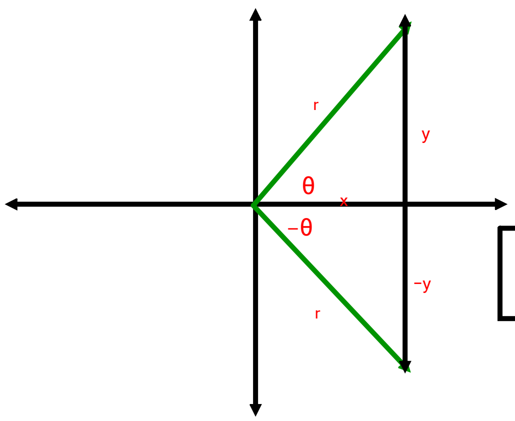
Negative Angles



$\sin\theta = \frac{y}{r}$ $\cos(\theta) = \frac{x}{r}$

$\sin(-\theta) =$ $\cos(-\theta) =$

$\boxed{\sin(-\theta) =}$ $\boxed{\cos(-\theta) = \cos\theta}$

Questions from Homework

① i) $y = 3\sin^4(2-x)^{-1} = 3[\sin(2-x)^{-1}]^4$

$y' = 12[\sin(2-x)^{-1}]^3 \cos(2-x)^{-1} \cdot -1(2-x)^{-2}(-1)$

$y' = 12\sin^3(2-x)^{-1}\cos(2-x)^{-1}(2-x)^{-2}$

$y' = \dfrac{12\sin^3(2x)^{-1}\cos(2-x)^{-1}}{(2-x)^2}$

h) $y = \dfrac{\sin x}{1+\cos x}$

$y' = \dfrac{\cos x(1+\cos x) - \sin x(-\sin x)}{(1+\cos x)^2}$

$y' = \dfrac{\cos x \boxed{+\cos^2 x + \sin^2 x}}{(1+\cos x)^2}$ ← Pythagorean

$y' = \dfrac{\cancel{1} + \cos x}{(1+\cos x)^{\cancel{2}}} = \dfrac{1}{1+\cos x}$

n) $y = \sin\left(\frac{1}{x}\right) = \sin(x^{-1})$

$y' = \cos(x^{-1}) \cdot (-x^{-2})$

$y' = \cos\left(\frac{1}{x}\right) \cdot -\frac{1}{x^2}$

$y' = -\frac{1}{x^2}\cos\left(\frac{1}{x}\right)$

Questions from Homework

m) $y = (1+\cos^2 x)^6 = (1+(\cos x)^2)^6$

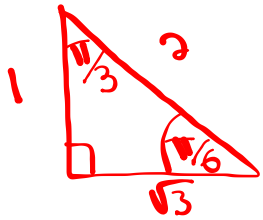
$y' = 6(1+(\cos x)^2)^5 \cdot 2(\cos x)(-\sin x)$

$y' = 6(1+\cos^2 x)^5 (-\underline{\underline{2\sin x\cos x}})$

$y' = -6(1+\cos^2 x)^5 (2\sin x\cos x)$

Double Angle Identity

Questions from Homework



③ a) $y = 2\sin x$ at $(\pi/6, 1)$

ⓘ Find derivative

$y' = 2\cos x$

ⓘⓘ Find $y'(\pi/6)$ = slope or "m"

$$y'(\pi/6) = 2\cos\left(\frac{\pi}{6}\right)$$

$$= \cancel{2}\left(\frac{\sqrt{3}}{\cancel{2}}\right)$$

$$= \sqrt{3}$$

$x_1 = \pi/6 \quad y_1 = 1 \quad m = \sqrt{3}$

ⓘⓘⓘ Find equation

$$y - y_1 = m(x - x_1)$$

$$y - 1 = \sqrt{3}\left(x - \frac{\pi}{6}\right)$$

$$6\cdot \quad y - 1 = \sqrt{3}x - \frac{\pi\sqrt{3}}{6}$$

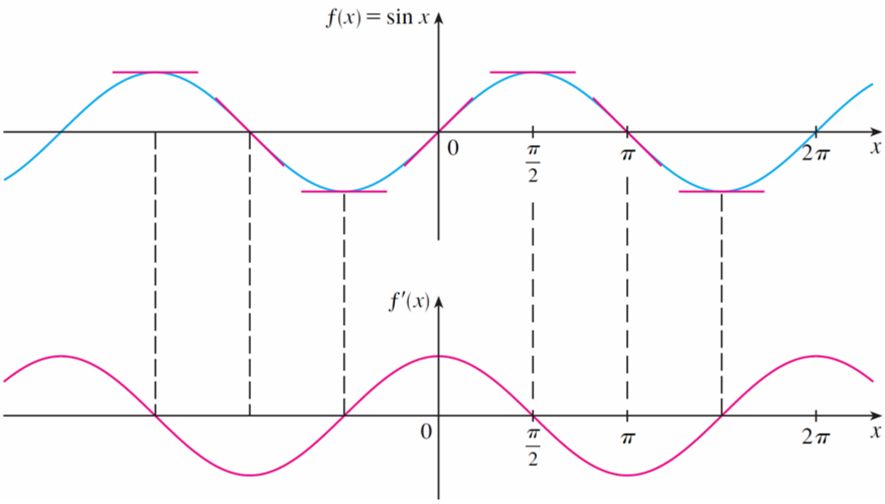
$$6y - 6 = 6x\sqrt{3} - \pi\sqrt{3}$$

$$0 = 6x\sqrt{3} - 6y - \pi\sqrt{3} + 6$$

Derivatives of Trigonometric Functions

The Sine Function

- We recall that the derivative $f'(x)$ of a function $f(x)$ gives the slope of the tangent.
- On the next slide we graph $f(x) = \sin x$ together with $f'(x)$, as determined by the slope of the tangent to the sine curve.
  - Note that x is measured in radians.
- The derivative graph resembles the graph of the cosine!



Let's check this using the definition of a derivative...

$$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h} = \lim_{h\to 0} \frac{\sin(x+h) - \sin x}{h}$$

$$= \lim_{h\to 0} \frac{\sin x \cos h + \cos x \sin h - \sin x}{h}$$

$$= \lim_{h\to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h\to 0} \left[\frac{\sin x \cos h - \sin x}{h} + \frac{\cos x \sin h}{h} \right]$$

$$= \lim_{h\to 0} \left[\sin x \left(\frac{\cos h - 1}{h} \right) + \cos x \left(\frac{\sin h}{h} \right) \right]$$

$$= \lim_{h\to 0} \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} + \lim_{h\to 0} \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h}$$

- Our calculations have brought us to four limits, two of which are easy:
  - Since x is constant while $h \to 0$,

$$\lim_{h\to 0} \sin x = \sin x \quad \text{and} \quad \lim_{h\to 0} \cos x = \cos x$$

- With some work we can also show that

$$\lim_{h\to 0} \frac{\sin h}{h} = 1 \quad \text{and} \quad \lim_{h\to 0} \frac{\cos h - 1}{h} = 0$$

- Thus our guess is confirmed:

$$f'(x) = \lim_{h\to 0} \sin x \cdot \lim_{h\to 0} \frac{\cos h - 1}{h} + \lim_{h\to 0} \cos x \cdot \lim_{h\to 0} \frac{\sin h}{h}$$

$$= (\sin x) \cdot 0 + (\cos x) \cdot 1 = \cos x$$

Rules to differentiate trigonometric functions:

Given that "u" represents some differentiable function...

$$\frac{d}{du}(\sin u) = \cos u \bullet du$$

$$\frac{d}{du}(\cos u) = -\sin u \bullet du$$

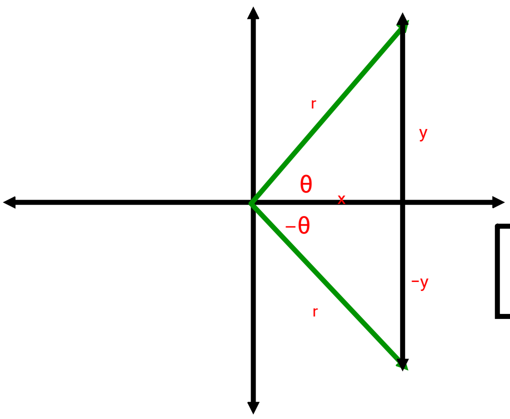
$$\frac{d}{du}(\tan u) = \sec^2 u \bullet du$$

$$\frac{d}{du}(\csc u) = -\csc u \cot u \bullet du$$

$$\frac{d}{du}(\sec u) = \sec u \tan u \bullet du$$

$$\frac{d}{du}(\cot u) = -\csc^2 u \bullet du$$

Negative Angles



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Ex: 7.2

① a) $y = \cos(-4x)$

$y' = -\sin(-4x) \cdot -4$

$y' = 4\sin(-4x)$

$y' = -4\sin(4x)$

Let's Practice...

Differentiate the following:

$$f(x) = \frac{1}{1+\tan x} = (1+\tan x)^{-1}$$

$$f'(x) = -1(1+\tan x)^{-2}(\sec^2 x) = \frac{-\sec^2 x}{(1+\tan x)^2}$$

Ex #2.

Differentiate:

$$f(x) = 2\csc^3(3x^2) = 2(\csc(3x^2))^3$$

$$f'(x) = 6(\csc(3x^2))^2(-\csc(3x^2)\cot(3x^2)) \cdot 6x$$

$$f'(x) = -36x\csc^2(3x^2)\csc(3x^2)\cot(3x^2)$$

$$f'(x) = -36x\csc^3(3x^2)\cot(3x^2)$$

Homework

Worksheet on derivatives of trigonometric functions

Page 314 #3 c, e Ex (7.2)

Page 319 #1 Ex (7.3)

Attachments

Derivatives Worksheet.doc