Questions from Homework

② e) $\sqrt{x+y} + \sqrt{xy} = 4$

$$(x+y)^{1/2} + (xy)^{1/2} = 4$$

$$\frac{1}{2}(x+y)^{-1/2}(1+y') + \frac{1}{2}(xy)^{-1/2}(xy'+y) = 0$$

$$\frac{1}{2\sqrt{x+y}}(1+y') + \frac{1}{2\sqrt{xy}}(xy'+y) = 0$$

$$\overset{2}{\cancel{4}}\sqrt{xy}\cancel{\sqrt{x+y}}\,\frac{1+y'}{\cancel{2\sqrt{x+y}}} + \overset{2}{\cancel{4\sqrt{xy}}}\sqrt{x+y}\,\frac{xy'+y}{\cancel{2\sqrt{xy}}} = 0 \quad 4\sqrt{xy}\sqrt{x+y}$$

$$2\sqrt{xy} + 2y'\sqrt{xy} + 2xy'\sqrt{x+y} + 2y\sqrt{x+y} = 0$$

$$2y'\sqrt{xy} + 2xy'\sqrt{x+y} = -2\sqrt{xy} - 2y\sqrt{x+y}$$

$$y'\left[2\sqrt{xy} + 2x\sqrt{x+y}\right] = -2\sqrt{xy} - 2y\sqrt{x+y}$$

$$y' = \frac{-2\sqrt{xy} - 2y\sqrt{x+y}}{2\sqrt{xy} + 2x\sqrt{x+y}}$$

Point $(2,2)$

$$y' = -\frac{\sqrt{xy} + y\sqrt{x+y}}{\sqrt{xy} + x\sqrt{x+y}}$$

$$y' = -\frac{\sqrt{(2)(2)} + 2\sqrt{(2)+(2)}}{\sqrt{(2)(2)} + 2\sqrt{(2)+(2)}}$$

$$= -\frac{2+4}{2+4}$$

$$\boxed{= -1}$$

7. $x^{2/3} + y^{2/3} = 1$

$$\frac{dy}{dx} = -\frac{\sqrt[3]{y}}{\sqrt[3]{x}} = -\sqrt[3]{\frac{y}{x}}$$

$$1 = -\sqrt[3]{\frac{y}{x}}$$

$$1 = -\frac{y}{x}$$

$$x = -y$$

$$\boxed{y = -x}$$

$$x^{2/3} + y^{2/3} = 1$$

$$x^{2/3} + (-x)^{2/3} = 1$$

$$x^{2/3} + x^{2/3} = 1$$

$$2x^{2/3} = 1$$

$$\left(x^{2/3}\right)^{3/2} = \left(\frac{1}{2}\right)^{3/2}$$

$$x = \pm\frac{1}{\sqrt{8}}$$

$$x = \pm\frac{\sqrt{8}}{8}$$

$$x = \pm\frac{2\sqrt{2}}{8} = \pm\frac{\sqrt{2}}{4}$$

$$\left(\frac{\sqrt{2}}{4}, -\frac{\sqrt{2}}{4}\right)$$

$$\left(-\frac{\sqrt{2}}{4}, \frac{\sqrt{2}}{4}\right)$$

Applications of Derivatives

Now that we know how to calculate derivatives, we use them in this unit to compute, *velocity, acceleration, and other rates of change.*

Suppose that an object moves along a straight line. (Think of a ball being thrown vertically upward or a car being driven along a road or a stone being dropped from a cliff.) The position function is $s = f(t)$, where s is the **displacement (directed distance)** of the object from the origin at time t. Recall that the **(instantaneous) velocity** of the object at time t is defined as the limit of average velocities over shorter and shorter time intervals.

In short, the ***velocity*** is the derivative of the position function and in Leibniz notation we write.

$$v = \frac{ds}{dt}$$

Velocity

If a stone is dropped from a cliff that is 122.5 m high, then its height in metres after *t* seconds is $h = 122.5 - 4.9t^2$

$h' = v(t) = -9.8t$

a) Find its velocity after 1 *s* and 2 *s*.
b) When will the stone hit the ground? (Let $h=0$)
c) With what velocity will it hit the ground? (Let $t=5$)

a) $t = 1s$ $\quad$ $t = 2s$

$v(1) = -9.8(1)$ $\quad$ $v(2) = -9.8(2)$

$= -9.8 m/s$ $\quad$ $= -19.6 m/s$

b) Let $h=0$

$h = 122.5 - 4.9t^2$

$0 = 122.5 - 4.9t^2$

$4.9t^2 = 122.5$

$t^2 = 25$

$t = \pm 5s$

$\therefore$ In 5 seconds the stone will hit the ground.

c) Let $t = 5s$

$v(t) = -9.8t$

$v(5) = -9.8(5)$

$= -49 m/s$

$\therefore$ The stone hits the ground with a velocity of $-49 m/s$.

The position of a particle moving on a line is given by the equation $s = 2t^3 - 21t^2 + 60t,$

where *t* is measured in seconds and *s* in meters

a) Find its velocity after 3 *s* and 6 *s*.
b) When is the particle at rest? (Let $v=0$)

a) $v = 6t^2 - 42t + 60$

$t = 3s$

$v = 6(3)^2 - 42(3) + 60$
$= 54 - 126 + 60$
$= -12 m/s$

$t = 6s$

$v = 6(6)^2 - 42(6) + 60$
$= 216 - 252 + 60$
$= 24 m/s$

b) $v = 6t^2 - 42t + 60$
$0 = 6(t^2 - 7t + 10)$
$0 = 6(t-5)(t-2)$

$t - 5 = 0$ | $t - 2 = 0$

$\boxed{t = 5s}$ | $\boxed{t = 2s}$

Applcations of Derivatives

Now that we know how to calculate derivatives, we use them in this unit to compute, *velocity, acceleration, and other rates of change.*

If an object moves along a straight line, its **acceleration** is the rate of change of velocity with respect to time. Therefore, the acceleration *a(t)* at time *t* is the derivative of the velocity function.

$$a(t) = v'(t) = \frac{dv}{dt}$$

Since the ***velocity*** is the derivative of the position function $s = f(t)$, it follows that the acceleration is the *second derivative* of the position function:

So

$$v(t) = s'(t) = \frac{ds}{dt}$$

$$a(t) = v'(t) = s''(t)$$

In Leibniz Notation,

$$a = \frac{d^2s}{dt^2}$$

Acceleration

The position function of a particle is $s = t^3 + 2t^2 + 2t$
given by where *s* is measured in meters and *t* in seconds

a) Find the velocity and acceleration as a funciton of time.
b) Find the acceleration at 3 *s*

a) $s = t^3 + 2t^2 + 2t$

$v = 3t^2 + 4t + 2$

$a = 6t + 4$

b) $t = 3s$

$a = 6(3) + 4$

$= 22 m/s^2$

If a ball is thrown upward with an initial velocity of 24.5 m/s, then its distance above the ground in meters after *t* seconds is

$$s = 24.5 - 4.9t^2$$

a) Find the acceleration of the ball.

$s = 24.5 - 4.9t^2$

$v = -9.8t$

$a = -9.8$ ← Constant

Notice that the acceleration in *Example 2* is a constant, and is called the *acceleration due to gravity*. The fact that it is negative means that **the ball slows down as it rises and speeds up as it falls.**

- In general, a negative acceleration indicates that the velocity is decreasing

- Likewise, a positive acceleration means that the velocity is increasing

Homework

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