# Warm-Up...

Given that (-2 , 5) is a point on the graph of $y = f(x)$, determine the coordinates of this point once the following transformations are applied...

(1) $y = 3f(x)$

$a = 3 \rightarrow$ vertical stretch by a factor of 3

$(x,y) \rightarrow (x, 3y)$

$(-2,5) \rightarrow \boxed{(-2,15)}$

(2) $y = f\left(\left(-\frac{1}{3}\right)x\right)$

$b = -\frac{1}{3} \rightarrow$ horizontal stretch by a factor of 3 & a horizontal reflection in the y-axis

$(x,y) \rightarrow (-3x, y)$

$(-2,5) \quad \boxed{(6,5)}$

(3) $y = 4f\left[\left(\frac{1}{2}\right)(x+5)\right] - 3$

$a = 4$ vertical stretch by a factor of 4

$b = \frac{1}{2}$ horizontal stretch by a factor of 2

$h = -5$ translated 5 units left

$k = -3$ translated 3 units down

$(x,y) \rightarrow (2x-5, 4y-3)$

$(-2,5) \rightarrow \boxed{(-9,17)}$

(4) $y - 5 = -2f(-2x+6)$ factor

$y = -2f(-2x+6) + 5$

$y = -2f[-2(x-3)] + 5$

$a = -2$ vertical stretch by a factor of 2 and a vertical reflection in the x-axis

$b = -2$ horizontal stretch by a factor of $\frac{1}{2}$ and a horizontal reflection in the y-axis

$h = 3$ translated 3 units right

$k = 5$ translated 5 units up

$(x,y) \rightarrow \left(-\frac{1}{2}x + 3, -2y + 5\right)$

$(-2,5) \rightarrow \boxed{(4,-5)}$

# Transformations:

2. The function $y = f(x)$ is transformed to the function $g(x) = -3f(4x - 16) - 10$. Copy and complete the following statements by filling in the blanks.

The function $f(x)$ is transformed to the function $g(x)$ by a horizontal stretch about the a by a factor of b. It is vertically stretched about the c by a factor of d. It is reflected in the e, and then translated f units to the right and g units down.

factor

$$g(x) = -3f(4x-16)-10$$

$$g(x) = -3f[4(x-4)]-10$$

$a=-3$ $b=4$ $h=4$ $k=-10$

a) y-axis

b) 1/4

c) x-axis

d) 3

e) x-axis

f) 4

g) 10

# Summary of Transformations...

| Transformations of the graphs of functions | | | |
|---|---|---|---|
| $f(x)+k$ | shift $f(x)$ up k units | vertical trans. | $(x,y)\to(x,y+k)$ |
| $f(x)-k$ | shift $f(x)$ down k units | " " | $(x,y)\to(x,y-k)$ |
| $f(x+h)$ | shift $f(x)$ left h units | horizontal trans | $(x,y)\to(x-h,y)$ |
| $f(x-h)$ | shift $f(x)$ right h units | " " | $(x,y)\to(x+h,y)$ |
| $f(-x)$ | reflect $f(x)$ about the y-axis | horizontal ref. | $(x,y)\to(-x,y)$ |
| $-f(x)$ | reflect $f(x)$ about the x-axis | vertical ref. | $(x,y)\to(x,-y)$ |
| $af(x)$ | When $0<a<1$ – vertical shrinking of $f(x)$<br>When $a>1$ – vertical stretching of $f(x)$<br>**Multiply the y values by a** | | $(x,y)\to(x,ay)$ |
| $f(bx)$ | When $0<b<1$ – horizontal stretching of $f(x)$<br>When $b>1$ – horizontal shrinking of $f(x)$<br>**Divide the x values by b** | | $(x,y)\to(\frac{1}{b}x,y)$ |

# Transformations:

$$y = f(x) \longrightarrow y = \underline{a}f\left(\underline{b}(x-\underline{h})\right)+\underline{k}$$

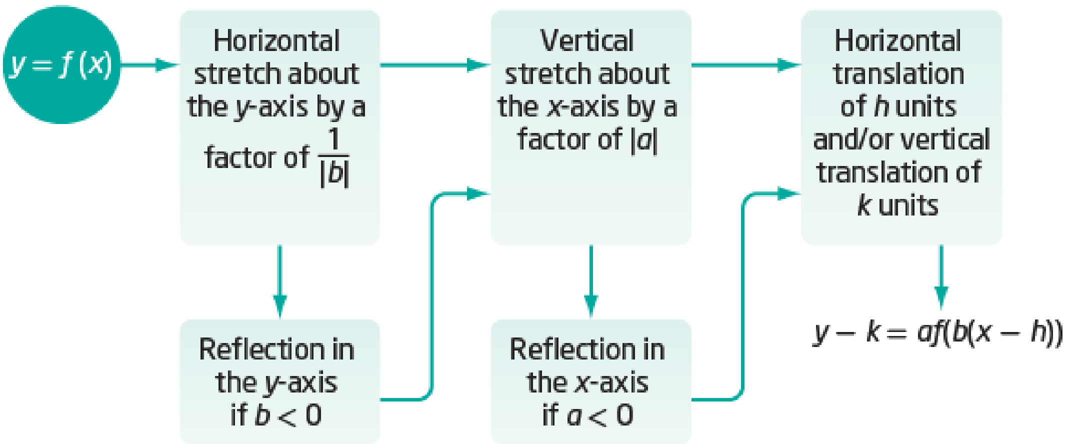
**Mapping Rule:** $$(x,y) \rightarrow \left(\frac{1}{b}x+h, ay+k\right)$$

Important note for sketching...

Transformations should be applied in following order:

1. Reflections
2. Stretches
3. Translations

Remember....RST



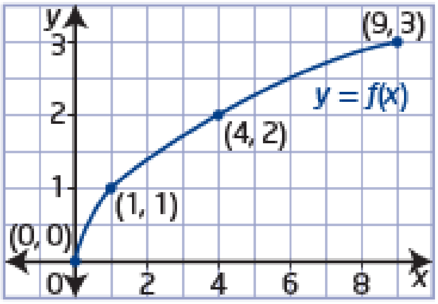
## Example 1

### Graph a Transformed Function

Describe the combination of transformations that must be applied to the function $y = f(x)$ to obtain the transformed function. Sketch the graph, showing each step of the transformation.

**a)** $y = 3f(2x)$

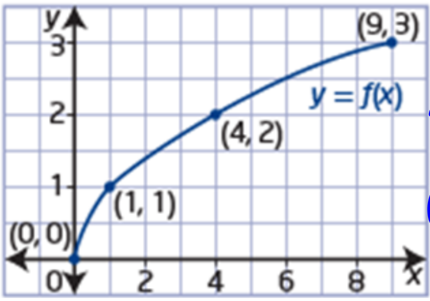
**b)** $y = f(3x + 6)$



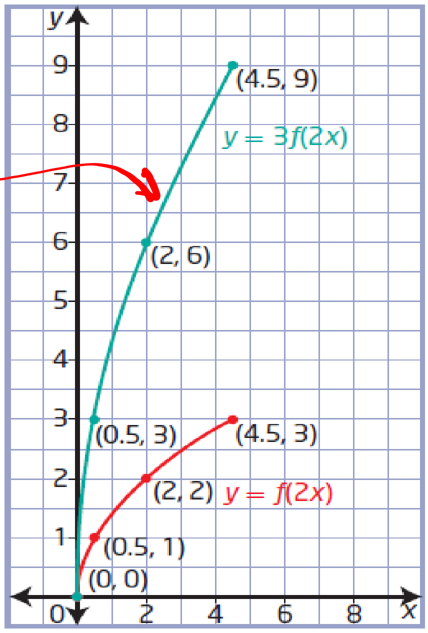
**a)** $y = 3f(2x)$ $a = 3$ $b = 2$ $h = 0$ $k = 0$

The graph of $y = f(x)$ is horizontally stretched about the $y$-axis by a factor of $\frac{1}{2}$ and then vertically stretched about the $x$-axis by a factor of 3.

$$(x, y) \to \left(\frac{1}{2}x + 0, 3y + 0\right)$$




| | |
|---|---|
| (0,0) | (0,0) |
| (1,1) | (1/2,3) |
| (4,2) | (2,6) |
| (9,3) | (9/2,9) |



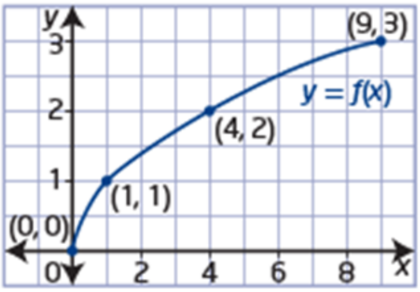
factor

**b)** $y = f(3x + 6)$ $a=1$ $b=3$ $h=-2$ $k=0$

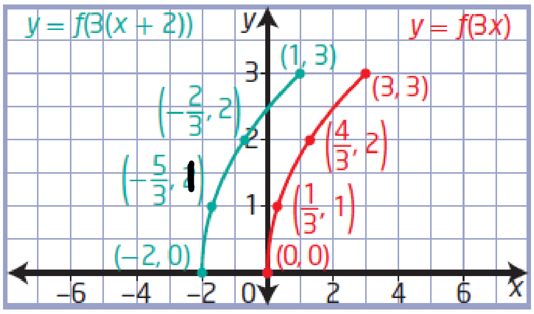
$y = 1f[3(x+2)]+0$

The graph of $y = f(x)$ is horizontally stretched about the $y$-axis by a factor of $\frac{1}{3}$ and then horizontally translated 2 units to the left.

$(x, y) \to (\frac{1}{3}x - 2, y)$




| | |
|---|---|
| $(0,0)$ | $(-2, 0)$ |
| $(1,1)$ | $(-\frac{5}{3}, 1)$ |
| $(4,2)$ | $(-\frac{2}{3}, 2)$ |
| $(9,3)$ | $(1, 3)$ |




$\frac{1}{3}\left(\frac{1}{1}\right) - \frac{2}{1}$

$\frac{1}{3} - \frac{6}{3}$

$\frac{-5}{3}$

$\frac{1}{3}\left(\frac{4}{1}\right) - 2$

$\frac{4}{3} - \frac{6}{3}$

$\frac{-2}{3}$

$\frac{1}{3}\left(\frac{9}{1}\right) - 2$

$\frac{9}{3} - 2$

$3 - 2$

$1$

# Questions From Homework

**3.** Copy and complete the table by describing the transformations of the given functions, compared to the function $y = f(x)$.

| | Function | Reflections | Vertical Stretch Factor | Horizontal Stretch Factor | Vertical Translation | Horizontal Translation |
|---|---|---|---|---|---|---|
| (i) | $y - 4 = f(x - 5)$ | – | – | – | 4 | 5 |
| (ii) | $y + 5 = 2f(3x)$ | – | 2 | $\frac{1}{3}$ | -5 | – |
| (iii) | $y = \frac{1}{2}f\left(\frac{1}{2}(x - 4)\right)$ | – | $\frac{1}{2}$ | 2 | – | 4 |
| (iv) | $y + 2 = -3f(2(x + 2))$ | ↑ | 3 | $\frac{1}{2}$ | -2 | -2 |

vertical reflection in x-axis

(i) $y = f(x-5) + 4$

$a=1$ $b=1$ $h=5$ $k=4$

(ii) $y = 2f(3x) - 5$

$a=2$ $b=3$ $h=0$ $k=-5$

(iii) $y = \frac{1}{2}f\left(\frac{1}{2}(x-4)\right)$

$a=\frac{1}{2}$ $b=\frac{1}{2}$ $h=4$ $k=0$

(iv) $y = -3f(2(x+2)) - 2$

$a=-3$ $b=2$ $h=-2$ $k=-2$

**6.** The key point $(-12, 18)$ is on the graph of $y = f(x)$. What is its image point under each transformation of the graph of $f(x)$?

e) $y+3 = -\frac{1}{3}f[2(x+6)]$

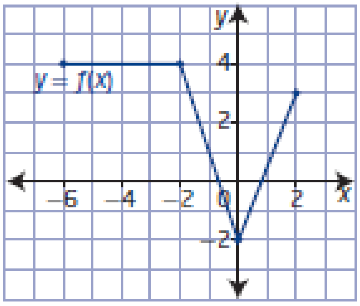
$y = -\frac{1}{3}f[2(x+6)] - 3$

$a = -\frac{1}{3}$ $b = 2$ $h = -6$ $k = -3$

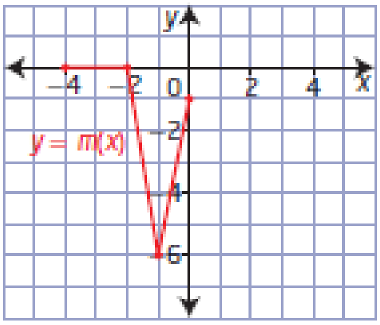
$(x, y) \rightarrow \left(\frac{1}{2}x - 6, -\frac{1}{3}y - 3\right)$

$(-12, 18) \rightarrow \boxed{(-12, -9)}$

**4.** Using the graph of $y = f(x)$, write the equation of each transformed graph in the form $y = af(b(x - h)) + k$.




b)




| $f(x)$ | $m(x)$ |
|---|---|
| $(-6,4)$ | $(-4,0)$ |
| $(-2,4)$ | $(-2,0)$ |
| $(0,-2)$ | $(-1,-6)$ |
| $(2,3)$ | $(0,-1)$ |

$$(x,y) \longrightarrow \left(\tfrac{1}{2}x - 1,\ y - 4\right)$$

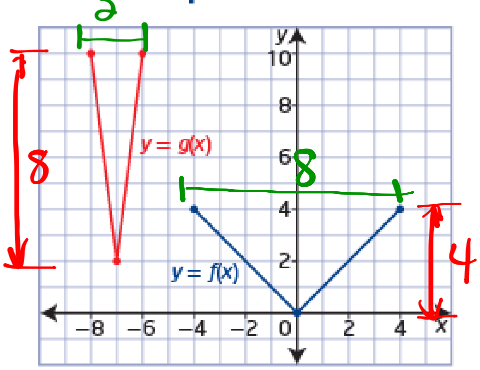
$a = 1$ $b = 2$ $h = -1$ $k = -4$

$$m(x) = 1f(2(x+1)) - 4$$

## Example 3

### Write the Equation of a Transformed Function Graph

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$. Explain your answer.




### Solution

Locate key points on the graph of $f(x)$ and their image points on the graph of $g(x)$.

$(-4, 4) \rightarrow (-8, 10)$
$(0, 0) \rightarrow (-7, 2)$
$(4, 4) \rightarrow (-6, 10)$

The equation of the transformed function is $g(x) = 2f(4(x + 7)) + 2$.

① Reflections: None

② Vertical stretch factor $= \frac{8}{4} = 2$ $\quad a = 2$
(Compare Range $\frac{\text{New}}{\text{old}}$)

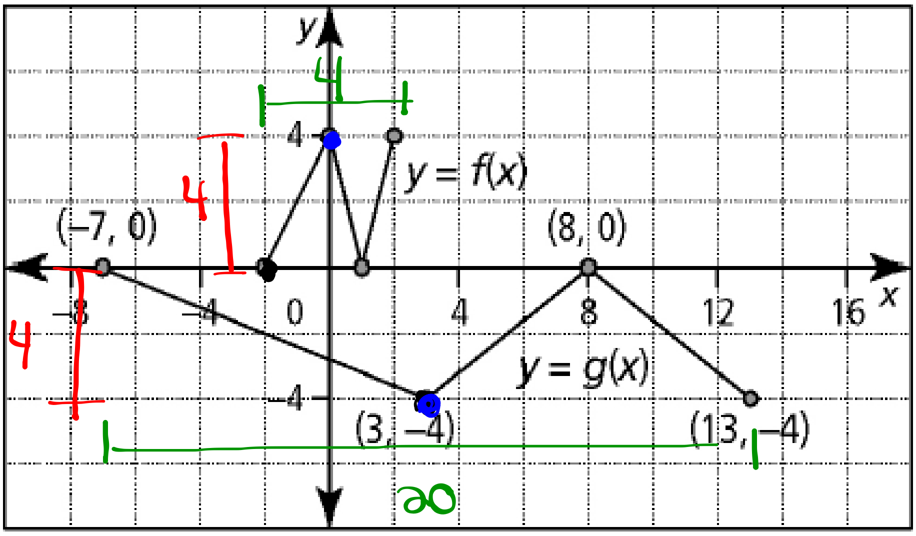
③ horizontal stretch factor $= \frac{2}{8} = \frac{1}{4}$ $\quad b = 4$
(Compare domain $\frac{\text{New}}{\text{old}}$)

④ horizontal translation: $(0,0) \rightarrow (-7,2)$ left 7 units
(Pick a point on $f(x)$ with an x-value of 0) $h = -7$

⑤ vertical translation: $(0,0) \rightarrow (-7,2)$ up 2 units
(Pick a point on $f(x)$ with an y-value of 0) $k = 2$

⑥ Equation: $g(x) = 2f(4(x+7)) + 2$

The graph of the function $y = g(x)$ represents a transformation of the graph of $y = f(x)$. Determine the equation of $g(x)$ in the form $y = af(b(x - h)) + k$.




① Reflection: vertical in x-axis $(a<0)$

② vertical stretch factor $= \frac{4}{4} = 1$ $\boxed{a=-1}$

③ horizontal stretch factor $= \frac{20}{4} = 5$ $\boxed{b=\frac{1}{5}}$

④ horizontal translation: $(0,4) \to (3,-4)$ right 3 $\boxed{h=3}$

⑤ vertical translation: $(-2,0) \to (-7,0)$ $\boxed{k=0}$

⑥ $g(x) = -1f\left(\frac{1}{5}(x-3)\right) + 0$

$$\boxed{g(x) = -f\left(\frac{1}{5}(x-3)\right)}$$

$$y = -f\left(\frac{1}{5}(x-3)\right)$$

# Homework

Page 38 # 3-6
Plus 7, 8, 9 (a, c, e) and 10

① f) $3y - 6 = f(-2x+12)$

Factor

$3y = f(-2x+12) + 6$

$\frac{3y}{3} = \frac{1}{3}f[-2(x-6)] + \frac{6}{3}$ (Only divide/multiply the a & K)

$y = \frac{1}{3}f[-2(x-6)] + 2$ $\quad (x,y) \rightarrow \left[\frac{1}{-2}x+6, \frac{1}{3}y+2\right]$

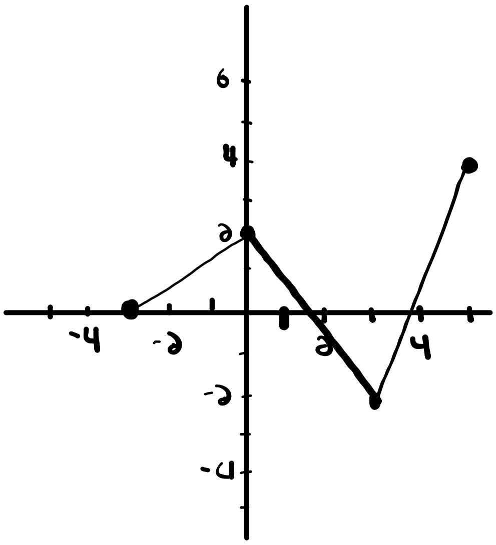
$a = \frac{1}{3}$ $\quad b = -2$ $\quad h = 6$ $\quad K = 2$

$a = \frac{1}{3} \rightarrow$ A vertical compression about the x-axis by a factor of $\frac{1}{3}$

$b = -2 \rightarrow$ A horizontal compression about the y-axis by a factor of $\frac{1}{2}$ and a horizontal reflection in the y-axis.

$h = 6 \rightarrow$ translated 6 units right

$K = 2 \rightarrow$ translated 2 units up.




Domain: $\{x \mid -3 \leq x \leq 5, x \in \mathbb{R}\}$

or $[-3, 5]$

Range: $\{y \mid -2 \leq y \leq 4, y \in \mathbb{R}\}$

or $[-2, 4]$

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6 2)

$y = -2f(-\frac{2}{3}x - 6) + 4$

$y = -2f(-\frac{2}{3}(x+9)) + 4$

$-6 \div -\frac{2}{3}$

$= -6 \times -\frac{3}{2}$

$= \frac{18}{2}$

$= 9$

$a = -2$ $b = -\frac{2}{3}$ $h = -9$ $k = 4$

$(x, y) \rightarrow \left[-\frac{3}{2}x - 9, -2y + 4\right]$