Questions from homework

4) $\sum_{n=2}^{6} \frac{3}{n-1} = \frac{3}{1} + \frac{3}{2} + \frac{3}{3} + \frac{3}{4} + \frac{3}{5}$

$= \frac{180+90+60+45+36}{60}$

$= \frac{411}{60} = \boxed{\frac{137}{20}}$

10) $3+6+12+24+48 = \boxed{\sum_{n=1}^{5} 3(2)^{n-1}}$

geometric

$a=3$

$r=2$

$t_n = 3(2)^{n-1}$

Limit (of a sequence - t_n)

A finite number *L* that the value of t_n gets closer and closer to, or "approaches," as *n* becomes very large, or "approaches infinity." The value of t_n can be made as close as you like to *L* by using a sufficiently large value for *n*.

The notation for a limit is $\boxed{\lim_{n\to\infty} t_n = L}$

Converging Sequence (Has a limit)
A seqence in which the terms approach a limit

For example, $\frac{1}{4}, \frac{2}{5}, \frac{3}{6}, \frac{4}{7}, \ldots$ converges to 1

0.25, 0.4, 0.5, 0.57

The above sequence was generated using the following general term.	$t_n = \frac{n}{n+3}$

What happens if "*n*" is a very large number?

$t_{10} = \frac{10}{13} = 0.77$

$t_{100} = \frac{100}{103} = 0.97$

$t_{1000} = \frac{1000}{1003} = 0.997$

$\lim_{n\to\infty} t_n = L$

$$\boxed{\lim_{n\to\infty} \frac{n}{n+3} = 1}$$

Diverging Sequence (Has no limit) → Limit does not exist

A seqence in which the terms do not approach a limit

For example, *1, 2, 3, 4,...* diverges. (no limit exists)

The above sequence was generated using the following general term.	$t_n = n$

What happens if "*n*" is a very large number?

$t_{10} = 10$

$t_{100} = 100$

$t_{1000} = 1000$

$\boxed{\lim_{n \to \infty} n = DNE}$ → diverging

Decide whether each sequence *converges* or *diverges* then state the limit.

2, 4, 8, 16, 32,... *diverges* $\lim_{n\to\infty} 2^n = \text{DNE}$

$t_n = 2(2)^{n-1}$

$t_n = 2^n$

3, 1.5, 0.75, 0.375,... *converges* $\lim_{n\to\infty} 3\left(\frac{1}{2}\right)^{n-1} = 0$

$t_n = 3\left(\frac{1}{2}\right)^{n-1}$

$t_{10} = 3\left(\frac{1}{2}\right)^9$

$= 3\left(\frac{1}{512}\right)$

$= \frac{3}{512}$

Infinite Sequences

Suppose we have a sequence defined by $t_n = \dfrac{n}{2n+1}, n \in N$

Generate the first 4 terms of the sequence

$\frac{1}{3}, \frac{2}{5}, \frac{3}{7}, \frac{4}{9}$

$0.\overline{3}, 0.4, 0.43, 0.\overline{4}$

You may notice that as "n" increases "t_n" approaches ½

Symbolically this is written $\lim\limits_{n\to\infty} \dfrac{1n}{2n+1} = \dfrac{1}{2}$

and is read "The limit as n approaches infinity of n over $(2n + 1)$ is ½."

Algebraically we solve by dividing the numerator and the denominator by the highest power of n.

if the degree of the numerator and denominator are the same, then your limit will be the quotient of the leading coefficients.

$$\lim_{n\to\infty} \frac{\underline{1}n^2-2}{4+\underline{3}n^2} = \frac{1}{3}$$

if the degree of the numerator is larger than the degree of the denominator, then your limit will not exist.

$$\lim_{n\to\infty} \frac{n^2+2n}{n-3} = \text{DNE}$$

if the degree of the denominator is larger than the degree of the numerator, then your limit will always equal 0.

$$\lim_{n\to\infty} \frac{1}{3n^5-2} = 0$$

Find the limit if it exists

$$t_n = n + 5 = \frac{n+5}{1} = \frac{n^1+5}{n^0}$$

$$\lim_{n\to\infty} n+5 = DNE$$

$$t_n = \frac{3n^1+1}{4n^1-2}$$

$$\lim_{n\to\infty} \frac{3n+1}{4n-2} = \frac{3}{4}$$

Homework

#1 b)
#2
#3
#4