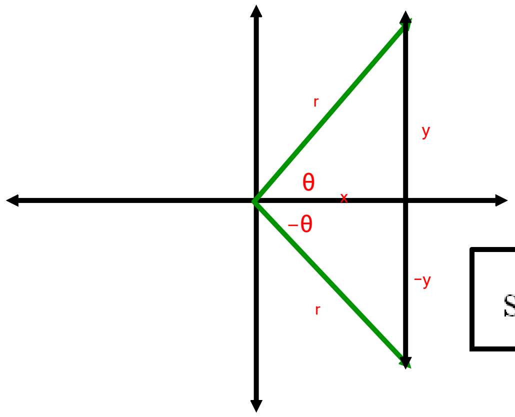
Negative Angles



$$\sin\theta = \frac{y}{r}$$ $$\cos(\theta) = \frac{x}{r}$$

$$\sin(-\theta) = -\frac{y}{r}$$ $$\cos(-\theta) = \frac{x}{r}$$

$$\boxed{\sin(-\theta) = -\sin\theta}$$ $$\boxed{\cos(-\theta) = \cos\theta}$$

Ex: 7.2

① a) $y = \cos(-4x)$

$y' = -\sin(-4x) \cdot -4$

$y' = 4\sin(-4x)$

$y' = -4\sin(4x)$

Ex: 7.3

① d) $y = -\frac{1}{4}\csc(-8x)$

$y' = -\frac{1}{4}(-\csc(-8x)\cot(-8x)) \cdot -8$

$y' = -2\csc(-8x)\cot(-8x)$

$y' = -2(-\csc 8x)(-\cot 8x)$

$y' = -2\csc(8x)\cot(8x)$

Ex 7.2

① o) $y = \sin(\cos x)$

$y' = \cos(\cos x)(-\sin x)$

$y' = -\sin x \cos(\cos x)$

p) $y = \cos^3(\sin x) = [\cos(\sin x)]^3$

$y' = 3[\cos(\sin x)]^2(-\sin(\sin x))\cos x$

$y' = -3\cos^2(\sin x)\sin(\sin x)\cos x$

q) $y = x\cos\frac{1}{x} = (x)(\cos x^{-1})$

$y' = \cos x^{-1} + x(-\sin x^{-1})(-x^{-2})$

$y' = \cos\left(\frac{1}{x}\right) + \frac{1}{x}\sin\left(\frac{1}{x}\right)$

s) $y = \frac{1+\sin x}{1-\sin 2x}$

$y' = \frac{\cos x(1-\sin 2x) - (1+\sin x)(-\cos 2x \cdot 2)}{(1-\sin 2x)^2}$

$y' = \frac{\cos x - \cos x \sin 2x + 2\cos 2x(1+\sin x)}{(1-\sin 2x)^2}$

$y' = \frac{\cos x - \cos x \sin 2x + 2\cos 2x + 2\sin x \cos 2x}{(1-\sin 2x)^2}$

Ex 7.3

① a) $y = 3\tan 2x$

$y' = 3\sec^2(2x) \cdot 2$

$y' = 6\sec^2(2x)$

g) $y = \sec\sqrt[3]{x} = \sec(x^{1/3})$

$y' = \sec(x^{1/3})\tan(x^{1/3}) \cdot \frac{1}{3}x^{-2/3}$

$y' = \sec(x^{1/3})\tan(x^{1/3}) \cdot \frac{1}{3x^{2/3}}$

$y' = \frac{\sec\sqrt[3]{x}\tan\sqrt[3]{x}}{3\sqrt[3]{x^2}}$

7.3

① d) $y = -\frac{1}{4}\csc(-8x)$

$y' = -\frac{1}{4}(-\csc(-8x)\cot(-8x)) \cdot -8$

$y' = -2\csc(-8x)\cot(-8x)$ *Negative Angle Identity

$y' = -2\csc(8x)\cot(8x)$

m) $y = 2x(\sqrt{x} - \cot x)$

$y = 2x^{3/2} - 2x\cot x$

$y' = 3x^{1/2} - [2\cot x + 2x(-\csc^2 x \cdot 1)]$

$y' = 3\sqrt{x} - 2\cot x + 2x\csc^2 x$

Ex: $u = x^2$

$y = \sin(x^2)$

$y' = \cos(x^2) \cdot 2x$

$y' = 2x\cos(x^2)$

① n) $u = \tan x$

$y = \sin(\tan x)$

$y' = \cos(\tan x) \cdot \sec^2 x$

$y' = \sec^2 x\,[\cos(\tan x)]$

k) $y = \dfrac{1}{\sqrt{(\sec 2x - 1)^3}} = \dfrac{1}{(\sec 2x - 1)^{3/2}} = (\sec 2x - 1)^{-3/2}$

$y' = -\frac{3}{2}(\sec 2x - 1)^{-5/2} \cdot \sec 2x \tan 2x \cdot (2)$

$y' = \dfrac{-3\sec(2x)\tan(2x)}{(\sec 2x - 1)^{5/2}}$

$y' = -\dfrac{3\sec(2x)\tan(2x)}{\sqrt{(\sec 2x - 1)^5}}$

① a) $y = \tan^2(\cos x) = [\tan(\cos x)]^2$

$y' = 2[\tan(\cos x)] \cdot \sec^2(\cos x) \cdot (-\sin x)(1)$

$y' = -2\sin x [\tan(\cos x)][\sec^2(\cos x)]$

b) $y = \frac{x^2 \tan x}{\sec x} = x^2\left(\frac{\sin x}{\cos x}\right) \cdot \left(\frac{\cos x}{1}\right) = x^2 \sin x$

$y' = x^2(\cos x)(1) + 2x(\sin x)$

$y' = x^2 \cos x + 2x \sin x$

$y' = x[x\cos x + 2\sin x]$

Final Review

① b) $f(x) = \dfrac{2x-2}{x+3}$ $\quad f(x+h) = \dfrac{2x+2h-2}{x+h+3}$

$$f'(x) = \lim_{h\to 0} \frac{(x+3)\cancel{(x+h+3)}\dfrac{2x+2h-2}{\cancel{x+h+3}} - \dfrac{2x-2}{\cancel{x+3}}\cancel{(x+3)}(x+h+3)}{h(x+3)(x+h+3)}$$

$$= \lim_{h\to 0} \frac{\cancel{2x^2} + \cancel{2xh} - \cancel{2x} + \cancel{6x} + 6h - \cancel{6} - (\cancel{2x^2} + \cancel{2xh} + \cancel{6x} - \cancel{2x} - 2h - \cancel{6})}{h(x+3)(x+h+3)}$$

$$= \lim_{h\to 0} \frac{8\cancel{h}}{\cancel{h}(x+3)(x+h+3)} = \boxed{\frac{8}{(x+3)^2}}$$

h) $y = (x^2)(\csc x)$

$y' = (x^2)(-\csc x \cot x)(1) + 2x\csc x$

$y' = -x^2 \csc x \cot x + 2x\csc x$

$y' = x\csc x(-x\cot x + 2)$

$y' = x\csc x(2 - x\cot x)$

$u = (1-2x)^2$

$du = 2(1-2x)(-2)$

i) $y = \cot^3(1-2x)^2 = [\cot(1-2x)^2]^3$

$y' = 3[\cot(1-2x)^2]^2[-\csc^2(1-2x)^2][2(1-2x)(-2)]$

$y' = 3\cot^2(1-2x)^2[-\csc^2(1-2x)^2][-4(1-2x)]$

$y' = 12(1-2x)\cot^2(1-2x)^2\csc^2(1-2x)^2$

⑥ Find the points on the curve $y = \frac{x}{x-1}$ where the tangent line is parallel to the line $x + 4y = 1$

① $x + 4y = 1$

$4y = -x + 1$

$y = -\frac{1}{4}x + \frac{1}{4}$

$m = -\frac{1}{4}$

② $y = \frac{x}{x-1}$

$x-1 \quad -x$

$y' = \frac{(x-1)(1) - (x)(1)}{(x-1)^2}$

$y' = \frac{-1}{(x-1)^2}$

③ $\frac{-1}{(x-1)^2} = -\frac{1}{4}$

$-(x-1)^2 = -4$

$(x-1)^2 = 4$

$x^2 - 2x + 1 = 4$

$x^2 - 2x - 3 = 0$

$(x-3)(x+1) = 0$

$x - 3 = 0 \quad | \quad x + 1 = 0$

$x = 3 \quad | \quad x = -1$

④ $y = \frac{x}{x-1}$

$y = \frac{3}{3-1} \quad | \quad y = \frac{-1}{-1-1}$

$y = \frac{3}{2} \quad | \quad y = \frac{1}{2}$

$\boxed{(3, 3/2)} \quad | \quad \boxed{(-1, 1/2)}$

⑥ Find the point on the curve $y = x\sqrt{x}$ where the tangent line is parallel to the line $6x - y = 4$

① $6x - y = 4$

$6x - 4 = y$

$y = 6x - 4$

$m = 6$

② $y = x\sqrt{x} = x(x)^{1/2} = x^{3/2}$

$y' = \frac{3}{2}x^{1/2} = \frac{3\sqrt{x}}{2}$

③ $\frac{3\sqrt{x}}{2} = \frac{6}{1}$

$3\sqrt{x} = 12$

$\sqrt{x} = 4$

$x = 16$

④ $y = x\sqrt{x}$

$y = (16)\sqrt{16}$

$y = 64$

$(16, 64)$