

Arithmetic  
(common difference "d")

$$t_n = a + (n - 1)d$$

$$S_n = \frac{n}{2}(2a + (n - 1)d)$$

$$S_n = \frac{n}{2}(a + t_n)$$

Geometric  
(Common Ratio "r")

$$t_n = ar^{n-1}$$

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

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$$S_n = \frac{a}{1 - r}$$

$$-1 < r < 1$$

Ex 1.7

$$\textcircled{1} \text{ b) } 1 - \frac{2}{3} + \frac{4}{9} - \frac{8}{27} + \dots$$

$$a = 1$$

$$r = -\frac{2}{3}$$

$$S_{\infty} = \frac{a}{1-r}$$

$$= \frac{1}{1 - (-\frac{2}{3})} \rightarrow 1 + \frac{2}{3}$$

$$= \frac{1}{\frac{5}{3}} \leftarrow \frac{3}{3} + \frac{2}{3}$$

$$= 1 \times \frac{3}{5}$$

$$= \boxed{\frac{3}{5}}$$

$$\text{c) } \frac{1}{4} - \frac{5}{16} + \frac{25}{64} - \frac{125}{256} + \dots$$

$$a = \frac{1}{4}$$

Divergent (r is too small)

$$r = -\frac{5}{4}$$

$$\textcircled{2} \text{ b) } \sum_{n=1}^{\infty} \left(-\frac{2}{5}\right)^n = \frac{-2}{5} + \frac{4}{25} - \frac{8}{125} + \frac{16}{625} - \dots$$

$$a = -\frac{2}{5}$$

$$r = \frac{4}{25} \div -\frac{2}{5}$$

$$= \frac{4}{25} \times -\frac{5}{2}$$

$$= -\frac{2}{5}$$

$$S_n = \frac{-\frac{2}{5}}{1 - (-\frac{2}{5})}$$

$$= \frac{-\frac{2}{5}}{\frac{7}{5}}$$

$$= -\frac{2}{5} \times \frac{5}{7}$$

$$= \boxed{-\frac{2}{7}}$$

$$\textcircled{2} \text{ b) } \sum_{n=1}^{\infty} \left(-\frac{2}{5}\right)^n = \left(-\frac{2}{5}\right) + \frac{4}{25} - \frac{8}{125} + \dots$$

$$t_1 = \left(-\frac{2}{5}\right)^1 = \frac{(-2)^1}{(5)^1} = -\frac{2}{5}$$

$$r = \frac{4}{25} \div -\frac{2}{5}$$

$$t_2 = \left(-\frac{2}{5}\right)^2 = \frac{(-2)^2}{(5)^2} = \frac{4}{25}$$

$$= \frac{4}{25} \times -\frac{5}{2}$$

$$t_3 = \left(-\frac{2}{5}\right)^3 = \frac{(-2)^3}{(5)^3} = -\frac{8}{125}$$

$$= \frac{-20}{50} = \left(-\frac{2}{5}\right)$$

$$S_n = \frac{a}{1-r}$$

$$S_n = \frac{-\frac{2}{5}}{1 - \left(-\frac{2}{5}\right)}$$

$$S_n = \frac{-\frac{2}{5}}{1 + \frac{2}{5}}$$

$$S_n = \frac{-\frac{2}{5}}{\frac{5}{5} + \frac{2}{5}} = \frac{-2}{5} \div \frac{7}{5} = \frac{-2}{5} \times \frac{5}{7}$$

$$= \frac{-10}{35} = \left(-\frac{2}{7}\right)$$

$$\textcircled{1} \text{ g) } \textcircled{0.1} + \underbrace{0.05}_{0.5} + \underbrace{0.025}_{0.5} + \underbrace{0.0125}_{0.5} + \dots$$

$$a = 0.1$$

$$r = 0.5$$

$$S_n = \frac{a}{1-r} = \frac{0.1}{1-0.5} = \frac{0.1}{0.5} = \boxed{0.2}$$

$$= \frac{2}{10}$$

$$= \frac{1}{5}$$

$$\textcircled{2} \text{ b) } \sum_{n=1}^{\infty} \left(-\frac{2}{5}\right)^n = -\frac{2}{5} + \frac{4}{25} - \frac{8}{125} + \dots$$

$$t_1 = \left(-\frac{2}{5}\right)^1 = \textcircled{-\frac{2}{5}}$$

$$r = \frac{4}{25} \div -\frac{2}{5}$$

$$r = \frac{4}{25} \times -\frac{5}{2} = -\frac{20}{50} = -\frac{2}{5}$$

$$t_2 = \left(-\frac{2}{5}\right)^2 = \frac{4}{25}$$

$$t_3 = \left(-\frac{2}{5}\right)^3 = -\frac{8}{125}$$

$$S_n = \frac{-\frac{2}{5}}{1 - \left(-\frac{2}{5}\right)}$$

$$S_n = \frac{-\frac{2}{5}}{1 + \frac{2}{5}}$$

$$S_n = \frac{-\frac{2}{5}}{\frac{5}{5} + \frac{2}{5}}$$

$$S_n = -\frac{2}{5} \div \frac{7}{5}$$

$$S_n = -\frac{2}{5} \times \frac{5}{7} = -\frac{10}{35} = \boxed{-\frac{2}{7}}$$

## Series and Sequence Review

① Given:

80000, —, —, —, 117128

$$a = 80000$$

$$t_5 = 117128$$

$$n = 5$$

$$r = ?$$

$$t_n = ar^{n-1}$$

$$117128 = 80000 r^{5-1}$$

$$\frac{117128}{80000} = \frac{80000}{80000} r^4$$

$$(1.4641)^{\frac{1}{4}} = (r^4)^{\frac{1}{4}}$$

$$\boxed{1.1 = r}$$

$$\begin{aligned}
 \text{AROI} &= 100(r-1) \\
 &= 100(1.1-1) \\
 &= 100(0.1) \\
 &= 10\%
 \end{aligned}$$

## Series and Sequence Review

② Given:

$$t_n = a + (n-1)d$$

$$a = 110$$

$$d = 80$$

$$t_5 = ?$$

$$t_5 = 110 + (5-1)(80)$$

$$t_5 = 110 + 320 = \boxed{\$430}$$

③ 60, 57, 54.150, 51.4425, ...

a) Geometric  $r = \frac{57}{60} = \frac{54.15}{57} = \frac{51.4425}{54.150} = 0.95$

b)  $t_n = ar^{n-1}$   
 $t_n = (60)(0.95)^{n-1}$

c)  $t_n = 22.64L$

$$t_n = ar^{n-1}$$

$$\frac{22.64}{60} = \frac{(60)(0.95)^{n-1}}{60}$$

$$0.377\bar{3} = (0.95)^{n-1}$$

$$(\cancel{0.95})^n = (\cancel{0.95})^{n-1}$$

$$\frac{\log(0.377\bar{3})}{\log(0.95)} = 19$$

$$\left. \begin{array}{l} 19 = n-1 \\ 20 = n \end{array} \right\}$$

In year 20 the sap production will be 22.64L

## Series and Sequence Review

$$\textcircled{4} \text{ a) } \lim_{n \rightarrow \infty} \frac{2n^4}{3n^4 + 4} = \frac{-1}{3} \quad \text{converging}$$

(degree is same.)

$$\text{b) } \lim_{n \rightarrow \infty} \frac{n^4}{n^5 + 9} = 0 \quad \text{converging}$$

(degree is larger in denominator)

$$\text{c) } \lim_{n \rightarrow \infty} (-1)^{n+1} \cdot n^2 = \text{DNE}$$

$$t_1 = (-1)^2 (1)^2 = 1$$

$$t_2 = (-1)^3 (2)^2 = -4$$

$$t_3 = (-1)^4 (3)^2 = 9$$

$$t_4 = (-1)^5 (4)^2 = -16$$

Diverging

## Series and Sequence Review

$$\textcircled{5} \text{ a) } \sum_{n=1}^5 n^2 + 1 = 2 + 5 + 10 + 17 + 26 = \boxed{60}$$

$$\text{b) } \sum_{n=1}^8 \underline{(3)} \left( \underline{\frac{1}{2}} \right)^{n-1} \quad \begin{array}{l} a=3 \\ r=\frac{1}{2} \end{array} \quad \begin{array}{l} S_n = \frac{a}{1-r} \\ = \frac{3}{1-\frac{1}{2}} \\ = \frac{3}{\frac{2}{2}-\frac{1}{2}} \\ = \frac{3}{\frac{1}{2}} = 3 \times 2 = \boxed{6} \end{array}$$

$$\text{c) } \underline{1} + \underline{5} + \underline{9} + \dots + \underline{77}$$

$$a=1$$

$$d=4$$

$$t_n=77$$

(i) Find  $n$ :

$$t_n = a + (n-1)d$$

$$77 = \underline{1} + (n-1)(4)$$

$$\frac{76}{4} = \frac{4(n-1)}{4}$$

$$19 = n - 1$$

$$20 = n$$

(ii) Find  $S_{20}$

$$S_n = \frac{n}{2}(a + t_n)$$

$$S_{20} = \frac{20}{2}(1 + 77)$$

$$S_{20} = 10(78) = \boxed{780}$$

$$\text{d) } \underline{2} - \underline{8} + \underline{32} \dots \quad S_8 = \frac{(2)[(-4)^8 - 1]}{(-4) - 1}$$

$$a=2$$

$$r=-4$$

$$S_8 = ?$$

$$= \frac{2[65535]}{-5}$$

$$= \boxed{26214}$$



## Series and Sequence Review

$$\textcircled{6} \quad \begin{array}{l|l} t_3 = 9 & t_7 = \frac{1}{9} \\ t_3 = ar^{3-1} & t_7 = ar^{7-1} \\ t_3 = ar^2 & t_7 = ar^6 \\ ar^2 = 9 & ar^6 = \frac{1}{9} \end{array}$$

Elimination by  
division

$$\frac{ar^6 = \frac{1}{9}}{ar^2 = 9}$$

$$(r^4)^{\frac{1}{4}} = \left(\frac{1}{81}\right)^{\frac{1}{4}}$$

$$r = -\frac{1}{3}$$

$$\frac{1}{9} \div 9 = \frac{1}{81}$$

$$ar^2 = 9$$

$$a\left(\frac{1}{3}\right)^2 = 9$$

$$a\left(\frac{1}{9}\right) = 9$$

$$a = 81$$

$$\text{if } r = \frac{1}{3}$$

$$S_5 = \frac{(81)\left[\left(\frac{1}{3}\right)^5 - 1\right]}{\left(\frac{1}{3}\right) - 1}$$

$$S_5 = \frac{81\left[\frac{1}{243} - \frac{243}{243}\right]}{\left(\frac{1}{3} - \frac{3}{3}\right)}$$

$$S_5 = 81\left(\frac{-242}{243}\right) \div \frac{-2}{3}$$

$$S_5 = 81\left(\frac{-242}{243}\right)\left(-\frac{3}{2}\right)$$

$$S_5 = \frac{58806}{486} = \boxed{121}$$

$$\text{if } r = -\frac{1}{3}$$

$$S_5 = \frac{(81)\left[\left(-\frac{1}{3}\right)^5 - 1\right]}{\left(-\frac{1}{3}\right) - 1}$$

$$S_5 = \frac{81\left[-\frac{1}{243} - \frac{243}{243}\right]}{\left(-\frac{1}{3} - \frac{3}{3}\right)}$$

$$S_5 = 81\left(\frac{-244}{243}\right) \div \frac{-4}{3}$$

$$S_5 = 81\left(\frac{-244}{243}\right)\left(-\frac{3}{4}\right)$$

$$S_5 = \frac{59292}{972} = \boxed{61}$$

## Series and Sequence Review

$$\begin{array}{l} \textcircled{1} \quad t_{12} = 15 \\ t_{12} = a + (12-1)d \\ t_{12} = a + 11d \\ a + 11d = 15 \end{array} \quad \left| \begin{array}{l} S_{15} = 105 \\ S_{15} = \frac{15}{2} [2a + (15-1)d] \\ S_{15} = 7.5 [2a + 14d] \\ S_{15} = 15a + 105d \\ 15a + 105d = 105 \quad \div: 15 \\ a + 7d = 7 \end{array} \right.$$

$$t_1 = -7$$

$$t_2 = -5$$

$$t_3 = -3$$

Elimination by subtracting

$$\begin{array}{r} a + 11d = 15 \\ \leftarrow a + 7d = 7 \\ \hline 4d = 8 \\ \underline{4} \quad \underline{4} \end{array}$$

$$\boxed{d = 2}$$

$$\begin{array}{r} a + 7d = 7 \\ a + 7(2) = 7 \\ a + 14 = 7 \\ \boxed{a = -7} \end{array}$$