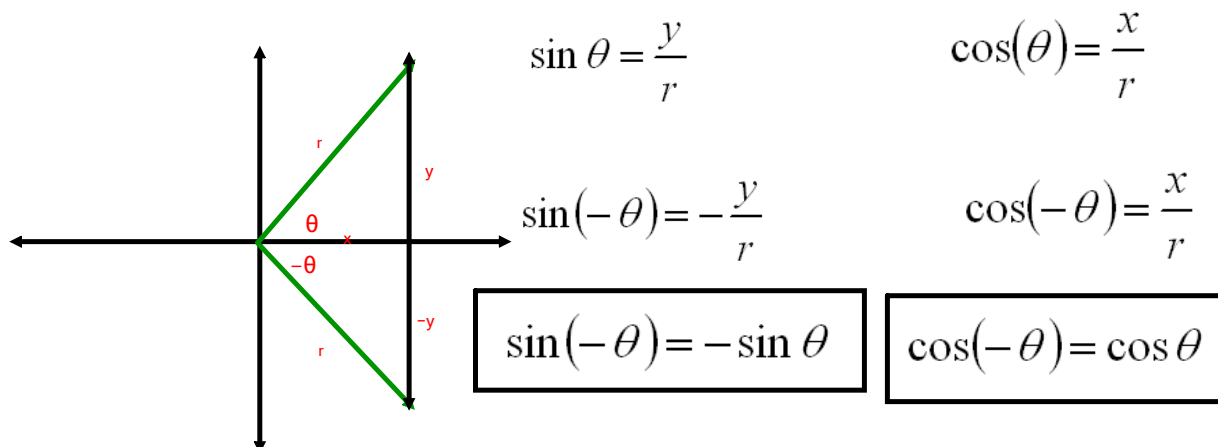


Negative Angles



Ex: 7.2

① a) $y = \cos(-4x)$
 $y' = -\sin(-4x) \cdot -4$
 $y' = 4\sin(-4x)$
 $y' = -4\sin(4x)$

Ex: 7.3

① d) $y = -\frac{1}{4}\csc(-8x)$
 $y' = -\frac{1}{4}(-\csc(-8x)\cot(-8x)) \cdot -8$
 $y' = -2\csc(-8x)\cot(-8x)$
 $y' = -2(-\csc 8x)(-\cot 8x)$
 $y' = -2\csc(8x)\cot(8x)$

Ex 7.2

$$\textcircled{1} \text{ a) } y = \sin(\cos x)$$

$$y' = \cos(\cos x) (-\sin x)$$

$$y' = -\sin x \cos(\cos x)$$

$$\text{p) } y = \cos^3(\sin x) = [\cos(\sin x)]^3$$

$$y' = 3[\cos(\sin x)]^2 (-\sin(\sin x)) \cos x$$

$$y' = -3 \cos^2(\sin x) \sin(\sin x) \cos x$$

$$\text{q) } y = x \cos \frac{1}{x} = (x)(\cos x^{-1})$$

$$y' = \cos x^{-1} + x (-\sin x^{-1}) (-x^{-2})$$

$$y' = \cos\left(\frac{1}{x}\right) + \frac{1}{x} \sin\left(\frac{1}{x}\right)$$

$$\text{s) } y = \frac{1 + \sin x}{1 - \sin x}$$

$$y' = \frac{\cos x (1 - \sin x) - (1 + \sin x)(-\cos x \cdot 1)}{(1 - \sin x)^2}$$

$$y' = \frac{\cos x - \cos x \sin x + 2 \cos x (1 + \sin x)}{(1 - \sin x)^2}$$

$$y' = \frac{\cos x - \cos x \sin x + 2 \cos x + 2 \sin x \cos x}{(1 - \sin x)^2}$$

Ex 7.3

$$\textcircled{1} \text{ a) } y = 3 \tan 2x$$

$$y' = 3 \sec^2(2x) \cdot 2$$

$$y' = 6 \sec^2(2x)$$

$$\text{g) } y = \sec^3 \sqrt{x} = \sec(x^{1/3})$$

$$y' = \sec(x^{1/3}) \tan(x^{1/3}) \cdot \frac{1}{3} x^{-2/3}$$

$$y' = \sec(x^{1/3}) \tan(x^{1/3}) \cdot \frac{1}{3x^{2/3}}$$

$$y' = \frac{\sec^3 \sqrt{x} \tan^3 \sqrt{x}}{3 \sqrt{x^2}}$$

7.3

$$\textcircled{1} \text{ d) } y = -\frac{1}{4} \csc(-8x)$$

$$y' = -\frac{1}{4} (-\csc(-8x) \cot(-8x)) \cdot -8$$

$$y' = -2 \csc(-8x) \cot(-8x)$$

$$y' = -2 \csc(8x) \cot(8x)$$

*Negative Angle
Identity

$$\text{m) } y = 2x(\sqrt{x} - \cot x)$$

$$y = 2x^{3/2} - 2x \cot x$$

$$y' = 3x^{1/2} - [2 \cot x + 2x(-\csc^2 x \cdot 1)]$$

$$y' = 3\sqrt{x} - 2 \cot x + 2x \csc^2 x$$

$$\text{Ex: } y = \sin(x^2) \quad u = x^2$$

$$y' = \cos(x^2) \cdot 2x$$

$$y' = 2x \cos(x^2)$$

$$\text{① n) } y = \sin(\tan x) \quad u = \tan x$$

$$y' = \cos(\tan x) \cdot \sec^2 x$$

$$y' = \sec^2 x [\cos(\tan x)]$$

$$\text{K) } y = \frac{1}{\sqrt{(\sec 2x - 1)^3}} = \frac{1}{(\sec 2x - 1)^{3/2}} = (\sec 2x - 1)^{-3/2}$$

$$y' = -\frac{3}{2} (\sec 2x - 1)^{-5/2} \cdot \sec 2x \tan 2x \cdot (2)$$

$$y' = \frac{-3 \sec(2x) \tan(2x)}{(\sec 2x - 1)^{5/2}}$$

$$y' = \frac{-3 \sec(2x) \tan(2x)}{\sqrt{(\sec 2x - 1)^5}}$$

$$\textcircled{1} \text{ a) } y = \tan^2(\cos x) = [\tan(\cos x)]^2$$

$$y' = 2[\tan(\cos x)] \cdot \sec^2(\cos x) \cdot (-\sin x)(1)$$

$$y' = -2\sin x [\tan(\cos x)] [\sec^2(\cos x)]$$

$$\text{b) } y = \frac{x^2 \tan x}{\sec x} = x^2 \left(\frac{\sin x}{\cos x} \right) \cdot \left(\frac{\cos x}{1} \right) = x^2 \sin x$$

$$y' = x^2 (\cos x)(1) + 2x (\sin x)$$

$$y' = x^2 \cos x + 2x \sin x$$

$$y' = x [x \cos x + 2 \sin x]$$

Final Review

$$\textcircled{1} \text{ b) } f(x) = \frac{2x-2}{x+3} \quad f(x+h) = \frac{2x+2h-2}{x+h+3}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{(x+3)(x+h+3)(2x+2h-2)}{x+h+3} - \frac{(2x-2)(x+3)(x+h+3)}{x+3}}{h(x+3)(x+h+3)}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{2x^2} + \cancel{2xh} - \cancel{2x} + \cancel{6x} + \cancel{6h} - \cancel{6} - (\cancel{2x^2} + \cancel{2xh} + \cancel{6x} - \cancel{2x} - \cancel{2h} - \cancel{6})}{h(x+3)(x+h+3)}$$

$$= \lim_{h \rightarrow 0} \frac{\cancel{8h}}{h(x+3)(x+h+3)} = \frac{8}{(x+3)^2}$$

$$h) y = (x^2)(\csc x)$$

$$y' = (x^2)(-\csc x \cot x)(1) + 2x \csc x$$

$$y' = -x^2 \csc x \cot x + 2x \csc x$$

$$y' = x \csc x (-x \cot x + 2)$$

$$y' = x \csc x (2 - x \cot x)$$

$$i) y = \cot^3(1-2x) = [\cot(1-2x)]^3$$

$$y' = 3[\cot(1-2x)]^2 [-\csc^2(1-2x)] [2(1-2x)(-2)]$$

$$y' = 3 \cot^2(1-2x) [\csc^2(1-2x)] [-4(1-2x)]$$

$$y' = 12(1-2x) \cot^2(1-2x) \csc^2(1-2x)$$

$$u = (1-2x)^2$$

$$du = 2(1-2x)(-2)$$

Final Review

$$= \frac{2(x+h)-2}{(x+h)+3}$$

$$\textcircled{1} \text{ b) } f(x) = \frac{2x-2}{x+3} \quad f(x+h) = \frac{2x+2h-2}{x+h+3}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\frac{2x+2h-2}{x+h+3} - \frac{2x-2}{x+3}}{h}$$

CD: $(x+3)(x+h+3)$

$$f'(x) = \lim_{h \rightarrow 0} \frac{(x+3)(2x+2h-2) - (2x-2)(x+h+3)}{h(x+3)(x+h+3)}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\cancel{2x^2} + \cancel{2xh} - \cancel{2x} + \cancel{6x} + \cancel{6h} - \cancel{6} - (\cancel{2x^2} + \cancel{2xh} + \cancel{6x} - \cancel{2x} - \cancel{2h} - \cancel{6})}{h(x+3)(x+h+3)}$$

$$f'(x) = \lim_{h \rightarrow 0} \frac{\cancel{8h}}{\cancel{h}(x+3)(x+h+3)} = \frac{8}{(x+3)^2}$$

$$\textcircled{4} \text{ b) } y = \frac{\sqrt{x}}{3+x^2} = \frac{x^{1/2}}{3+x^2}$$

$$y' = \frac{\frac{1}{2}x^{-1/2}(3+x^2) - 2x(x^{1/2})}{(3+x^2)^2}$$

$$y' = \frac{\frac{3+x^2}{2x^{1/2}} - \frac{2x^{3/2}}{1}}{(3+x^2)^2}$$

CD: $2x^{1/2}$

$$y' = \frac{3+x^2 - 4x^2}{2x^{1/2}(3+x^2)^2} = \frac{3-3x^2}{2\sqrt{x}(3+x^2)^2}$$

Final Review:

$$\textcircled{1} \text{ a) } y = \left[\frac{2x+1}{x-1} \right]^5$$

$$y' = 5 \left[\frac{2x+1}{x-1} \right]^4 \left[\frac{2x-2 - 1(2x+1)}{(x-1)^2} \right]$$

$$y' = 5 \cdot \frac{(2x+1)^4}{(x-1)^4} \cdot \frac{-3}{(x-1)^2} = \frac{-15(2x+1)^4}{(x-1)^6}$$

② Find the equation of the tangent line

$$y = (x^2 - 3)^8 \quad \text{at } x = \underline{2}, y = 1$$

(2, 1)

① Find y

$$y = (2^2 - 3)^8$$

$$y = (4 - 3)^8 = 1$$

② Find y'

$$y' = 8(x^2 - 3)^7 (2x)$$

$$y' = 16x(x^2 - 3)^7$$

③ Find m

$$y'(2) = 16(2)(2^2 - 3)^7$$

$$y'(2) = 32(1) = 32$$

m = 32

④ Find equation

$$y - y_1 = m(x - x_1)$$

$$y - 1 = 32(x - 2)$$

$$y - 1 = 32x - 64$$

$$y = 32x - 63$$

⑥ Find the points on the curve $y = \frac{x}{x-1}$ where the tangent line is parallel to the line $x+4y=1$

$$\textcircled{1} x+4y=1$$

$$4y = -x+1$$

$$y = \left(-\frac{1}{4}\right)x + \frac{1}{4}$$

$$m = -\frac{1}{4}$$

$$\textcircled{2} y = \frac{x}{x-1}$$

$$y' = \frac{(x-1)(1) - (x)(1)}{(x-1)^2}$$

$$y' = \frac{-1}{(x-1)^2}$$

$$\textcircled{3} \frac{-1}{(x-1)^2} = -\frac{1}{4}$$

$$-(x-1)^2 = -4$$

$$(x-1)^2 = 4$$

$$x^2 - 2x + 1 = 4$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$$\begin{array}{l|l} x-3=0 & x+1=0 \\ x=3 & x=-1 \end{array}$$

$$\textcircled{4} y = \frac{x}{x-1}$$

$$y = \frac{3}{3-1}$$

$$y = \frac{3}{2}$$

$$\boxed{(3, \frac{3}{2})}$$

$$y = \frac{-1}{-1-1}$$

$$y = \frac{1}{2}$$

$$\boxed{(-1, \frac{1}{2})}$$

⑥ Find the point on the curve $y = x\sqrt{x}$ where the tangent line is parallel to the line $6x - y = 4$

$$\textcircled{1} 6x - y = 4$$

$$6x - 4 = y$$

$$y = 6x - 4$$

$$m = 6$$

$$\textcircled{2} y = x\sqrt{x} = x(x)^{1/2} = x^{3/2}$$

$$y' = \frac{3}{2}x^{1/2} = \frac{3\sqrt{x}}{2}$$

$$\textcircled{3} \frac{3\sqrt{x}}{2} = \frac{6}{1}$$

$$3\sqrt{x} = 12$$

$$\sqrt{x} = 4$$

$$x = 16$$

$$\textcircled{4} y = x\sqrt{x}$$

$$y = (16)\sqrt{16}$$

$$y = 64$$

$$(16, 64)$$

Do Ex. 2.9 on page 112 of Text
Questions 1, 4, 6, 9, 11, 12, 14

↑
a, d

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③ Find $\frac{dy}{dx}$ at $x=4$ if $y = u^2 - 2u^5$ and $u = x - \sqrt{x}$

① Find u $u = x - \sqrt{x}$ $u = (4) - \sqrt{4}$ $u = 2$	② Find $\frac{dy}{du}$ $y = u^2 - 2u^5$ $\frac{dy}{du} = 2u - 10u^4$	③ Find $\frac{du}{dx}$ $u = x - x^{1/2}$ $\frac{du}{dx} = 1 - \frac{1}{2}x^{-1/2}$
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$$\begin{aligned} \frac{dy}{dx} \Big|_{x=4} &= (2u - 10u^4) \left(1 - \frac{1}{2\sqrt{x}}\right) \\ &= (2(2) - 10(2)^4) \left(1 - \frac{1}{2\sqrt{4}}\right) \\ &= (4 - 160) \left(1 - \frac{1}{4}\right) \\ &= (-156) \left(\frac{3}{4}\right) \\ &= \frac{-468}{4} \\ &= -117 \end{aligned}$$

④ If $F(x) = f(g(x))$, where $\begin{cases} g(a) = 4 \\ g'(a) = 3 \\ f'(4) = 5 \end{cases}$ find $F'(a)$

$$\begin{aligned} F'(x) &= f'(g(x)) \cdot g'(x) \\ F'(a) &= f'(g(a)) \cdot g'(a) \\ F'(a) &= f'(4) \cdot g'(a) \\ F'(a) &= 5 \cdot 3 = \underline{\underline{15}} \end{aligned}$$