Sequences

Find the first 5 terms of the following sequences:

general term

$t_n = 3^n$

$t_1 = 3^1 = 3$

$t_2 = 3^2 = 9$

$t_3 = 3^3 = 27$

$t_4 = 3^4 = 81$

$t_5 = 3^5 = 243$

3, 9, 27, 81, 243, ...

geometric ($r = 3$)
multiplying by a constant

$t_n = n + 5$

$t_1 = 1 + 5 = 6$

$t_2 = 2 + 5 = 7$

$t_3 = 3 + 5 = 8$

$t_4 = 4 + 5 = 9$

$t_5 = 5 + 5 = 10$

* 6, 7, 8, 9, 10, ...

arithmetic ($d = 1$)
adding a constant

$t_n = (n+2)(n-1)$

$t_1 = (1+2)(1-1) = 0$

$t_2 = (2+2)(2-1) = 4$

$t_3 = (3+2)(3-1) = 10$

$t_4 = (4+2)(4-1) = 18$

$t_5 = (5+2)(5-1) = 28$

0, 4, 10, 18, 28, ...

Arithmetic Sequences

adding a constant → "common difference"

Ex: 2, 5, 8, 11, 14 $d = t_2 - t_1 = t_3 - t_2$

3 3 3 3

- The difference between each term is constant.
- In the sequence 2, 5, 8, 11, 14. the difference between each term is 3.
- The difference is called "**d**". $d = t_2 - t_1$
- The first term is called "**a**" or "$\mathbf{t_1}$".
- The second term is called "$\mathbf{t_2}$".
- The last term or an indicated term is called "$\mathbf{t_n}$". (general term)
- The position of a term or the number of terms is called "**n**".

$a = 2$

$d = 3$

Arithmetic Sequences

To find any given term in an arithmetic sequence we use the following formula:

$$t_n = a + (n-1)d$$

common difference

$d = t_2 - t_1 = t_3 - t_2 = t_4 - t_3 \ldots$

first term

# of terms

Example I.

Find the indicated term of the following sequence

$a = 1$

$d = 4-1 = 7-4 = 3$

1, 4, 7...

3 3

a) Find t_7

1, 4, 7, 10, 13, 16, 19

$t_n = a + (n-1)d$

$t_7 = 1 + (7-1)(3)$

$t_7 = 1 + 6(3)$

$t_7 = 1 + 18$

$t_7 = 19$

b) Find t_{50} ← $n = 50$

$t_{50} = 1 + (50-1)(3)$

$t_{50} = 1 + 49(3)$

$t_{50} = 1 + 147$

$t_{50} = 148$

We can also determine the number of terms in the sequence.

$$t_n = a + (n-1)d$$

Example II.

How many terms are in the following sequences? (Solve for "n")

1, 3, 5,... 71

$a = 1$

$d = 2$

$t_n = 71$

$t_n = a + (n-1)d$

$71 = 1 + (n-1)(2)$ (−1 / −1)

$\frac{70}{2} = \frac{(n-1)(2)}{2}$

$35 = n - 1$ (+1 / +1)

$\boxed{36 = n}$

$x, x+3, x+6, \ldots, x+33$

$a = x$

$d = x+3-x$

$d = 3$

$t_n = x+33$

$t_n = a + (n-1)d$

$x+33 = x + (n-1)(3)$ (−x / −x)

$\frac{33}{3} = \frac{(n-1)(3)}{3}$

$11 = n - 1$ (+1 / +1)

$\boxed{12 = n}$

arithmetic

$t_n = a + (n-1)d$

Find "a", "d", and "t_n" for the following sequence

4, 7, 10, 13, 16, 19, 22, 25

$t_5 = 16$, $t_8 = 25$

$t_5 = a+(5-1)d$	$t_8 = a+(8-1)d$
$t_5 = a+4d$	$t_8 = a+7d$
$16 = a+4d$	$25 = a+7d$
$a+4d = 16$	$a+7d = 25$

Elimination

$a+7d = 25$

$a+4d = 16$

$3d = 9$

$\frac{3d}{3} = \frac{9}{3}$

$d = 3$

$a+7d = 25$

$a+7(3) = 25$

$a+21 = 25$ (−21 −21)

$a = 4$

general term = "t_n"

$t_n = a+(n-1)d$

$t_n = 4+(n-1)(3)$

$t_n = 4+3n-3$

$t_n = 3n+1$

Homework

#1
#2
#3
#6
#9