometric $r = \frac{57}{60} = \frac{54.15}{57} = \frac{51.4425}{54.150} = 0.95$

b) $t_n = ar^{n-1}$

$t_n = (60)(0.95)^{n-1}$

c) $t_n = 22.64L$

$t_n = ar^{n-1}$

$\frac{22.64}{60} = \frac{\cancel{(60)}(0.95)^{n-1}}{\cancel{60}}$

$0.377\bar{3} = (0.95)^{n-1}$

$(\cancel{0.95})^{19} = (\cancel{0.95})^{n-1}$

$\frac{\log(0.377\bar{3})}{\log(0.95)} = 19$

$19 = n-1$

$\boxed{20 = n}$

In year 20 the sap production will be 22.64L

Series and Sequence Review

(4) a) $\lim_{n\to\infty} \frac{2-n^4}{3n^4+4} = \frac{-1}{3}$ converging

(degree is same.)

b) $\lim_{n\to\infty} \frac{n^4}{n^5+9} = 0$ converging

(degree is larger in denominator)

c) $\lim_{n\to\infty} (-1)^{n+1} \cdot n^2 = \text{DNE}$

$$\left.\begin{aligned} t_1 &= (-1)^2(1)^2 = 1 \\ t_2 &= (-1)^3(2)^2 = -4 \\ t_3 &= (-1)^4(3)^2 = 9 \\ t_4 &= (-1)^5(4)^2 = -16 \end{aligned}\right\} \text{Diverging}$$

Series and Sequence Review

5) a) $\sum_{n=1}^{5} n^2+1 = 2+5+10+17+26 = \boxed{60}$

b) $\sum_{n=1}^{\infty} (3)(\frac{1}{2})^{n-1}$

$a=3$
$r=\frac{1}{2}$

$S_n = \frac{a}{1-r}$

$= \frac{3}{1-\frac{1}{2}}$

$= \frac{3}{\frac{2}{2}-\frac{1}{2}}$

$= \frac{3}{\frac{1}{2}} = 3\times 2 = \boxed{6}$

c) $1+5+9+\ldots+77$

$\underbrace{1+5}_{4}\underbrace{+9}_{4}$

$a=1$
$d=4$
$t_n=77$

(i) Find n:

$t_n = a+(n-1)d$

$77 = 1+(n-1)(4)$

$\frac{76}{4} = \frac{\cancel{4}(n-1)}{\cancel{4}}$

$19 = n-1$

$20 = n$

(ii) Find S_{20}

$S_n = \frac{n}{2}(a+t_n)$

$S_{20} = \frac{20}{2}(1+77)$

$S_{20} = 10(78) = \boxed{780}$

d) $2-8+32\ldots$

$\underbrace{2-8}_{-4}\underbrace{+32}_{-4}$

$a=2$
$r=-4$
$S_8 = ?$

$S_8 = \frac{(2)[(-4)^8-1]}{(-4)-1}$

$= \frac{2[65535]}{-5}$

$= \boxed{26214}$

Series and Sequence Review

⑥ $t_3 = 9$

$t_3 = ar^{3-1}$

$t_3 = ar^2$

$ar^2 = 9$

$t_7 = 1/9$

$t_7 = ar^{7-1}$

$t_7 = ar^6$

$ar^6 = 1/9$

Elimination by Division

$$\frac{ar^6 = 1/9}{ar^2 = 9}$$

$r^4 = \frac{1}{81}$

$\boxed{r = \pm\frac{1}{3}}$

$ar^2 = 9$

$a\left(\frac{1}{3}\right)^2 = 9$

$a\left(\frac{1}{9}\right) = 9$

$\frac{1a}{9} = 9$

$\boxed{a = 81}$

if $r = 1/3$

$$S_5 = \frac{(81)\left[\left(1/3\right)^5 - 1\right]}{\left(1/3\right) - 1}$$

$$S_5 = \frac{81\left[\frac{1}{243} - \frac{243}{243}\right]}{\frac{1}{3} - \frac{3}{3}}$$

$$S_5 = 81\left(-\frac{242}{243}\right)\left(-\frac{3}{2}\right)$$

$S_5 = 121$

if $r = -1/3$

$$S_5 = \frac{(81)\left[\left(-1/3\right)^5 - 1\right]}{\left(-1/3\right) - 1}$$

$$S_5 = \frac{81\left[\frac{-1}{243} - \frac{243}{243}\right]}{\frac{-1}{3} - \frac{3}{3}}$$

$$S_5 = 81\left(-\frac{244}{243}\right)\left(-\frac{3}{4}\right)$$

$S_5 = 61$

Series and Sequence Review

① $t_{12} = 15$

$t_n = a + (n-1)d$

$t_{12} = a + (12-1)d$

$t_{12} = a + 11d$

$\boxed{a + 11d = 15}$

$S_{15} = 105$

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{15} = \frac{15}{2}(2a + (15-1)d)$

$S_{15} = 7.5(2a + 14d)$

$S_{15} = 15a + 105d$

$\boxed{15a + 105d = 105}$

Elimination Method:

$$\begin{array}{rl} & a + 11d = 15 \\ (-) & a + 7d = 7 \\ \hline & 4d = 8 \\ & \boxed{d = 2} \end{array}$$

$a + 7d = 7$

$a + 7(2) = 7$

$a + 14 = 7$

$\boxed{a = -7}$

-7 -5 -3